olfactory bulb is not as diffusely organized as was first thought. Indeed, he has, with the aid of electrophysiological and 2-deoxyglucose histological methods, identified a subset of olfactory bulb glomeruli that may represent the labeled line responsible for detecting nipple odor cues that direct infant suckling behavior. The book concludes with an interesting comparison by Robert Johnston of the function of the olfactory and vomeronasal chemosensory systems.

All of the chapters in this collection are well written, and together they form a fitting tribute to a remarkable career in chemosensory science.

RICHARD C. ROGERS
Department of Physiology,
University of Nevada School of Medicine, Reno 89557

Three-Manifold Topology

The Smith Conjecture. JOHN W. MORGAN and HYMAN BASS, Eds. Academic Press, Orlando, Fla., 1984. xvi, 245 pp., illus. $49.50. Pure and Applied Mathematics, 112. From a symposium, New York, 1979.

What is the nature of the fixed point set of a periodic diffeomorphism of the three-dimensional sphere? Or, more specifically, can the fixed point set be a knotted circle? This is not a question likely to stir familiar and vivid images in the minds of most people or to cause public debate and conjecture. Nevertheless, it is a question that has had a tremendous influence on mathematics and that has for several years engaged some of the best research mathematicians in an attempt to find an answer.

Now, nearly 40 years after it was first clearly posed, the question has been answered. The answering of the question has gone far beyond the resolving of a fundamental question in the topology of manifolds. It is the result of a beautiful confluence of technique and knowledge from diverse areas of mathematics, and it has had a profound influence on the methods of analyzing three-dimensional manifolds. Since the publication of the seminal work that played a major role in answering the question, it is possible to find articles in popular magazines describing the study of three-dimensional manifolds. These articles attract considerable interest, particularly if they provide a metaphorical relation to the shape of the universe. Recently molecular chemists have become concerned with the symmetries of the three-dimensional sphere that leave knotted circles invariant (fixed with respect to set rather than with respect to point), and they speak of molecular graphs in the language of three-manifold topology.

At the turn of the century and into the 1920's, mathematicians were formalizing the basic concepts of topology and beginning a topological study of several mathematical objects. From the point of view of topology it is natural to study the transformations and symmetries of mathematical objects, and so it was natural to study periodic transformations that deform an object. These deformations may be arbitrary, but they never cut, tear, or puncture the object. Preceding this work by several years and using both analytical and geometrical methods, a systematic study and classification of surfaces had been carried out. Surfaces fit into the scheme of topology now called manifold topology, in which they represent two-dimensional manifolds. And so there was interest in a study of periodic diffeomorphisms of surfaces. Most relevant to this review is the study of periodic diffeomorphisms of the two-sphere.

An understanding of periodic diffeomorphisms of the two-sphere followed the work of L. E. J. Brouwer and B. Kerekjarto done in the 1920's (although there were gaps that were filled by S. Eilenberg in 1934). Brouwer and Kerekjarto's work showed that an orientation-preserving, periodic diffeomorphism of the two-sphere is equivalent to a rotation about an axis. In other words, any orientation-preserving, periodic diffeomorphism of the two-sphere is precisely as one would imagine it to be.

In a sense, the study of three-manifolds, as well as of all higher-dimensional manifolds, can be thought of as a generalization of the study of two-manifolds. It was in this context that P. A. Smith wrote a series of papers in the 1930's and 1940's in which he examined periodic homeomorphisms of the n-dimensional sphere. He discovered that certain topological invariants of the fixed point set of a periodic diffeomorphism of an n-sphere were the same as those of a lower-dimensional sphere. Interpreted in the special case of a nontrivial, orientation-preserving, periodic diffeomorphism of the three-sphere, the work of Smith means that a nonempty fixed point set is precisely a circle. (The word "circle" is used in the general sense here, as a space topologically equivalent to a round circle in the plane.) For every positive integer $n > 1$ there are orientation-preserving diffeomorphisms of the three-sphere, periodic with period n and having as a fixed point set an unknotted circle. A circle in the three-sphere is considered unknotted if there is a self-diffeomorphism of the three-sphere taking the given circle onto a round circle determined by the intersection of the three-sphere and a two-dimensional linear subspace of Euclidean four-space. Smith then asked if there are knotted circles that arise as fixed point sets of orientation-preserving, periodic diffeomorphisms. The negative answer to this question became known as the Smith conjecture.

In the fall of 1978, the Smith conjecture was proved to be correct. In the spring of 1979, a symposium was held at Columbia University on the solution of the Smith conjecture. It was an exciting time. The principal participants in arriving at the solution presented the various pieces of the proof. There was an electricity in the air, for it was recognized that this was the beginning of a new era in three-dimensional topology. The proof, as its beauty finally emerged, represented ideas and techniques from several areas of mathematics, including classical three-manifold topology, hyperbolic geometry, Kleinian groups, the algebra of matrixes, and minimal surface theory. The symposium was also an occasion for a tribute to Smith, who had spent most of his mathematical life at Columbia University and lived in retirement in the neighborhood.

The volume edited by Morgan and Bass contains written versions of the presentations given at the symposium. In addition, there are two papers containing generalizations of the methods used in the proof of the Smith conjecture, as well as one presenting a discussion of the situation in higher dimensions.

The history of the Smith conjecture and the outline of its proof, explaining, with a "flow diagram," how the various pieces of the proof fit together, provide a spectacular example of the powerful techniques employed in the proof and of the relation among the various areas of mathematics. This subject is capably dealt with in the book by Morgan.

The general theme of proof is credited to W. Thurston. However, several other individuals were involved in creating the final solution, and at least three new and major contributions to the theory were introduced. These new contributions, combined with recent discoveries from classical three-manifold topology, provided the pieces necessary for an answer.

The proof begins with the assumption that one has a nontrivial, orientation-preserving, periodic diffeomorphism of the three-sphere that has prime power period. By Smith's theorem, such a diffeomorphism has a circle of fixed points if it has any at all. Of course, the idea is to prove that the circle is unknotted. The proof proceeds by study of the three-manifold that is the complement of the circle in the three-sphere. First, several reductions are made disallowing various possibilities if the circle were knotted. The proof then branches into two distinct considerations. One branch can be labeled algebraic and the other geometric. The algebraic branch begins with the deep and fundamental uniformization theorem for three-manifolds announced by Thurston. An algebraic interpretation of this theorem says that for a large class of three-manifolds the fundamental group, a principal algebraic invariant, can be represented by a very special group of 2×2 matrixes. It is at this point that the work of Bass and P. Shalen entered. Bass described the structure of these special groups of 2×2 matrixes, and Shalen, using classical methods of three-manifold topology, parlayed the algebraic statements into information about the structure of the three-manifold under consideration.

As all the preceding pieces were unfolding, it was the geometric branch of the argument that seemed to be the stumbling block. Some progress had been made on the problems occurring in this situation when a special case was understood by C. Gordon and R. Litherland. However, in completely independent work, primarily done by W. Meeks and S.-T. Yau, the methods and techniques of minimal surface theory were introduced to the study of three-dimensional manifolds. This work had an enormous impact on the understanding of three-manifolds, answering many questions. It also had an immediate application to the remaining gap in the solution of the Smith conjecture.

The Smith conjecture was solved. A circle of fixed points of a periodic diffeomorphism of the three-sphere can only be unknotted. In other words, any periodic diffeomorphism of the three-sphere having a circle of fixed points is precisely as one would imagine it to be.

Most of the material appearing in the papers in the book can be found in other publications in which the authors present their independent work in a broader context. The papers prepared for this volume provide the concise statements and supporting proofs applicable to the considerations of the Smith conjecture. Having papers written specifically for this volume certainly makes it easier to understand the logical structure and details needed for proving the Smith conjecture; moreover, it makes for a wonderful introduction to the various broader theories relevant to the study of three-manifolds.

The paper by Morgan on Thurston's uniformization theorem may be absolutely the best source there is on this subject and the closest approximation to a complete proof. It is based on lectures by Thurston and several personal conversations between Morgan and Thurston. Morgan has done an excellent job of giving logical organization to the proof of this important theorem. There are substantial exclusions; however, the exclusions are clearly acknowledged, and a remarkably clear picture appears in Morgan's presentation. One of the unusual and unfortunate phenomena resulting from the intense activity in low-dimensional topology is that such an important and fundamental result as the uniformization theorem has gone without a complete and written proof for such a long period of time. The paper by Morgan is a noteworthy attempt to correct this situation. Some of the exclusions remain; other gaps are being filled, and papers on them are appearing in the literature. These gaps will surely be filled and a complete and accessible proof will be available. But for now this book presents the best picture available of the uniformization theorem, and, in total, the material in the book represents the frontier of research in three-manifold topology.

WILLIAM JACO
Department of Mathematics, Oklahoma State University, Stillwater 74078

Fluvial Geomorphology

The Colorado River. Instability and Basin Management. WILLIAM L. GRAF. Association of American Geographers, Washington, D.C., 1985. viii, 86 pp., illus. Paper, $5. Resource Publications in Geography.

In October 1983 the Santa Cruz River of southeastern Arizona was the source of a major flood, the largest in over 70 years of continuous observation. As is the case with most urban communities in the United States, Tucson, Arizona, had prepared for such an event through engineering hydraulic surveys of its "floodplains." The surveys had clearly delineated zones of hazardous overbank flow to be expected from large, rare floods.

In the 1983 flood the Santa Cruz River did not overflow its banks as the surveys had predicted. Like other streams in the Colorado River basin, the Santa Cruz had experienced a complex metamorphosis over the past century. Water withdrawals, vegetation changes, climatic shifts, and basin development all induced instabilities in this arid-region river. In the late 1800's the Santa Cruz incised to form a classic arroyo. From 1920 to the 1960's it was relatively stable in a period characterized by unusually small floods. The 1983 flood encountered an incised channel that generated little overbank flow, but instead widened by spectacular scour of channel banks. Houses collapsed into the river as the flood flow undercut their foundations. Bridges were left on one bank as the river migrated into and beyond the opposite abutment.

Tucson's floodplain managers, indeed water-resource managers throughout the Southwest, would benefit from W. L. Graf's concise and lucid analysis of fluvial system complexities in the Colorado River basin. Graf provides a scientific analysis, emphasizing the important fluvial influences of flooding, riparian vegetation, incision, channel widening, high dams, and chemical changes. His conclusion is that engineering experience with the humid-region rivers of America is being grossly misapplied to the sensitive stream systems of the West. Too much emphasis is being placed on the design of the Colorado River basin to suit human whims without attention's being paid to the unique fluvial geomorphic processes of that region.

In one of several in-depth discussions, Graf presents an excellent overview of the influence of vegetation on channel stability. With the advent of Anglo settlement in the 1800's, native vegetation growing on river banks was cut for lumber and fence posts. Large floods, which had had little effect on banks stabilized by plants, proved to be destructive in the absence of dense riparian vegetation. In the 1900's exotic species invaded the region, replacing the decimated native riparian communities and stabilizing banks. Concern over water loss by evapotranspiration then led to an expensive and unsuccessful program of eradication for the exotics during the last 40 years. The program became obsolete when rapid groundwater withdrawal destroyed both native and exotic riparian communities. Today major channel changes are occurring on rivers like the Santa Cruz, which flow in fine-grained unconsolidated alluvial fills.

Perhaps those who strive to control the modern environment of the Colorado basin would do well to study the experience of the prehistoric Hohokam, Mogollon, and Anasazi peoples of 1000 years