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Seismology of the Sun

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The sun is an oscillating star. The oscillations are imperceptible to the naked eye, but they have been revealed by Doppler shifts in spectral lines that are formed in the solar atmosphere. The pattern of motion is complicated and for a long time was not understood, but now we know that it is a result of the interference between about 10^7 resonant modes of vibration, many of which are coherent over the entire surface of the sun. The modes have wavelengths greater than a few thousand kilometers and periods from a few minutes to several hours. The frequencies carry information about the structure and dynamics of the region where the modes have appreciable amplitudes, which in many cases spans much of the solar interior. The unravelling of this information, which is analogous to seismological studies of Earth, is rapidly evolving into a new branch of solar physics known as helioseismology.

Summary. Oscillations of the sun make it possible to probe the inside of a star. The frequencies of the oscillations have already provided measures of the sound speed and the rate of rotation throughout much of the solar interior. These quantities are important for understanding the dynamics of the magnetic cycle and have a bearing on testing general relativity by planetary precession. The oscillation frequencies yield a helium abundance that is consistent with cosmology, but they reinforce the severity of the neutrino problem. They should soon provide an important standard by which to calibrate the theory of stellar evolution.

Solar physicists grapple with a complex dynamical problem. The fluid in the outer layers of the sun is highly turbulent and carries magnetic fields, which it distorts into intricate configurations. In some places the field is concentrated into small regions that darken into sunspots; elsewhere the field erupts, causing a temporary local brightening or flare. Charged particles are ejected that subsequently disturb Earth's ionosphere, causing magnetic storms and inhibiting radio communication in particularly severe instances. The rotation of the sun is not uniform, and this feature is commonly believed to be an essential ingredient of a dynamo that maintains and modulates the magnetic activity and is responsible for the 11-year cycle of sunspots. Such magnetic activity may influence the terrestrial climate. Solar variability with longer characteristic time scales is also suspected. Yet despite this complexity of behavior, the sun is believed to be among the simplest of stars.

One of the major assumptions of stellar physics is that these complicated phenomena are not relevant to our understanding of the large-scale structure of the solar interior. Therefore relatively simple theoretical models suffice to describe the structure and evolution of a star such as the sun. In these models, the sun is treated as a perfect sphere powered by thermonuclear reactions that gradually convert hydrogen into helium in the high-temperature core. The energy is liberated in the inner 30 percent (by radius) and is transported outward by radiative diffusion. In the outer layers the energy is carried predominantly by turbulent convection, because the stellar material is too opaque to permit the radiation to pass. The thickness of the convecting region is poorly determined by theory, for it depends on the uncertain ratio of the abundances of hydrogen and helium at the time the sun was formed; values of the thickness ranging from less than 20 percent to about 30 percent of the radius of the sun have been commonly believed. But this uncertainty is not all: theoretical predictions of the rate of production of neutrinos by the nuclear reactions in the core are three times greater than the measured value. Is this a minor error, or is it a symptom of a fundamental flaw in the theory of stellar evolution? Helioseismological studies may play a major role in answering that question.

In the past few years we have started to consolidate our picture of the inside of the sun. Initially, seismological inferences were used to calibrate theoretical solar models. Thus it became possible to estimate the solar helium abundance, a quantity of considerable cosmological interest. It also led to an estimate of the depth of the base of the convection zone. This is important for our understanding of the dynamics of the outer envelope of the sun and is pertinent to theories of both the solar cycle and the long-term variability. More recently, seismological deductions have been made by means of so-called inverse methods, which are free from the uncertain details of the theory of stellar structure. In particular, it has been possible to measure the angular velocity throughout much of the solar interior and so to evaluate the oblateness of the sun's gravitational field. A knowledge of the latter is required for testing theories of gravity from observations of planetary orbits. It has also been possible to infer the sound speed throughout much of the sun.

It is the purpose of this article to describe the seismological inferences

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that have already been made and to provide a basis for anticipating what is likely to be learned in the foreseeable future. The knowledge is important not merely for establishing a more accurate picture of the inside of the sun but to provide a standard with which to compare our theoretical models of other stars.

First Helioseismological Inferences

Solar oscillations were discovered in 1960 by Leighton, Noyes, and Simon (*1*), who found that typically about half the surface of the sun is occupied by patches that oscillate intermittently with periods near 5 minutes and with amplitudes of about 1 km sec^{-1}. The oscillations persist for six or seven periods with a spatial coherence of about 30,000 km, or 2 percent of the diameter of the sun; consequently they were for a long time regarded as a local phenomenon, triggered possibly by eruptions in the convection zone. Some 10 years later Ulrich, and independently Leibacher and Stein, proposed that these oscillations are a superposition of coherent acoustic modes (*2*). What excites the modes is an issue of unresolved debate (*3*).

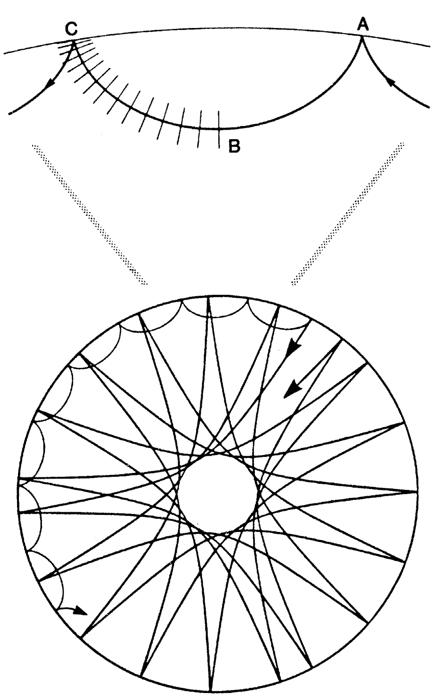
The major restoring forces responsible for solar oscillations are pressure and buoyancy. Which is the more important force depends largely on frequency and to a lesser extent on wavelength. Pressure fluctuations dominate at high frequency, producing acoustic waves; at low frequency buoyancy dominates, producing internal gravity waves. Waves propagating through the sun may undergo reflection where appropriate conditions are satisfied and can thus be confined within a cavity. A resonant mode of oscillation is the result of constructive interference between such internally reflected waves. Thus it is analogous to a standing wave in, say, an organ pipe. And, as in an organ, resonance can occur only at particular frequencies, which depend on conditions in the cavity. But in an organ, as in almost all musical instruments, the oscillations take place in only one dimension; the sun oscillates in three dimensions, and this clearly permits a far richer spectrum of tones.

Standing acoustic waves are known as *p* modes. In the sun, they have periods between about 3 minutes and 1 hour. It is these modes that have been the most widely observed and have provided nearly all the useful diagnostic information. Consequently they are the principal subject of our discussion. Standing internal gravity waves are called *g* modes. Their periods exceed about 40 minutes; they depend on buoyancy, whose magnitude is determined by the stratification of density and pressure and is sensitive to the distribution of chemical elements within the sun.

Two typical ray paths of acoustic waves are shown in Fig. 1. They lie in a plane that passes through the center of the sun (*4*). The surface layers of the sun, where the density of the solar material varies rapidly with depth, act as a reflecting boundary. Downward propagating waves are refracted back upward, unless the motion is precisely vertical. This comes about because temperature, and consequently sound speed, increase with depth, causing deeper parts of the wave fronts to travel more quickly (*5*). Thus the waves are confined within a cavity that excludes a central region of the sun. The more nearly vertical a wave is as it is reflected at the surface, the greater is the depth of the acoustic cavity. On the whole, if the wavelength λ of the disturbance on the surface of the sun is shorter, the waves travel more nearly horizontally, and the cavity within which they are confined is shallower.

For each value of λ there is a fundamental acoustic frequency and a sequence of overtones. They are identified by an integer n, which increases with increasing frequency. Theoretical calculations by Ulrich (*2*), and subsequently by Ando and Osaki (*6*), showed how the frequencies ν were expected to depend on n and λ.

The theory was confirmed observationally by Deubner (*7*), who in 1975 made continuous measurements lasting many hours of Doppler velocity in an equatorial strip on the solar disk (*8*). Fourier transforms in longitude and time resulted in the determination of ν as a function of the horizontal wave number $2\pi\lambda^{-1}$. The latter is related to an integer l, called the degree of the mode, which measures the number of wavelengths that make up the circumference of the sun (*9*). A power spectrum of subsequent, more accurate observations is displayed in Fig. 2.



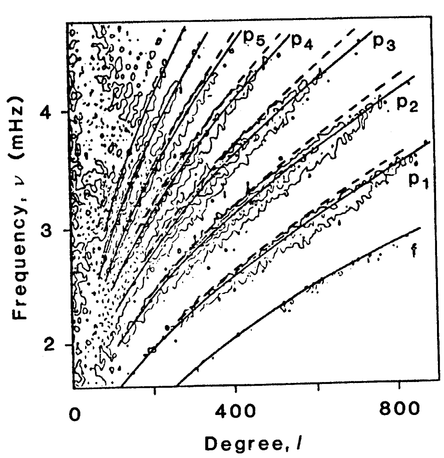


Fig. 1 (left). Ray paths of acoustic waves in the sun. The more deeply penetrating rays refer to waves with $n/l = 5$; the waves confined to the shallower cavity have $n/l = 1/20$. The circle represents the surface of the sun. In the enlargement, the lines intersecting the ray path represent wave fronts at equal intervals of phase; they are most closely spaced near the surface where the sound speed is least. At the level B the wave propagates horizontally and travels at the same angular phase speed $\lambda\nu$ about the center of the sun, as does the disturbance on the surface. Fig. 2 (right). Spectrum of Doppler data (*43*) showing contours of constant $l^{1/2}$ times power. Superposed on the ridges are the eigenfrequencies of two theoretical solar models. The continuous lines are for a model of the present sun whose helium abundance Y was 0.25 at the time the sun was formed; the dashed lines are for $Y = 0.19$. The theoretical curves are labeled p_n for p modes of order n and f for the f modes. The theoretical values of the latter for the two solar models are indistinguishable at the high values of l for which they are plotted. The continuous lines pass through the regions of maximum power, suggesting that $Y \simeq 0.25$. [After (*44*)]

The frequencies depend also on the sound speed in the acoustic cavity. This is illustrated in Fig. 2, where the eigenfrequencies of two theoretical models of the sun are superposed on the power spectrum of the observations. Evidently the data select one of the models and suggest that the initial helium abundance Y of the sun is about 25 percent (by mass).

It is clear from Fig. 2 that only the frequencies of modes with quite large values of l can be discerned. These modes have small wavelengths λ and are therefore confined to only a shallow cavity; the depth to which the p modes

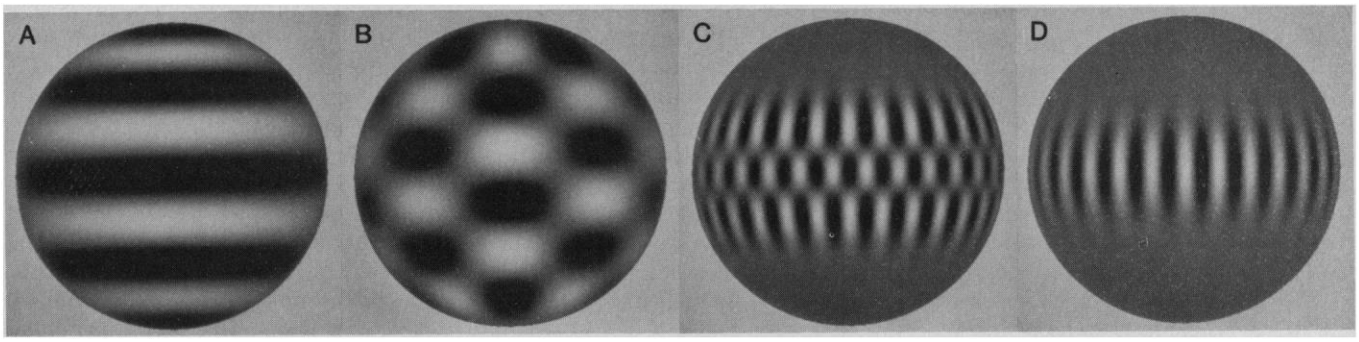


Fig. 3. Doppler velocities of solar p modes. The shading represents the line-of-sight component of velocity: dark regions are approaching the observer and light regions are receding (or vice versa). The motion is almost radial, so that the mid-grey at the edges of the sun's image represents zero velocity. (A) The zonal mode ($m = 0$) of degree $l = 10$; (B and C) tesseral modes with $(l, m) = (20, 10)$ and $(32, 30)$; (D) a sectoral mode ($m = l$) of degree 30.

penetrate is about $(2n + 3)/l$ of the radius of the sun. Therefore the determination of Y is indirect, resting heavily on the theory of stellar evolution from which the models of the sun were computed. What is measured directly is the upper part of the convection zone, and from that it has been possible to estimate that the thickness of the convection zone is 30 percent of the solar radius (*10, 11*). This was the first diagnosis from helioseismology.

Interest in the solar helium abundance had been stimulated partly by the neutrino problem. Only if Y were very low could solar models be constructed with neutrino fluxes that agreed with observation, unless the assumptions of stellar evolution theory were modified. But such low values are in conflict with many theories of cosmology, which predict Y to be in excess of 20 percent immediately after the Big Bang and to increase slowly afterward. The helioseismological calibration illustrated in Fig. 2, and a calibration from low-degree modes that will be discussed later, are consistent with cosmology. But they reinforce the severity of the neutrino problem and the doubt that is cast on the premises of stellar evolution theory (*12*).

Classification of Solar Oscillations

Because the amplitudes of solar oscillations are small, they can safely be treated as small perturbations about a spherical equilibrium state. This permits the separation of the oscillations into normal modes, which simplifies the theoretical analysis. In spherical polar coordinates (r, θ, ϕ), the radial component of the velocity of a mode with cyclic frequency ν can be expressed as

$$V_{nl}(r)\, P_l^m(\cos\theta)\, \cos(m\phi - 2\pi\nu t) \qquad (1)$$

where t is time and P_l^m is the associated Legendre function of degree l and order m. The coordinate r is the distance from the center of the sun, θ is colatitude, and ϕ is longitude.

Figure 3 shows how several individual modes might appear to an observer. At any given instant of time, the pattern of Doppler velocities of an individual mode is one of alternating regions of approaching (shown as light tones) and receding (dark tones) flow. For zonal modes ($m = 0$), all the nodal lines of the spherical harmonics are lines of latitude; for sectoral modes ($m = l$) they are lines of longitude. More common are the tesseral modes such as those in Fig. 3, B and C. Of course in reality the motion of the sun is a superposition of many such modes, and the observer's signal is much more complicated.

The functions V_{nl} are the eigenfunctions of a system of linear ordinary differential equations, the associated eigenvalues ν_{nl} being the frequencies ν of the modes. There are two discrete spectra of modes for given values of l and m: the p modes and the g modes (when $l = 0$ only the p sequence exists). The modes in each spectrum can be arranged in ascending order of frequency and period, respectively, and labeled with the integer n, which is the order of the mode. Typically the eigenfunctions $V_{nl}(r)$ possess n zeros.

In addition there is a mode whose frequency lies between those of the p modes and the g modes; it is called the f mode and exists only for $l \geq 2$. Except when l is low, it has no nodes. It is essentially a surface gravity wave, and when l is large its frequency is independent of the stratification of the equilibrium state of the sun. It can therefore be easily identified in a power spectrum such as that depicted in Fig. 2. Were the sun to be perfectly spherical, ν_{nl} and V_{nl} would be independent of m. This comes about because m depends on the axis of coordinates, and with spherical symmetry all choices must be equivalent.

The computation of theoretical eigenfrequencies must be performed numerically. For a given solar model, frequencies can now be calculated without undue computational effort to a precision as high as the observational accuracy. Results from a typical solar model are illustrated in Fig. 4; frequencies with like values of n are connected with straight lines. In all cases ν increases with l at fixed n; for the f and p modes ν varies roughly as a positive power of l, but for the g modes ν approaches a finite limit. That limit is the maximum value of the buoyancy frequency (*13*) beneath the convection zone, which in the model illustrated here is 0.47 mHz. This general behavior continues to higher values of l, as can be seen for the f and p modes in Fig. 2.

Acoustic Modes of Low Degree

Although quantitative assessment of theoretical solar models requires the numerical solution of the full eigenvalue problem, it is helpful also to carry out simpler approximate analyses. The physical understanding that is derived provides a valuable guide to designing useful comparisons between theory and observation. Moreover, it also enables us to appreciate the physical significance of any discrepancies that are found. Thus, after Deubner announced that the solar frequencies were lower than theoretical predictions, it was from a simple analysis of the oscillations of a polytrope that the principal implication was first recognized: the convection zones in the theoretical models were too shallow (*10*). The repercussions concerned not only the large-scale structure of the sun but also the dynamics of the convection zone. Of course more accurate numerical computations were required to estimate by how much the depth of the convection zone needed to be augmented (*11*).

After obtaining the high-degree oscillation spectrum, the next step forward came in 1979 when Isaak and colleagues detected a uniformly spaced sequence of frequencies in the range of 2 to 4 mHz in Doppler shifts in light integrated from the entire disk of the sun (*14*). The observations employed resonance scattering from gaseous sodium or potassium, which has high spectral stability and a low noise level. Since there was no spatial resolution, a direct determination of l and m was not possible. Nevertheless, it could be calculated that only modes with $l \leq 3$ were visible (*15*), which implies that n is quite large (Fig. 4). For such modes it was known that (*16*)

$$\nu_{nl} = \left[n + \frac{1}{2}\left(l + \frac{1}{2}\right) + \alpha\right]\nu_0 + \epsilon(n,l) \quad (2)$$

where

$$\nu_0 = \left(2\int_0^R \frac{dr}{c}\right)^{-1} \quad (3)$$

and α is a constant of order unity, c is the speed of sound, R is the radius of the sun, and ϵ is a small correction term. The functional form of ϵ had not yet been derived, but it was known from numerical computations to be too small to have been resolved by the observations. Thus it was deduced that modes of degrees 0 and 2 and modes of degrees 1 and 3 contributed alternately to the peaks in the power spectrum of the data (*15*). The spacing between the peaks must therefore be $\frac{1}{2}\nu_0$, which measures the sound travel time from the center to the surface of the sun.

Not long after, Fossat and colleagues (*17*) improved the temporal resolution by making similar observations from the South Pole. They obtained an uninterrupted record lasting 120 hours; it was possible to distinguish between different modes with like values of $n + \frac{1}{2}l$ and so to measure ϵ. Fortuitously at about the same time, Tassoul (*18*) presented an asymptotic analysis that gave not only the functional dependence of ϵ on n and l but also implied that ϵ is a measure of conditions close to the center of the sun. Her results predicted that the separation Δ between visible modes of degrees 0 and 2 with like $n + \frac{1}{2}l$ is three-fifths of the separation of the corresponding modes of degrees 1 and 3, whatever the structure of the sun. The 3:5 ratio was evident in the South Pole data (*19*) and, together with a comparison between power in the spectrum and the relative sensitivity of the measurements to modes of different l, provided the determination of the degrees of the modes (*20*). Subsequently, even more highly resolved data were obtained by Isaak and colleagues (*21*), who combined observations from Hawaii and Tenerife over a period of 3 months. Figure 5 is a power spectrum of their data.

The low-degree modes are commonly considered to provide a more robust means of calibrating solar models than the high-degree modes, for they depend on integrated properties of the entire star. Initially there was some ambiguity in fitting the data, for although l had been identified, a direct determination of n had not been possible without the observation of intermediate-degree modes. Moreover there was a systematic discrepancy between theory and observation (*20*). Therefore a determination of the initial solar helium abundance Y by fitting theory to observation, for example, must be considered somewhat uncertain. We judge from the comparisons, however, that $Y = 0.25 \pm 0.02$, which agrees with the high-degree calibration illustrated in Fig. 2.

Another conclusion that can be drawn from the low-degree data concerns the structure of the core. As a possible way out of the neutrino problem, some have advocated that the processed material in the core has been mixed with its surroundings (*22*). This would have increased the abundance of unburned hydrogen in the core and raised the sound speed. However, the standard unmixed solar model with $Y = 0.25$ reproduces the observed properties of ϵ, whereas models with a lesser variation of chemical composition do not. In particular, a chemically homogeneous solar model predicts a separation Δ that is 50 percent greater than the value observed. Thus a substantial degree of homogenization appears to be ruled out.

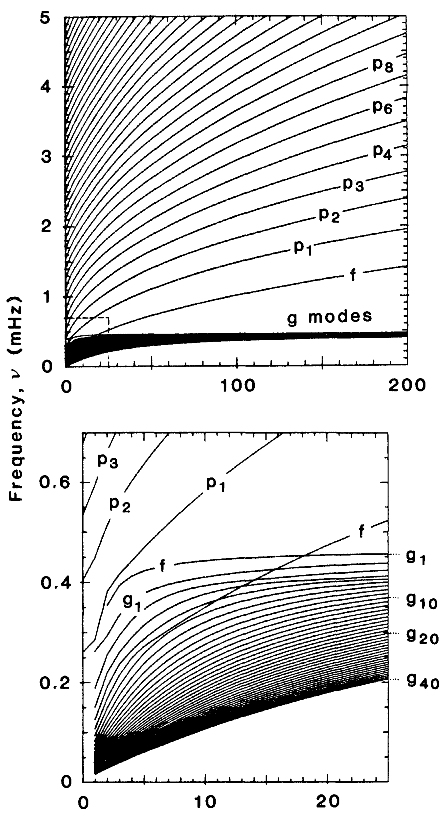


Fig. 4. Eigenfrequencies of solar model 1 (*45*) plotted against degree l; modes of like order n are connected by straight lines. The p and g modes are labeled p_n and g_n. The lower panel is an enlargement of the bottom left-hand corner of the upper panel; the connecting lines never cross, although the scale of the diagram is too small to resolve that in some cases. Plotting of g modes was stopped arbitrarily at $n = 40$; formally the order of the g modes is unbounded, and the eigenfrequencies accumulate at $\nu = 0$.

Acoustic Modes of Intermediate Degree

Much of the uncertainty in the calibration of solar models was removed by the observation of modes of intermediate degree. By averaging light in the east-west direction and appropriately projecting Doppler velocities onto Legendre functions, Duvall and Harvey (*23*) were able to identify zonal models such as that illustrated in Fig. 3A. The resulting power spectrum (Fig. 6) is markedly similar to the theoretical eigenfrequencies illustrated in Fig. 4A. At higher l it overlaps Fig. 2, and the order of each ridge can therefore be determined. After careful analysis it was possible to identify modes with degree as low as unity and to confirm the identifications that had previously been made by model-fitting to the low-degree data.

The data can be interpreted by considering the properties of the rays illustrated in Fig. 1. A wave can be regarded as being locally plane, with wave number **k** whose horizontal component is $k_h = L/r$, where $L = [l(l + 1)]^{1/2}$. The magnitude k of the wave number is related to ν and c by the simple acoustic dispersion relation $2\pi\nu = kc$. Since c increases with depth, k must decrease, until the radius r_t is reached where $k = k_h$. This is the radius of a caustic surface, which bounds a sphere into which the waves cannot propagate (Fig. 1). It is given by

$$\frac{c(r_t)}{r_t} = \frac{2\pi\nu}{rk_h} = \frac{2\pi\nu}{L} \quad (4)$$

Thus the depth of penetration depends on the angular phase speed $2\pi\nu/L$ of the disturbance. At a given frequency, waves of smaller degree penetrate more deeply, because c/r is a decreasing function of r. Spherically symmetric waves, with $l = 0$, are not refracted at all according to this simple description; they propagate vertically, passing right through the center of the sun. An oscillation mode results from the interference between many such traveling waves, giving the structure exhibited in Eq. 1.

If Fig. 4A were superposed on the observations of Fig. 6, the two would be barely distinguishable. Therefore in Fig. 7 we display the differences between the observed and calculated frequencies. Although they are small, they are far from random; furthermore they are substantially larger than the errors in the observations. Thus we must conclude that there are significant errors in this theoretical model. To discover what they are it is expedient to examine the eigenfunctions.

The vertical component of velocity in several oscillations is illustrated in Fig. 8. Two sets of modes have been chosen, each with roughly the same frequency but with different degrees. For the p modes the variation in the penetration depth is in accordance with Eq. 4. Just below the surface, the structures of p modes of different degree are quite similar. Here propagation is predominantly vertical, and the dynamics is insensitive to degree. Thus modes of the same frequency sample the outer layers of the sun in much the same way; only near the caustic surface, where the waves propagate nearly horizontally, do the eigenfunctions sense the degree.

With these properties in mind we notice three salient features of the discrepancies plotted in Fig. 7. The first is that there is a significant discrepancy at the largest values of l; from this we infer that the very surface layers, where these modes are confined, cannot be adequately represented by the theoretical model. The second is that at high l there is a systematic variation of the discrepancy with l that ceases at $l \simeq 20$, beneath which the discrepancy is independent of l to within the observational accuracy. From this we infer that there is no substantial error deeper in the model than the turning points r_t of the $l = 20$ modes; modes of lower degree, penetrating more deeply, sense the erroneous regions in an l-independent fashion. The $l = 20$ turning point varies somewhat with frequency but is near $r \simeq 0.45\ R$. The third feature is that the discrepancy is least at the lowest values of l. Thus the error immediately above $r \simeq 0.45\ R$ tends to compensate the error in the surface layers, contributing negatively to the theoretical frequencies and implying that here the theoretical model underestimates the sound speed.

The behavior seen in Fig. 7 is specific to the solar model considered here for illustration and is not shared in detail by all other models. However, qualitatively similar results were obtained from the frequencies computed by Ulrich and Rhodes (24) for a model with a more complicated treatment of the physical properties of matter within the sun. Yet there are detailed differences in frequencies predicted by various models that exceed the uncertainties of the observed frequencies. Such differences should allow helioseismology to test and refine the physics used in the calculation of the evolution of stars.

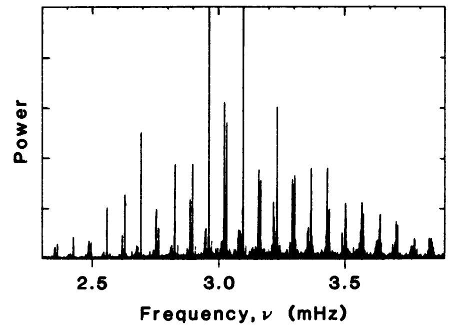


Fig. 5. Power spectrum of low-degree whole-disk Doppler measurements carried out over a period of 3 months at Tenerife and Hawaii with up to 22 hours of coverage per day (21). The greatest amplitude is 15 cm sec^{-1}. The separations Δ between modes with like $n + \frac{1}{2}l$ are just discernible.

The Internal Sound Speed

Fitting theoretical models by adjusting uncertain parameters is a simple and sometimes enlightening procedure for estimating the structure of the sun. Nevertheless it suffers the severe deficiency that the range of possibilities considered may have been too tightly constrained by the untested assumptions of the theory. This is exemplified by the attempts to determine Y; the frequencies of the optimal model deviate significantly from the observations (20). Attempts have been made to improve the fit by adjusting additional parameters, but with only limited success (24). Our discussion of Fig. 7 provides some guidance for estimating the changes that are required, but it is not adequate. It is preferable to employ a so-called inverse method, whereby the structure of the sun is estimated directly as a function of the data.

The frequency of a p mode is determined by the condition that acoustic waves interfere constructively. This requires that the phase difference between the end points A and C of a ray (see Fig. 1) should be the same, up to an integral multiple n of 2π, whether it is evaluated along the ray or along the surface. When buoyancy is neglected, this condition is

$$2\pi(n + \alpha) \simeq 2L \int_{r_t(\nu/L)}^{R} \left(\frac{4\pi^2 r^2 \nu^2}{c^2 L^2} - 1 \right)^{1/2} \frac{dr}{r}$$
$$\equiv 2\pi L\, F\!\left(\frac{\nu}{L}\right) \qquad (5)$$

which defines the function F; α is a constant that accounts for the phase changes at the caustic and on reflection at the surface (25). This condition exhibits a remarkably simple property of the spectrum of acoustic modes: namely, that $(n + \alpha)/L$ depends on l and ν only in the combination ν/L, a property first noticed by Duvall (26) in the solar data. As is evident in Fig. 9, with a suitable choice of α the many ridges in Figs. 2 and 6 collapse onto a single curve with perhaps surprising accuracy. Then the observed quantity $(n + \alpha)/L$ measures the function F.

When $l \ll n$, the radius r_t of the lower turning point, where the integrand in Eq.

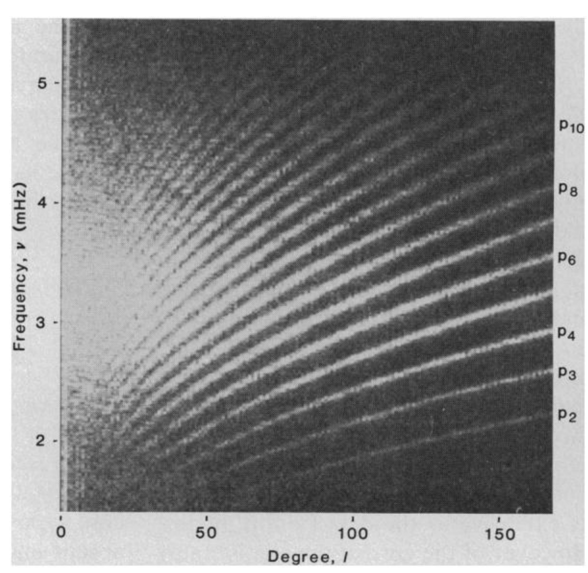


Fig. 6. Power spectrum of zonal modes observed by Duvall and Harvey (23). Lighter tones represent greater power. The orders n of the modes contributing to the ridges were identified by comparison with Fig. 2 in which power from f modes, whose frequencies at high l are independent of the detailed structure of the sun, is clearly visible.

5 vanishes, is small. Moreover, the integrand is dominated by the first term in the square root throughout most of the star. If the integral is expanded about the value obtained by ignoring the second term in the square root and setting $r_t = 0$, Eqs. 2 and 3 are recovered, except that $l + \frac{1}{2}$ is replaced by L.

Equation 5 can be regarded as an integral equation for $c(r)$ in terms of F. It can be inverted analytically to yield $c(r)$ without recourse to a solar model. However, the simple acoustic dispersion relation on which the analysis depends is of questionable applicability; hence the procedure must be tested. When applied to computed eigenfrequencies of a solar model, the inversion has been shown to be accurate to within about 1 percent at radii between 0.4 and 0.9 R. In this part of the model, at least, it is possible to determine the sound speed from the set of modes available (*27*).

Figure 10 illustrates the result of the inversion. The base of the convection zone occurs where the second derivative of temperature, and hence of sound speed, changes almost discontinuously, producing a visible kink in the curve. It is located at $r/R \simeq 0.7$, close to the position of the bottom of the convection zone in typical solar models. Indeed, the sound speed deviates little from the model used to compute the frequencies in Fig. 4. Only in a region below the convection zone in the vicinity of $r/R = 0.4$ does the difference exceed the estimated error in the inversion. Here the sun appears to be hotter than the model by about 2 percent, which is in accordance with our discussion of Fig. 7. Unfortunately, the inversion fails in the core of the sun, where information relating to solar evolution and to the neutrino problem is to be found.

Gravity Modes

The frequencies of the g modes also have a distinctive pattern. The modes can be regarded as a superposition of locally plane gravity waves, just as the p modes can be decomposed into plane acoustic waves. Gravity waves can propagate only in convectively stable regions, namely in the solar atmosphere and in the interior beneath the convection zone. We confine attention to the interior modes. Except when l is low their amplitudes decrease rapidly through the convection zone, and they cannot be observed in the photosphere. Therefore the g-mode data will never be as extensive as the data from p modes. However, if the core of the sun is stably

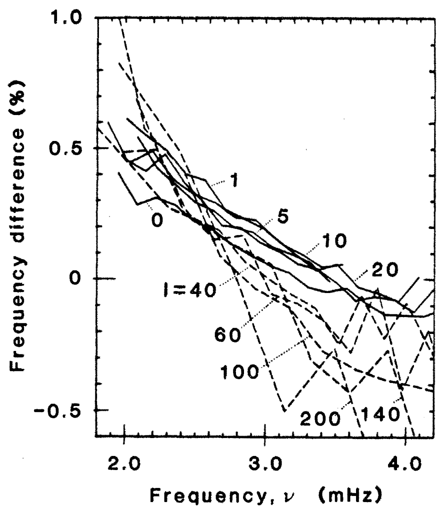


Fig. 7. Percentage differences, $100 \times (\nu_o - \nu_t)/\nu_t$, between corresponding observed frequencies ν_o deduced from Fig. 6 and the theoretical frequencies ν_t of Fig. 4, plotted against ν_t. Values corresponding to modes with like degree l are joined by continuous straight lines if $l \leq 20$ and dashed lines if $l \geq 40$. Observational uncertainties are about ± 0.07 percent if $l \leq 20$ and $\pm$ 0.3 percent if $l \geq 40$. [After (*46*)]

stratified, as most theoretical models predict, the g modes approach the center more closely than do the p modes (Fig. 8). Therefore low-degree g modes are potentially a more powerful diagnostic of the innermost regions.

High-order g modes satisfy a dispersion relation analogous to Eq. 5 for p modes, namely (*28*)

$$\frac{n + \beta}{L} = G(\nu) \equiv \int_{r_1}^{r_2} \left(\frac{N^2}{4\pi^2\nu^2} - 1\right)^{1/2} \frac{dr}{r} \qquad (6)$$

which defines the function G; here N is the buoyancy frequency, the limits of integration r_1 and r_2 are the levels at which the integrand vanishes, and β is a constant of order unity (*29*). When $n \gg l$, the frequency $2\pi\nu$ is much less than N, and the integrand is dominated by its first term. The turning points r_1 and r_2 are then roughly the radii at which N vanishes (that is, the center of the sun and the base of the convection zone, $r = r_c$, in typical solar models). It may further be shown that

$$(n + \tfrac{1}{2}l + \tilde{\beta})\, \nu_{n,l} \simeq L/P_0$$

where

$$P_0 = 2\pi^2 \left(\int_0^{r_c} N \frac{dr}{r}\right)^{-1} \qquad (8)$$

and $\tilde{\beta}$ is a constant related to β and to the nature of the variation of N near $r = r_c$. The presence of $\frac{1}{2}l$ in this formula is a manifestation of the radiative core; if the core were convective this term would be absent and ν would depend on l only through L on the right-hand side of Eq. 7. The analysis predicts that periods are uniformly spaced as the order n varies; the spacing is P_0/L and hence decreases with increasing degree. For typical solar models, the value of P_0 is 33 to 36 minutes.

In contrast to the relations for the p modes, the asymptotic Eq. 7 is not obviously confirmed by observation. The power spectra obtained from g-mode data are quite complicated. The periods are substantial fractions of a day, and hence the sidelobes produced by interruptions in the data cause greater confusion than they do for the p modes; in addition, the splitting induced by rotation is comparable with the separation between modes of different degree and order, which further complicates mode identification. Nonetheless there have been tentative identifications of sequences of peaks in power that are uniformly spaced in period (*30, 31*), with separations proportional to L^{-1} in accordance with Eq. 7. The values inferred for P_0 are somewhat higher than those obtained for normal theoretical solar models. If the gradients in chemical composition were to be reduced in the theoretical models, so would N, and according to Eq. 8 this would increase P_0. Only a modest degree of diffusive mixing is required to bring the computed value of P_0 into agreement with the values suggested by the observations (*32*). However, as noted above, mixing may not be consistent with the observed frequency separations between 5-minute modes of low degree. Moreover, with the available data the presence of approximately uniformly spaced sequences of peaks may be purely fortuitous; indeed, alternative identifications of the observed frequencies can be made that are consistent with normal solar models (*33*). Nevertheless the sensitivity of P_0 to mixing is an indication of the potential for diagnosing the solar core from the periods of g modes.

Solar Rotation

Judging from the angular velocities of young stars, we believe that the surface layers of the sun were once rotating much more rapidly than they are today. This belief is strengthened by the observation that the sun is now losing angular momentum via the solar wind. But by how much has the center of the sun spun down? Here opinions differ, for it is not known how strong the coupling is between the surface and the core. Some have argued that the coupling is weak and that, although the surface rotates with a period of about four weeks, the core is rotating much faster, with a period of only a few days. Others believe that large-scale circulations, rotationally induced instabilities, or wave motion provide effective coupling and that the core is really rotating not much faster than the surface. There are also those who maintain that the sun is pervaded by a magnetic field that gives it sufficient rigidity to be rotating almost uniformly. Among this diversity of opinion, a recent seismological measurement of the internal angular velocity is of considerable interest.

Rotation breaks the spherical symmetry of the sun. If the entire sun rotates about a common axis, it can still oscillate in essentially normal modes with structure close to that given by Eq. 1, provided that the coordinate axis is chosen as the axis of rotation. But now ν depends on m. The principal reason is that rotation causes the patterns illustrated in Fig. 3 to rotate, which modifies the apparent frequencies of especially the high-m modes when viewed from Earth. The extent to which the m degeneracy is split provides a measure of the angular velocity $\Omega(r)$ in the cavity within which the modes are confined (*34*).

Most of the observations from which Ω has been estimated are of sectoral ($m = l$) acoustic modes (*35*) with degrees up to 100. These modes are concentrated near the equator (*9*), particularly when l is large (Fig. 3), and cannot provide information about the latitudinal dependence of Ω. But they can reveal how Ω varies with radius r near the equatorial plane. The magnitude of the degeneracy splitting of the acoustic modes is an average of $\Omega(r)$ weighted approximately as the square of the velocity eigenfunctions scaled as in Fig. 8. With data from a sufficient variety of modes, those averages can be inverted to estimate $\Omega(r)$.

The results of such an inversion are shown in Fig. 11. Perhaps the most startling feature is the inward decline of Ω in the solar envelope. As we have already mentioned, the solar wind braking of the outermost layers has led most theorists to infer that Ω increases everywhere with depth. To be sure, the core appears to be rotating more rapidly, but why is it surrounded by a region of such slow rotation?

The abrupt increase of Ω at the edge of the energy-generating core appears to occur just where the variation of chemical composition produced by the nuclear reactions is predicted to give the strongest stabilization to vertical motion. However, the resolution of the splitting data is poorest in this region, so that the details of the variation of Ω are uncertain.

A secure determination of the angular velocity of the core appears to require knowledge of the splitting of g modes that penetrate almost to the center of the sun. There is a report of rotational splitting of dipole g modes (*30*), but there is some doubt about its interpretation. Were we to accept the result and to combine it with the p mode splitting data, we would deduce that the mean angular velocity of the core is between about three and six times the angular velocity at the surface. This is consistent with a recent report of rotational splitting of dipole p modes by Isaak and colleagues (*36*).

The rotational flattening of the gravitational potential outside the sun depends predominantly on the value of Ω near the equatorial plane at radii between 0.3 and 0.9 R. This is the very region where Ω has been most securely determined by the splitting data. The values of the quadrupole moment J_2 inferred from the two curves in Fig. 11 are both 1.7×10^{-7} and are too small to influence significantly the interpretation of current measurements of the precession of the perihelion of Mercury (*35*). Unless the oscillation data have been misinterpreted, which seems unlikely, these measurements are consistent with Einstein's theory of general relativity.

As in any rapidly developing field, there is some controversy over the interpretation of data. For example, Hill and colleagues (*37*) have interpreted mea-

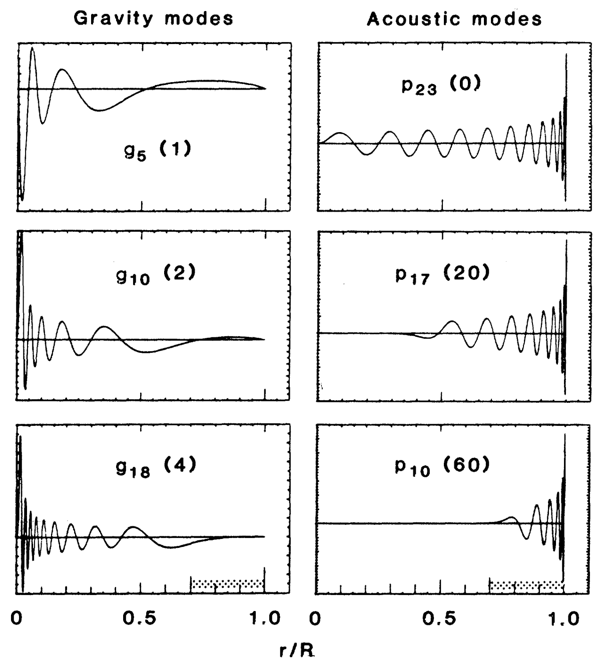
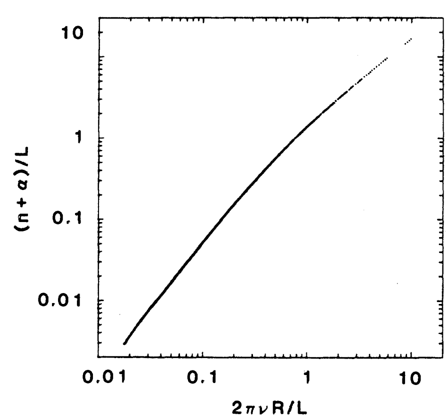


Fig. 8. Vertical component of velocity of several modes of oscillation, scaled by $r\rho^{1/2}$ where $\rho(r)$ is density. The ordinate scales are arbitrary; the horizontal line is at zero. For each value of l (given in parentheses), the order n was chosen such that the g-mode frequencies are approximately 0.10 mHz (periods of 165 minutes) and the p-mode frequencies are approximately 3.3 mHz (periods of 5 minutes). The shaded bands indicate the extent of the convection zone.



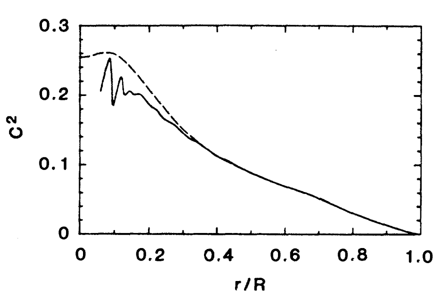


Fig. 9 (left). The function $(n + \alpha)/L$ plotted against the surface phase velocity $2\pi\nu R/L$ (which is in units of Mm $\sec^{-1}$). The data include the high-degree modes considered by Duvall (*26*), which are similar to those of Fig. 2, and the modes of low and intermediate degree discussed by Duvall and Harvey (*23*) and illustrated in Fig. 6. The value $\alpha = 1.57$ was chosen to minimize the scatter of the data about a single curve. Fig. 10 (right). The square of the sound speed c obtained by inverting Eq. 5, using a smooth curve through the points in Fig. 9 to determine the function F, is shown as a continuous line. The actual value of c^2 of solar model 1 (*45*) is shown as a dashed line. The large discrepancy at $r \lesssim 0.3\ R$ results mainly from the inaccuracy of the simple asymptotic theory. The unit of speed is Mm $\sec^{-1}$. [After (*27*)] surements of fluctuations at the periphery of the solar image to be the result of acoustic and gravity modes with high rotational splitting. Thus they have concluded that most of the sun is rotating substantially more rapidly than the surface. When considered in relation to the precession of the orbit of Mercury, this raises difficulties with Einstein's theory of general relativity, but it is apparently consistent with some other covariant theories of gravity. From earlier observations of the oblateness of the solar image, Dicke had postulated that the interior of the sun is rotating rapidly and is distorted by an intense magnetic field (*38*). We must await more detailed seismological probing before these issues are resolved.

The analysis used to obtain Fig. 11 did not permit the determination of the rotation close to the solar surface. However observations of modes of higher degree (*39*), which provide better depth resolution near the surface, have suggested that the rotation rate increases with depth in the outer part of the convection zone. Such observations have also shown indications of day-to-day variations in the subphotospheric velocity field. These could be caused by the passage through the field of view of giant convective cells, with associated velocities of order 100 m sec^{-1}. Although these results are tentative, they hint at the potential of helioseismology for probing the dynamics of the solar convection zone.

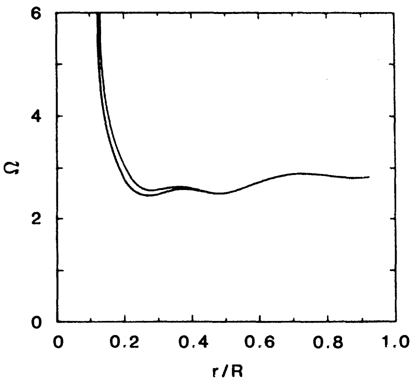


Fig. 11. Examples of the equatorial angular velocity $\Omega(r)$ of the sun, in units of 10^{-6} sec^{-1}, inferred from the rotational splitting of 5-minute p modes reported by Duvall and Harvey (*35*), using an asymptotic inversion procedure (*47*). The lower curve was computed under the assumption that Ω is independent of colatitude θ except very close to the surface; the upper curve results from the assumption that the θ-dependence of Ω is at all values of r the same as the observed surface variation. The curves are uncertain for $r \lesssim 0.3\ R$ because the data from the deeply penetrating low-degree modes are imprecise (*35*); yet the difference between them represents well the sensitivity to the assumptions about the θ-dependence. The p-mode data provide no information about the rotation of the very center of the sun. The photospheric equatorial angular velocity $\Omega(R)$ is 2.86×10^{-6} sec^{-1}.

Conclusions

Ten years ago there was little empirical information about the solar interior. What was available, the neutrino flux, disagreed with the theoretical models. The neutrino problem is still with us, but the observational data from the solar interior have expanded tremendously. As a result we can now study the sun at a level of detail that would have seemed hopelessly out of reach only a decade ago.

Helioseismology has provided, through the observations of 5-minute oscillations, an estimate of the sound speed throughout much of the solar interior. It has also provided a test of the structure of the deepest layers of the sun predicted by the theory of stellar evolution. There are certainly discrepancies between our present best models and the observations; these must be studied and eliminated. Nevertheless it is perhaps surprising how well normal solar models fit the data. In contrast, models with a low helium abundance seem to be excluded, as do models with substantial mixing of the core with its environment. Among standard solar models the seismological data appear to favor those with a high neutrino flux. This may strengthen the case for searching for a physical, as opposed to an astrophysical, solution to the neutrino problem, such as the possibility that neutrinos have mass and undergo transitions from one form to another. Nevertheless, it is essential to explore the astrophysical options, including that of the sun being in a transient interlude of thermal imbalance after a sudden overturning in the core. Such processes would almost certainly not be restricted to the sun.

It may have been thought, perhaps with some justification, that theory has provided a reasonably reliable estimate of the structure of the solar interior. The same can certainly not be said about the internal rotation, where all theoretical models must be highly uncertain. Indeed, none of them has predicted a rotation law such as the one that has recently been measured.

What are the requirements for the future? It behooves us to develop more subtle theory, particularly for extracting more detailed information about the core of the sun. And as always there is a need for better and more extensive data. Modes of low frequency, particularly the g modes, are sensitive to conditions in the core and would provide valuable information. But we also need better data from high-degree p modes to measure the region immediately beneath the solar surface. These will permit us to study the giant cells in the convection zone, which may control much of the magnetic activity in the solar atmosphere. The region sampled by high-degree modes also influences the frequencies of the p modes that penetrate more deeply, for all acoustic modes have their upper reflection point just below the solar surface. Reliable information about the interior requires accounting for the influence of this region on the frequencies of the lower-degree modes.

It is of great interest to study temporal variations in the stratification of the sun. A change in the solar radius causes a change in the p-mode frequencies of roughly the same relative magnitude. Given the high accuracy of some of the observed frequencies of the 5-minute oscillations (*40*), this may provide a sensitive method for detecting structural changes associated with solar activity and the solar cycle. We must also look for variations with time in the rotation. Could it be that the curious rotation law shown in Fig. 11 is a time-dependent phenomenon associated with torsional oscillations of the solar interior, perhaps under the influence of a magnetic field?

We may be getting near the limit of what can be attained from single ground-based observatories. The presence of night-time gaps in the data produces confusing aliases in the power spectra. Such gaps might be eliminated by observing from the South Pole during austral summer, but it is rare for continuous periods of clear weather to last more than a week. Therefore observations are being combined from widely separated sites (*21*); it has been estimated that nearly continuous coverage could be ensured with six stations suitably placed around the world (*41*). Such networks are now being established by investigators from Birmingham and Nice using Doppler measurements in integrated sunlight to study the low-degree modes. More ambitious efforts, to be carried out by the Global Oscillations Network Group and coordinated by the National Solar Observatory, are aimed at studying solar oscillations of higher degree by making spatially resolved Doppler measurements with identical instruments from a network of observatories.

Harder to eliminate are the effects of the terrestrial atmosphere. They can introduce daily modulations of the observed signal that hinder the combining of data from several observatories, particularly at the long periods of g modes. Modes with periods of 30 to 60 minutes may be overpowered by atmospheric noise; these modes have considerable diagnostic potential, but their amplitudes are so low that they have not been detected. Finally, atmospheric seeing distortions have particularly serious ramifications for measuring high-degree modes whose length scales are comparable with the scale of the seeing. Yet to study dynamics within the convection zone, we must observe these modes with a high signal-to-noise ratio, and this must be achieved within a time sufficiently short that the underlying convection cells do not change or move out of the field of view. It may not be possible to do this from the ground. Therefore several helioseismology instruments are now under study by the European Space Agency and by the National Aeronautics and Space Administration to fly on the satellite SOHO. They would acquire uninterrupted observations of extremely high quality, limited only by the intrinsic solar "noise" in the form of random surface velocity and intensity fluctuations.

Stars similar to the sun might be expected to exhibit a similar spectrum of oscillations. Analogs of the solar 5-minute oscillations may already have been detected in the stars HR 1217, α Centauri A, and ε Eridani (*42*). Rich spectra of oscillations with substantial diagnostic value have also been found in other classes of stars, notably the white dwarfs. Thus we are now seeing the birth of asteroseismology. Continued developments in the acquisition of better data and improvements in theoretical understanding will undoubtedly lead to the establishment of a firm empirical basis for the study of solar and stellar evolution and activity.

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5. The rapid increase with depth of molecular weight (that is, mean mass per particle) brought about by thermonuclear transformations in the core causes the sound speed to decrease with depth close to the center of the sun (Fig. 10). However, c/r is monotonic in normal solar models, so that the solution to Eq. 4, which determines the bottom of the acoustic cavity, is unique.
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9. If the sun were cylindrical, exactly l wavelengths of a sectoral mode such as that in Fig. 3D would fit around the circumference $2\pi R$. But because the sun is a sphere, the mode varies away from the equatorial plane between latitudes of $\pm \cos^{-1} [l/(l+1)]^{1/2}$. This effectively reduces the wavelength of the surface disturbance to $2\pi R/L$, where $L^2 = l(l+1)$. More complicated modes, such as those in Fig. 3, A, B, and C, can be regarded as superpositions of sectoral modes in different planes with the same value of l.
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12. Neutrinos are usually considered to be massless. If that were not the case (a possibility that has received considerable attention in the last few years), some of the neutrinos produced by the nuclear reactions in the sun could have been transformed during their passage to Earth into a form that would not have been detected. The neutrino observations may then be consistent with the seismological calibration.
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