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Torsional Oscillations of the Sun

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The cycle of magnetic activity of the sun is a long-outstanding puzzle of solar research. Starting from solar minimum, at which time the sun has a weak dipole magnetic field aligned with its rotation axis, activity begins at mid-latitudes with the emergence of localized regions of cycle, in which the dipole returns to its initial orientation, takes about 22 years.

Since magnetic fields suppress the convective flow of energy, the points where the fields of maximum intensity erupt are cooler and thus darker than the rest of the solar surface. These darker

Summary. The sun's differential rotation has a cyclic pattern of change that is tightly correlated with the sunspot, or magnetic activity, cycle. This pattern can be described as a torsional oscillation, in which the solar rotation is periodically sped up or slowed down in certain zones of latitude while elsewhere the rotation remains essentially steady. The zones of anomalous rotation move on the sun in wavelike fashion, keeping pace with and flanking the zones of magnetic activity. It is uncertain whether this torsional oscillation is a globally coherent ringing of the sun or whether it is a local pattern caused by and causing local changes in the magnetic fields. In either case, it may be an important link in the connection between the rotation and the cycle that is widely believed to exist but is not yet understood.

intense magnetic field. The active regions, during their evolution, contain field features that come and go on time scales ranging from a few days to a few months. As the cycle progresses, the number and complexity of these active regions increases, and the zone of activity moves toward lower latitudes. In the meantime, the original dipole field weakens, and toward solar maximum (roughly 4 years into the cycle) it vanishes and reappears with the opposite sign. The number of active regions then begins to decrease, returning the sun after about seven more years to a condition of solar minimum with a reversed axial magnetic dipole. Because the second half of the cycle follows a similar pattern, the full

areas, called sunspots, are occasionally large enough to be seen with the naked eye and are readily seen with a telescope. They were thus observed and studied long before solar magnetism was discovered, with recorded sightings dating back to ancient times. Galileo, in 1613, was the first to study them through a telescope, and he used his findings to support the Copernican revolution (*1*). By studying the transit of sunspots across the solar disk, he deduced that the sun rotates with a period of roughly 27 days. Twenty years later, Christophorus Scheiner (*2*) noted that the spots nearer the equator take less time to traverse the disk and deduced that the sun does not rotate as a rigid body. The sunspots then vanished and were only rarely seen for a period of some 70 years (*3*). Since their abrupt reemergence around 1715, their occurrence has been more or less regular, but the cyclic nature of the phenomenon went unnoticed until 1843 (*4*). Sunspot data for the most recent cycles are given in Fig. 1.

A detailed study of the sun's nonrigid, or differential, rotation was carried out in the last century by Carrington (*5*). Still using sunspots as indicators, he found that the angular velocity of the rotation varies smoothly with solar latitude, being fastest at the equator (period, ~25 days) and slowest at the poles (period, ~32 days). Studies of solar rotation continue to the present by means of a variety of indicators (*6*).

It has long been thought that the activity cycle is causally linked with differential rotation, but neither the link nor the basic mechanism governing either phenomenon is well understood. In 1908, George Ellery Hale, founder of the Mount Wilson Observatory, discovered the intense magnetic fields contained in sunspots (*7*). In 1925 he found a pattern in the polarities of these fields (*8*) that has become known as the Hale polarity law. By the 1950's, Harold and Horace Babcock (father and son), also at Mount Wilson, observed the far weaker overall solar magnetic fields (*9*) and found a corresponding polarity pattern for them. These discoveries both spurred theoretical interest and gave rise to numerous studies and speculations in solar-terrestrial phenomena (*10*).

The subsequent discovery of chromospheric emission lines whose strength is correlated with magnetic activity has enabled astronomers to identify and study solar-like activity cycles in other stars (*11*). This study is called the H-K project after the bright H and K lines of calcium. A result suggesting a connection between rotation and activity cycle is that the more rapidly rotating stars tend to have shorter and more intense cycles.

Mount Wilson Observational Program

Much of this century's investigation of the solar cycle and differential rotation has taken place at the Mount Wilson Observatory of the Carnegie Institution of Washington, located in the mountains overlooking Pasadena, California. The 150-foot tower telescope has been dedi-

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cated to this project since its erection by Hale in 1910. Although the telescope itself has changed only slightly, the instrumentation developed for it has kept it the state of the art for this branch of solar research.

Whereas the telescope serves to focus the sun's image, the primary tool for analysis is the spectrograph. Directly beneath the Mount Wilson tower is a pit 75 feet deep with a reflection grating at the bottom. The grating reflects the sun's light back up with a dispersion of 1.3 cm/Å to the exit slit in the observing room, where the photometry makes possible the measurement of wavelength to an accuracy of about 0.0001 Å, that is, about 1 part in 50 million (*12*).

The observations concentrate on Fraunhofer lines, the dark lines that show up against the continuous solar background spectrum as a result of the absorption of photons by atoms in the sun's atmosphere. By the Doppler effect, wavelength shifts in these lines indicate the line-of-sight component of the solar atmospheric velocity. In addition, certain lines are magnetically active, meaning that in the presence of a magnetic field they become split into circularly polarized components with slightly different wavelengths. This phenomenon, known as the Zeeman effect (*13*), provides a means for measuring the magnetic fields on a distant object such as the sun.

A milestone in instrumental development was Horace Babcock's invention of the magnetograph (*9*). This device enables the observer to measure simultaneously, for a particular Fraunhofer line and a small region on the sun's surface, both the line-shift and line-splitting and hence respectively the line-of-sight component of the material velocity and the magnetic field. Since 1966, modern computers have been used to obtain, store, and partially analyze (reduce) daily full-disk maps, called magnetograms, of solar magnetic and velocity patterns (*14*). The line of choice has been λ5250.216 Å, created in a transition from the ground state of neutral iron. This line is magnetically active and is produced near the top of the photosphere. The data are collected from roughly 24,000 contiguous regions covering the solar disk and are reduced through a program that removes all known systematic background effects, such as the earth's motion and the tilt of the sun's rotation axis. The precision of the spectrograph enables the determination of line-of-sight velocity and magnetic field to within ±5 m/sec and ±0.5 gauss, respectively. These synoptic data now span roughly 18 years, almost a full activity cycle (*15*).

As this observational program continues, our knowledge of what is happening at the top of the photosphere becomes more precise. The program is not, however, fully automated; each day's observation begins, as it has for the past 75 years, with a tracing done by hand of the exact locations and field strengths of the sunspots. This procedure is still the best method for overcoming the uncertainties resulting from the smearing of the sun's image by motions in the earth's atmosphere.

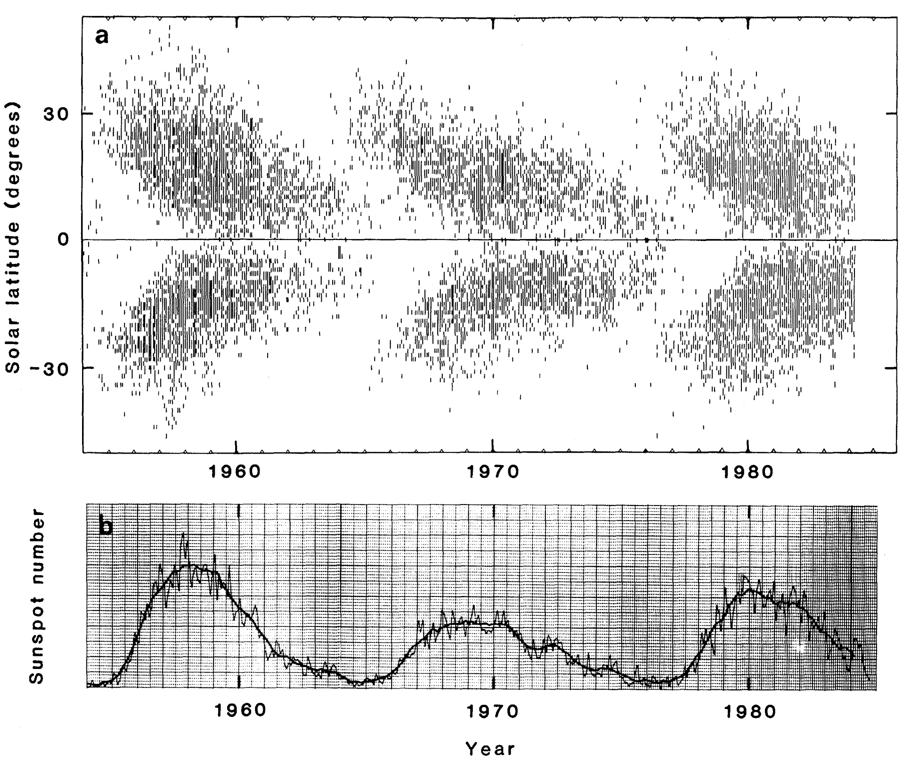


Fig. 1. Charts of recent solar magnetic activity cycles as revealed through sunspot data taken from daily drawings at Mount Wilson. (a) Butterfly diagram showing daily locations of sunspots. (b) Daily sunspot number as a function of time.

The Sun as a Rotating Convective Dynamo

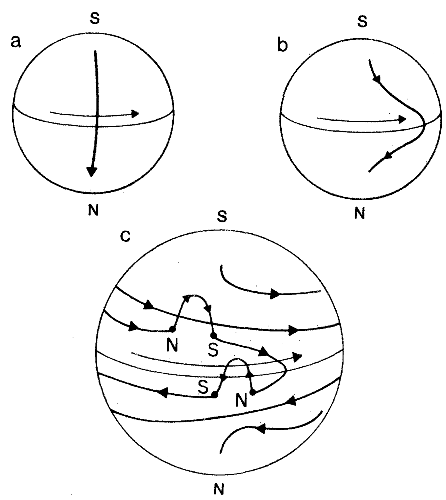
The polarity law discovered by Hale (*8*) is as follows. Before solar maximum, sunspots tend to occur in dipolar pairs, with their dipole (that is, north-to-south) axes slanted but nearly parallel to the equator. The north-south orientations of these pairs of sunspots are almost invariably the same for all the pairs in one hemisphere but are opposite on opposite sides of the equator, and there is an overall reversal (that is, from north-south to south-north) after each (11-year) half-cycle. The Babcocks' study of the weak but more extensive polar fields (*9*) revealed that the fields also reverse sign with each cycle and, furthermore, that the polarity of each leading spot (in the sense of solar rotation) in a dipolar pair is the same as the polarity of the pole for the hemisphere in which it is located. Putting all this together, the younger Babcock theorized (*16*) that the differential rotation tends to drag, and hence wind up, the magnetic field lines, which are envisaged as lying just inside the solar surface (see Fig. 2). In the process, the field lines become crowded together and, because of magnetic buoyancy (*17*), are pushed from the solar surface to form the regions of visible magnetic activity.

This marked a breakthrough in solar dynamo theory, which was proposed in 1955 by Parker (*18*) to describe the differential rotation and magnetic cycle as coupled phenomena. Babcock's model, modified and extended by Leighton (*19*), could account for the transformation of the poloidal field into a toroidal field, for the emergence of active regions, and for the Hale polarity law, but it could not explain the differential rotation itself or account properly for the length of the cycle or for the poleward drift and break-up of the following-polarity spots.

The thesis of modern dynamo theories is that convective motions, which take over the energy transport in the outer 30 percent (by radius) of the sun, play the decisive role in generating both magnetic fields and differential rotation. One approach, called kinematic α-ω dynamo theory, describes the magnetic cycle in terms of the differential rotation and internal motions. The equations are then essentially linear (*20*). Other projects start with the sun's mass, composition, angular momentum, and rate of energy production and use the basic equations of magnetohydrodynamics to construct a model to approximate the sun's behavior, in order to predict both the differential rotation and magnetic cycle (*21*). Because these equations are complicated and nonlinear, the self-consistent approach is an enormous computational effort that uses extremely large and fast computers. Neither approach has yielded definite or specific results.

The interpretation of the observational data is itself a complex and difficult problem. In measuring the solar rotation rate, the solar radius (which is the conversion factor from the measured linear speeds to the theoretically important angular velocity) is a critical factor, but its value depends on how the limb-darkening is measured, what part of the spectrum is being studied, and the particular Fraunhofer line being used. In the spectroscopic (Doppler shift) determination, which gives the rotation of the photospheric plasma (that is, the visible, partially ionized surface of the solar atmosphere), other motions must be taken into account and corrections made for the effects of scattered light. Equally difficult problems attend the determination by tracking sunspots. Furthermore, the rate of rotation and shape of the differential rotation curve depend somewhat on the indicator. It is now generally agreed, for example, that the plasma rate is slightly slower than either the magnetic rate, found by cross-correlation of synoptic magnetic maps (*22*), or the sunspot rate, found by measuring white-light photographs (*23*). These different differential rotation curves (Fig. 3) can be explained in terms of conservation of angular momentum in the upper layers of the convection zone (*24*).

Perhaps the most controversial aspect of solar rotation is the rotation of the sun's interior. Whereas most kinetic dynamo theories assume that the sun's interior rotates faster than the surface, the prediction of both differential rotation theories and self-consistent dynamo theories is that the interior rotates more slowly. Speculations range from the notion that the interior rate of rotation is several times faster than at the surface (*25*) to the notion that it is appreciably



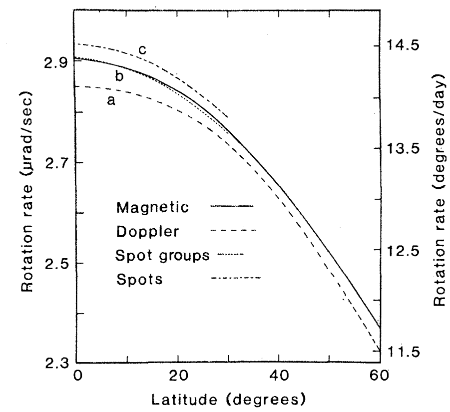


Fig. 2 (left). Schematic illustration of the Babcock model relating the activity cycle to differential rotation. (a) A typical magnetic field line in the surface at solar minimum. (b) Winding of the field line as it is dragged by the differentially rotating solar atmosphere. (c) Eruption of active regions as a result of overcrowding of the woundup lines. A north pole (N) is created at the point where the field line leaves the surface, and a south pole (S) is created at the point where it enters again. Fig. 3 (right). Curves illustrating the (globally fit) differential rotation of the sun, as determined (a) by the Doppler shifts of the Fraunhofer line of wavelength 5250.216 Å, (b) by the cross-correlation of solar magnetic fields, and (c) by tracking the motions of sunspots.

slower (*18*). The solar global 5-minute oscillations are generally regarded as the best observational estimate of the interior rotation rate, although much of the controversy has been generated by studies giving different interpretations of the oscillation data. However, the most recent study (*26*) suggests that, within the convection zone, the interior rate is not much different from the surface rate.

The Discovery of Torsional Oscillations

Torsional oscillations were discovered quite by accident in 1980. Mount Wilson astronomers Robert Howard and Barry LaBonte were searching the data for traces of giant cells, which had been predicted to exist by theorists working on self-consistent dynamo models (*27*). In the standard data reduction procedure, a set of globally defined functions is used to fit and extract the known velocity fields, leaving a residual array that Howard and LaBonte had expected would contain the evidence they were seeking. These arrays, however, yielded nothing identifiable as such, and this led to the suspicion that the anticipated giant cell signal was being extracted along with the fit. A study of the fit-determined coefficients was therefore in order, but this presented a problem. It was known that these coefficients had a large and apparently random day-to-day variation that was larger than anything expected from the sun and was tightly intercorrelated; it thus was thought to be a systematic instrumental error. It was as if someone were fiddling with a gain knob on the daily calibration, yet no one was able to track down the error (*28*). However, Howard and LaBonte realized that, because the error was correlated, it would cancel in the subtraction of two different fits to the data, and they chose to examine the remainder in the subtraction of a global from a latitude-by-latitude, or zonal, fit to the differential rotation.

To their surprise, the difference contained nothing identifiable as a giant cell signal but instead a remarkably regular pattern of traveling waves (*29*). This pattern, brought up to date and averaged over 20 Carrington rotations (*30*) is illustrated in Fig. 4. There are two waves with amplitudes of about 3 m sec^{-1} in each hemisphere, and each wave takes about 22 years to travel from where it begins at the pole to where it ends near the equator. However, the pattern shown in Fig. 4 comes from taking a difference between two measures of solar rotation, and this difference does not, therefore, indicate in absolute terms a speeding up or slowing down. The zones indicating faster or slower rotation at any particular time must be interpreted as faster or slower relative to the smooth (globally fit) differential rotation curve (see Fig. 3) determined at that same time.

In view of the identical 22-year periods, there is a connection between this surface velocity pattern and the magnetic activity cycle. In juxtaposition, the wave emerging at the pole first appears roughly at solar minimum and remains in polar regions until shortly before the subsequent minimum (*31*). It then begins its sweep toward the equator roughly as the next wave is forming at the pole (this is seen to be true even when the pattern, shown in projection in Fig. 4, is drawn on a sphere). The emergence of active regions has its centroid on the poleward side of the wave crest approaching the equator, which agrees with the basic idea in Babcock's original model. That is, the poleward side of the torsional wave crest is the region where the torsional wave enhances the differential rotation's shear and thus slightly tightens the winding up of the magnetic field lines.

The discovery of torsional waves was totally unanticipated by theorists, and it immediately set a number of them to work. Independent results (*32, 33*) showed that the α-ω dynamo models can be tailored to generate a single torsional wave that moves toward the equator, with the zone of activity on the poleward side of the torsional wave crest. However, these results did not explain the double-wave nature of the pattern or its polar origin. The models used in these studies indicated that, although the torsional wave may begin at latitudes higher than those where the greatest amount of magnetic activity would occur, it should be confined to latitudes less than about 45°.

In contrast, in self-consistent dynamo theories the dynamics comes from a detailed balance of forces, and torsional oscillations are of too high an order to be predicted. Furthermore, a feature independent of the model is that the 22-year period exhibited by the pattern means that the second half of the cycle begins during the first half, in the torsional wave near the pole. Although there is a flow of information between one half-cycle and the next, it is unclear why it should manifest itself in this way.

The regularity of the torsional pattern also met with a degree of suspicion. It was pointed out that such a pattern could arise as a mathematical artifact of the analysis itself (*34*). For example, in attempts to fit data containing a single isolated feature with a small set of smoothly varying fit functions (see Fig. 5), the fit must compromise between the functions adding together where the feature is present and canceling where it is absent. If this imperfect fit is subtracted from the actual data, then the difference tends to be a function with two peaks, one where the feature is present and the other toward the opposite side of the domain, where it is absent.

This objection was studied carefully (*35*). It was reasoned that, because the magnetic fields rotate more rapidly than the plasma, the regions of magnetic activity may actually drag the plasma along to some extent. An artificial data set was created by adding onto the smooth but differentially rotating background the single feature of a bulge in the rotational velocity whose amplitude was proportional to the total magnetic flux. This data set was analyzed by subtraction of fits. The result did contain the artifact anticipated earlier (*34*), but it was weak and in no way resembled the actual pattern seen in Fig. 4.

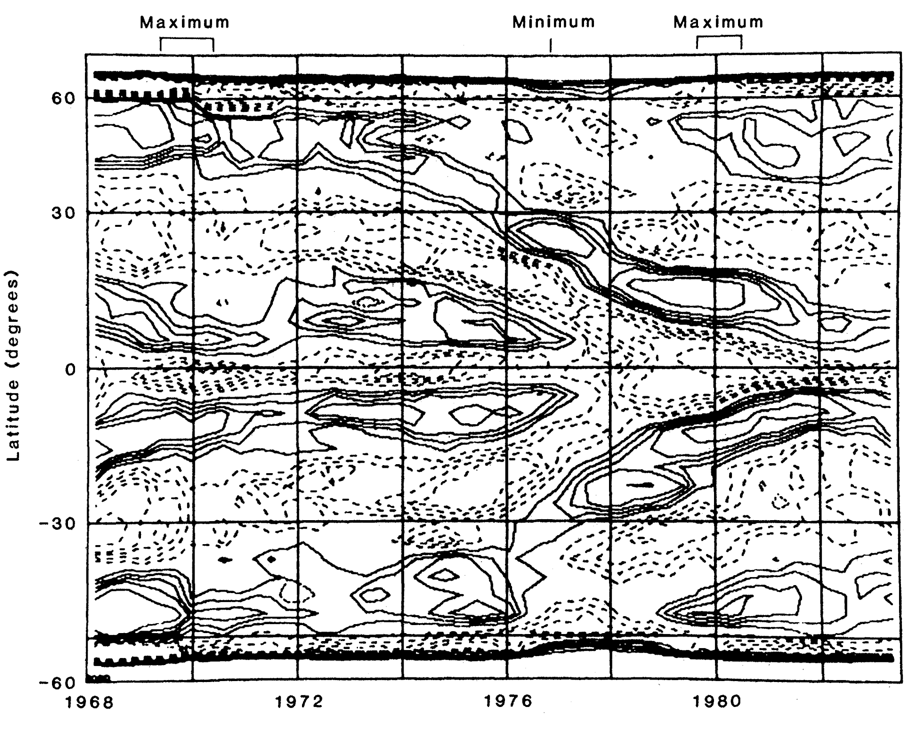


Fig. 4. A synoptic chart of the originally discovered torsional oscillation pattern found by subtracting the global fit from the zonal fits to solar rotation and averaging over periods of 20 Carrington rotations (~1.5 years). Each time period's results have also been adjusted so that the average over latitude in each hemisphere vanishes. Contour levels are ±1, 2, 3, 4, and 6 m sec^{-1}.

A Global Analysis of Torsional Oscillations

The smooth, continuous, pole-to-equator pattern seen in Fig. 4 suggests a global torsional oscillation mode having a wave number k of 2 per hemisphere (that is, two full waves per hemisphere). This suggests also that there are other such torsional modes, perhaps an infinite set. The dominance of $k = 2$ seen in this analysis might result either from the particular nature of the convective and turbulent driving forces that are exciting it or from the extraction of the lower modes along with the global fit to the differential rotation. The global fit for the standard data reduction represents the solar rotation as

$$\omega(\theta) = A + B\sin^2(\theta) + C\sin^4(\theta) \qquad (1)$$

where ω denotes the angular velocity in microradians per second and θ is the solar latitude. The coefficients A, B, and C are determined through a least-squares fit to each day's Doppler map.

That this fit does extract torsional oscillations has become apparent since the coefficients B and C have been found to oscillate with approximately 11-year periods (*12, 31*). The smoothness of the functions of Eq. 1 makes it clear that the extracted oscillations are of low mode, but their nonorthogonality makes further analysis difficult. A new global fit that uses orthogonal (that is, mathematically independent) functions (*36*) has been introduced. This fit replaces Eq. 1 by

$$\omega(\theta) = \bar{A} + \bar{B}T_2^1(\sin\theta) + \bar{C}T_4^1(\sin\theta) \qquad (2)$$

where the fit functions T_m^n belong to a class of functions known as Gegenbauer polynomials (see Fig. 5a). With these functions, the solar rotation is essentially represented as a sum of (i) rigid-body rotation ($k = 0$; $\bar{A}$ coefficient), (ii) one-half wave per hemisphere ($k = 1/2$; $\bar{B}$ coefficient), and (iii) one wave per hemisphere ($k = 1$; $\bar{C}$ coefficient). Thus most of the differential rotation shear is represented by the $k = 1/2$ term with modifications by the $k = 1$ term.

Because each fit is a linear combination of the other, they each span the same mathematical space. The new fit, however, clearly identifies the torsional modes that are being extracted. The values determined daily for the new coefficients are shown in Fig. 6. The higher degree of noise in the $\bar{A}$ coefficient before mid-1982 is the systematic error noted above. [In 1982, a new grating and exit-port slit assembly were installed at the telescope (*12*), and this error disappeared. Even with the error present, however, this is one of the most accurate Doppler-determined rates available.] The slight (~11-year) undulation of the coefficients $\bar{B}$ and $\bar{C}$, most evident for $\bar{C}$, represents standing waves with $k = 1/2$ and $k = 1$, respectively. When these modes are smoothed to reduce the random scatter and summed, they produce a flexing of the differential rotation curve with a flatter overall curve (that is, faster rotation at the poles in relation to the rate at the equator) at solar maximum (*37*). The amplitude is roughly 0.15 percent of the angular velocity, which is about 3 m sec^{-1} at the equator and is comparable to that for the $k = 2$ mode of Fig. 4.

The concept of torsional oscillation implies twisting, or change in shear. The $k = 0$ mode is not torsional in this sense, although it is of equal interest because all changes in the photospheric rotation rate are important to our understanding of the dynamics of the convection zone. The degree of noise in the $\bar{A}$ coefficient before 1982 prevents assessment of the behavior of this mode. However, a repeating pattern of cyclic variation in the sunspot rate was found (*23*) and seen to be correlated at least in part with yearly averages of $\bar{A}$. The variation was that faster rates of rotation occurred both at solar maximum and at solar minimum, with slower rotation between.

The Full Torsional Oscillation Pattern

A study of the systematic error shown in Fig. 5 suggests an alternative (*38*) to the original analysis of Howard and LaBonte (*29*). This error, which is large in the $\bar{A}$ coefficient and in the slopes (or zonal fits, the rotation rates determined separately in each strip of latitude), is essentially absent in the smaller $\bar{B}$ and $\bar{C}$ coefficients. Its amplitude is roughly proportional to the coefficient's size and is highly correlated from coefficient to coefficient. This suggests an instrumental problem in which (recall LaBonte's "gain knob" analogy) the random variation of a single instrumental parameter causes a proportional shifting of all the Doppler values. The effect is not apparent unless the coefficient is large enough for it to be above background solar noise (roughly the same for all coefficients); hence its absence in $\bar{B}$ and $\bar{C}$.

With this assumption, the daily variation of the $\bar{A}$ coefficient can be used to provide a daily value for the instrumental variation parameter that is part of the relative corrections to the slopes. In the absence of a reliable determination of a cyclic variation pattern for $\bar{A}$, however, the reference value from which these variations are determined must be the overall average of $\bar{A}$. Thus we are still

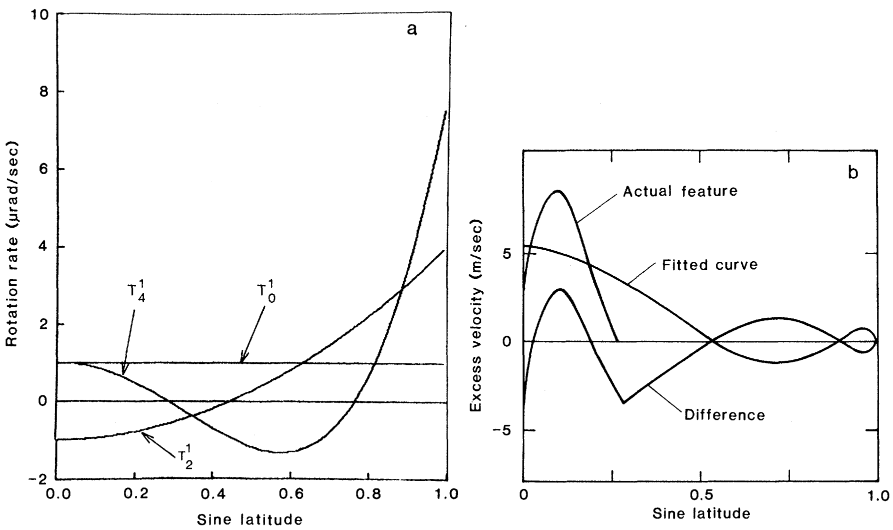


Fig. 5. An illustration of the production of a mathematical artifact by subtracting a global fit from a local phenomenon. (a) The functions used for fitting the data (see Eq. 2 in the text). (b) Appearances of a hypothetical localized feature in the data, the curve determined by the least-squares fit, and the remainder found in subtracting the fitted curve from the actual data.

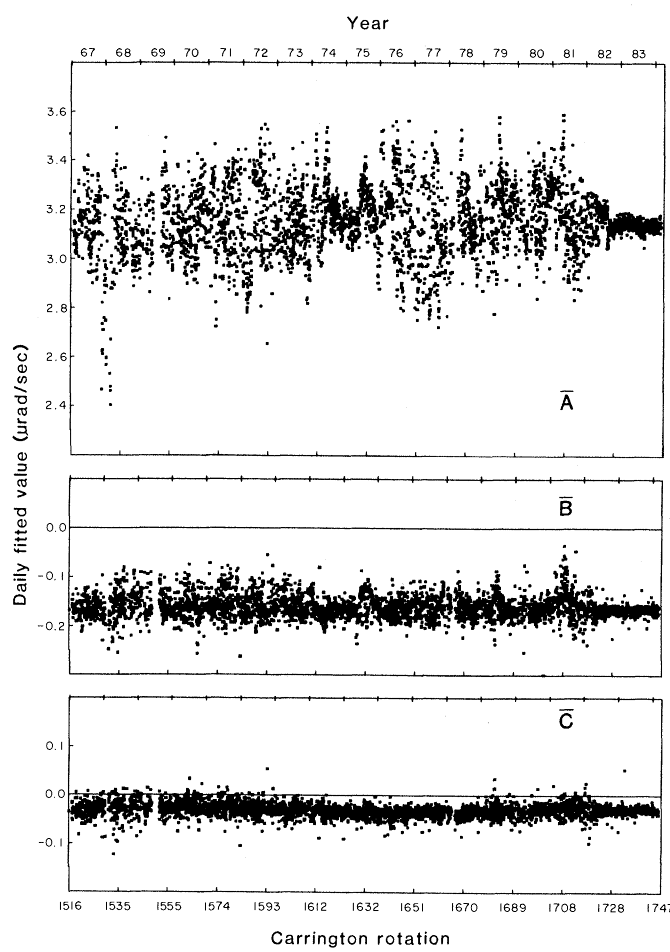


Fig. 6. Daily values for the coefficients $\bar{A}$, $\bar{B}$, and $\bar{C}$ for the period from 1 January 1967 to 5 February 1984. These values are determined by a least-squares fit of Eq. 2 to the observational data, after removing all known nonsolar effects. [From (*36*), courtesy of Reidel Publishing Company]

limited to determining the shear, but we do not extract any of the modes other than $k = 0$. In this sense, the pattern obtained is a full torsional oscillation pattern.

The result is given in Fig. 7, a and b. The full torsional pattern is not a double-wave pattern and appears to lack the global continuity in the double-wave pattern of Fig. 4. There is a speeding up of the rotation at high latitudes (that is, a polar spin-up) during and just after solar maximum; a single traveling wave then emerges at mid-latitudes (~45°) during solar minimum and moves toward the equator as the next cycle gets under way. This traveling wave creates a steepened shear in the differential rotation on its poleward side, and as it approaches the equator it grows in amplitude until the magnetic activity begins to break out along a path that is centered in the region of shear enhancement. The wave weakens and dies roughly at solar maximum but with the suggestion that its torsional impulse is transferred to the poles as the polar spin-up begins. The maximum torsional amplitude both in the polar spin-up and in the traveling wave is roughly 6 m sec^{-1}, or twice the amplitude of the $k = 2$ wave.

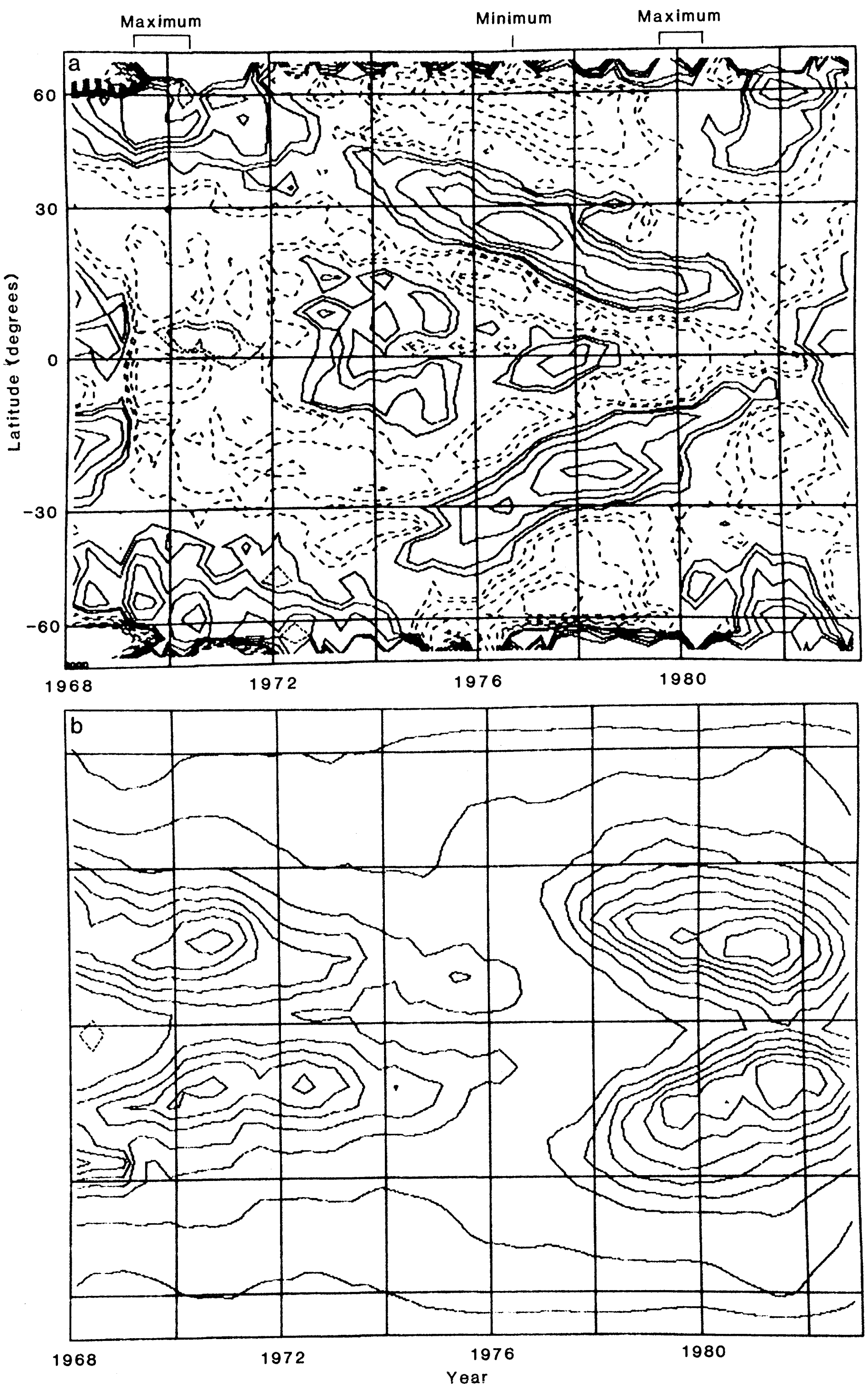


Fig. 7. Synoptic charts of (a) the full torsional oscillation pattern, determined by subtracting the overall average of the absolute differential rotation from the differential rotation curve determined from the corrected slopes, and (b) the total (positive plus negative) magnetic flux. All curves represent 20 Carrington rotation averages. The contour levels in (a) are ±2, 4, 6, 8, and 10 m sec^{-1}, and in (b) they are 0.5, 1, 2, . . . , 9×10^{20} Mx.

Are Torsional Oscillations a Global Phenomenon?

The use of global terminology in our description of torsional oscillations does not imply that we have shown them to be a global phenomenon. If they are, the decomposition into normal modes is unique. There is no guarantee—indeed, it is unlikely—that our functions of Eq. 2 are the functions appropriate for the representation of these modes. Thus, all assignments of wave numbers as given above should be regarded only as suggesting the shapes of the patterns of surface velocity and should not be taken as characterizing an actual normal mode decomposition. Furthermore, if Rosner and Brown were right (*34*), such a decomposition would be inappropriate.

An alternate way of producing the original $k = 2$ pattern of Fig. 4 is to fit the full pattern of Fig. 7a with Eq. 2 and then to subtract the fit (*38*). From a global point of view, the explanation seems simple: the full pattern is the superposition of global modes with $k = 0$, 1/2, 1, and so forth (all symmetric about the equator), and the fit contains the 0, 1/2, and 1 modes; thus the subtraction leaves a remainder dominated by $k = 2$. The discontinuity in the full pattern (the separation between polar spin-up and traveling wave) thus results from wave interferences among these various modes.

However, the production of the $k = 2$ pattern of Fig. 4 by subtracting a global fit from the full pattern, which in Fig. 7a appears to be more like a $k = 1$ pattern, is reminiscent of the demonstration (Fig. 5) of the production of a $k = 2$ pattern as an artifact. By examining Fig. 7a, one can see that Rosner and Brown's argument (*34*) was so easily dismissed because they did not include the polar spin-up at solar maximum (*39*). Furthermore, the model proposed in LaBonte and Howard's refutation assumed a bulge in the rotational velocity that was proportional to the magnetic flux and was thus wider and offset from the actual bulge in the full pattern (compare Fig. 7a with

7b). In the correct picture of Fig. 7a, we have exactly the pattern that can produce the $k = 2$ pattern of Fig. 4 as its artifact.

This does not prove, however, that it is an artifact. If it is, it arises from the alternation in the real pattern between the polar spin-up and the low-latitude traveling wave, each doubling to the opposite half of the hemisphere, and it is shaped further through the filling-in of separations in the full pattern where the global fit does not match perfectly the total time-averaged, zonally fit differential rotation curve. But in this case the continuity in Fig. 4 becomes an almost uncanny coincidence. However, if the pattern in Fig. 4 is the $k = 2$ mode (uncovered by extracting the lower modes) in a global torsional ringing of the sun, some serious questions remain. What has become of the $k = 3/2$ mode? Why is the $k = 2$ remainder a traveling wave when the modes extracted in the fit are standing waves? And how, since its amplitude is so weak (~3 m $\sec^{-1}$) in comparison with the amplitudes in the random turbulent motions (~300 m $\sec^{-1}$), is it able to survive and remain coherent during its 22-year passage from pole to equator?

If the pattern in Fig. 4 is an artifact, then one possibility is that the torsional oscillations are not a global phenomenon but rather a succession of more localized happenings that have been made, in the original analysis, to look global by being fitted with the globally defined mathematical functions of Eq. 1 or 2. Another possibility, mentioned earlier, is that they are a global phenomenon but are not being fit with functions that represent correctly the normal modes. Thus the full pattern of Fig. 7a may be essentially a $k = 1$ mode that is being straddled by the fit, with the fit subtraction simply revealing the straddling. A further study (*38*) that supports the artifact hypothesis has shown that the extraction of higher-order global fits, by including terms of higher order in Eq. 2, leads to higher-order continuous and wavelike patterns in the remainders, which appear to have pole-to-equator travel times of about $11 \times N$ years, where N is the order. These patterns are most complex near the poles, with a crossing-over of the higher modes by the lower modes, which all settle down to add constructively at low latitudes. Furthermore, the amplitudes of these higher modes do not decrease rapidly with N, as would be expected if Eq. 2 did provide a reasonable approximation to a modal decomposition of the torsional oscillations.

Abandoning the notion that the torsional oscillations are globally coherent on the solar surface would resolve the problem of the second half of the cycle being somehow determined or at least informed by a pattern of motions in the polar regions during the first half. The basic period would be about 11 years, as seen in Fig. 7. The waves would no longer traverse the fuel hemisphere, which would correspond better to the predictions of the α-ω models (*32, 33*). However, even if the torsional patterns of polar spin-up and traveling wave in Fig. 7 are not globally coherent on the surface, they may still be causally connected. Just as the traveling wave seems to send a rotational spin-up toward the poles as it ends, so also the polar spin-up may create an internal turbulence that moves toward the equator and emerges later as the traveling wave.

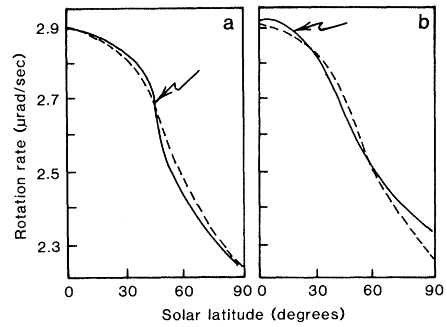


Fig. 8. Schematic illustration of the changing differential rotation profile during the activity cycle. The features are exaggerated for clarity, and each curve is given in comparison with an average differential rotation curve (dashed line). (a) Approximately 1 year after solar minimum. (b) Solar maximum. The arrows indicate the centroid of magnetic activity.

The Relation Between Torsional Oscillations and the Magnetic Cycle

The high conductivity of the solar plasma relates the turbulent and rotational atmospheric motions to the generation and motion of the magnetic field patterns; whether or not a global description of the torsional oscillations is appropriate, the net motion of the plasma is the important feature. Therefore, it is the full pattern (Fig. 7a) that should be studied in relation to the magnetic cycle (Fig. 7b).

In a rotating object, there is a natural tendency due to frictional forces for the angular velocity to be everywhere the same. It is widely held that the principal cause for the sun's differential rotation comes from its internal fluid dynamics through the combination of energy outflow and Coriolis effects. Self-consistent dynamo theories infer a causal link in the giant cells whose existence they predict, but, as noted above, no traces of these giant cells have been detected in the observations (*40*). The question therefore remains open.

There is also an interaction of the rotating sun with the interplanetary medium primarily through the magnetic fields that extend far out into space and sweep through the medium as the sun turns. This creates a drag on the rotation, and one might imagine that the absence of this drag as the polar fields pass through zero at solar maximum could account for the coincident polar spin-up. But the coupling is orders of magnitude too weak to produce any of the effects that we are considering. Furthermore, the nature of the solar convection zone (*41*) is such that even a localized external influence would produce a global $k = 0$ effect—that is, the sun would respond (on a time scale of roughly a year) essentially as a rigid body. Any localized, or $k > 0$, persistent pattern, including the differential rotation itself, must be maintained by internal dynamics.

However, as noted earlier, the overall rotation rate does show an increase both at solar maximum and at solar minimum (*23*). Because at both times a significant portion of the sun's magnetic field is absent, it may be, in fact, that an external magnetic influence takes part. It has been argued that, in the case of star formation, the reaction to the magnetic acceleration of bipolar winds can account fully for the needed reduction of the specific angular momentum in the accretion disk of an evolving protostar (*42*). If the generally accepted notion that the solar magnetic field is deeply rooted (*43*) could be abandoned, so that only a tiny fraction of the sun's total angular momentum takes part, a similar effect could be inferred.

The original Babcock model shows a way in which the global magnetic fields of the magnetic dipole at solar minimum can be twisted by the differential rotation so that they later erupt with great intensity at low latitudes (*17*). The traveling wave increases the steepness of the differential rotation on its poleward side and thus enhances the twisting of the field (Fig. 7a). This steepening, shown schematically in Fig. 8, is not so dramatic that it produces a maximum slope in the differential rotation curve. It may, however, provide the necessary extra push to thrust the already highly twisted field outward from the photosphere. That is, the traveling torsional wave may serve as a sort of catalyst. This may help clarify why the torsional wave precedes in both latitude and time the outbreak of magnetic activity.

Our understanding of this phenomenon is far from complete. It is not known, for example, whether the traveling wave is more or less uniformly distributed on the solar surface at any particular time, or whether it really consists of localized regions that move abnormally so that the averaged motion appears as a torsional wave. There are two reasons for suggesting this alternate possibility: (i) during the cycle, regions of magnetic activity erupt over and over in the same place (*44*), and (ii) there is in the rotation data an autocorrelation pattern that seems to be strongest when the traveling wave is strongest (*45*).

The resolution of these questions depends on a better theoretical understanding of the solar dynamo. If the torsional oscillations are not just smeared-out averages of locally eruptive phenomena, then they must be deeply rooted if they are to survive the surface turbulence on the sun. If they are a globally coherent phenomenon, an important question is whether the length of the cycle is determined by the natural periods of the modes that are excited by the internal driving forces or, alternately, whether the modes that are excited are those that have the right period. It would be useful, therefore, to analyze the possible normal modes of torsional oscillation near the solar surface.

Ultimately, however, it will be the observations that decide the issue. The amplitude for the torsional oscillations is so small that, for most of the data collected so far, their detection depends on extensive averaging and complicated statistical analysis. Accurate tracking of the absolute rate over a full cycle is needed, and this, as seen in Fig. 6, is now possible because of the improvements to the spectrograph at the Mount Wilson 150-foot tower telescope. The instrument now seems to be optimized because, in the new data (collected since the 1982 improvements), the noise level is now roughly what would be expected from the known small-scale velocity fields on the sun itself. The accuracy is now sufficient (*46*) to examine the torsional oscillations without having to do a subtraction of fits.

References and Notes

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