Reports

Around-the-World Relativistic Sagnac Experiment

Abstract. *In 1971 Hafele and Keating carried portable atomic clocks east and then west around the world and verified the Sagnac effect, a special relativity effect attributable to the earth's rotation. In the study reported here observations of the effect were made by using electromagnetic signals instead of portable clocks to make clock comparisons. Global Positioning System satellites transmit signals that can be viewed simultaneously from remote stations on the earth; thus an around-the-world Sagnac experiment can be performed with electromagnetic signals. The effect is larger than that occurring when portable clocks are used. The average error over a 3-month experiment was only 5 nanoseconds.*

Time signals from satellites having accurate atomic clocks on board now provide opportunities for comparing time standards worldwide. Since Global Positioning System (GPS) satellites can be observed from any location on the earth's surface, and in fact can be viewed simultaneously from two or more remotely situated sites where time standards may be located, it is possible to synchronize clocks by utilizing electromagnetic signals, which effectively close on themselves as they circumnavigate the globe (*1*, *2*).

In a rotating system of reference, such as on the rotating earth, the principle of the constancy of the speed of light leads directly to the prediction that light (or electromagnetic signals) traversing a closed path will take a different amount of time to complete the circuit depending on whether the direction of propagation is in the general direction of, or opposite to, the rotation. This has been observed in the laboratory (*3*) and is known as the Sagnac effect (*4*). The Sagnac effect is the basis for the sensitive ring-laser gyroscope (*5*).

It can be shown that the same effect occurs when one uses slowly moving portable atomic clocks on the earth's surface to synchronize remote time standards. A portable clock transported slowly eastward once around the earth's equator will lag a master clock at rest on the earth's surface by 207.4 nsec; if transported westward the portable clock will lead by about 207.4 nsec (*6*). This effect was actually observed by Hafele and Keating (*7*), who used commercial jet aircraft to transport an ensemble of cesium clocks eastward and then westward around the globe.

The Sagnac effect has the same form and magnitude whether slowly moving portable clocks or electromagnetic signals are used to complete the circuit. For slowly moving portable clocks, the effect can be viewed from a local nonrotating geocentric reference frame as being due to a difference between the second-order Doppler shift (time dilation) of the portable clock and that of the master clock whose motion is due to the earth's rotation. For electromagnetic signals the effect arises from a well-known consequence of the special theory of relativity—the relativity of simultaneity—which follows from the principle of the constancy of the speed of light. If we imagine two clocks fixed a small east-west distance x apart on the earth, then viewed from the nonrotating frame they will be moving with approximately equal speeds $v = \omega r$, where ω is the angular rotation rate of the earth and r is the distance of the clocks from the rotation axis. If a clock synchronization process involving electromagnetic signals were carried out by earth-fixed observers who ignored the earth's rotation, then the two clocks would not be synchronous when viewed from the nonrotating frame. The magnitude of the discrepancy is approximately $vx/c^2 = (2\omega/c^2)\,(rx/2)$. In general, such discrepancies depend on the path along which the light signals travel relative to the rotation axis. An acceptable way to avoid this problem is to synchronize the clocks in the nonrotating frame.

Thus in synchronizing clocks on the surface of the rotating earth by means of electromagnetic signals or portable clocks, it is necessary to apply a correction, arising from the Sagnac effect, to the clock readings in order to avoid problems of nontransitivity of the synchronization process. The Consultative Committee for the Definition of the Second and the International Radio Consultative Committee have agreed that, in order to obtain consistently synchronized clocks on the earth's surface at the subnanosecond level, the correction term to be applied is of the form

$$\Delta t = 2\omega/c^2 \times A_E$$
$$= 1.6227 \times 10^{-21}\ \mathrm{sec/m^2}\ A_E$$

where A_E is the projected area on the earth's equatorial plane swept out by the vector whose tail is at the center of the earth and whose head is at the position of the portable clock or the electromagnetic signal pulse. The A_E is taken as positive if the head of the vector moves in the eastward direction. In other words, if two clocks located on the earth's surface are compared by using portable clocks or electromagnetic signals in the rotating frame of the earth, then Δt must be

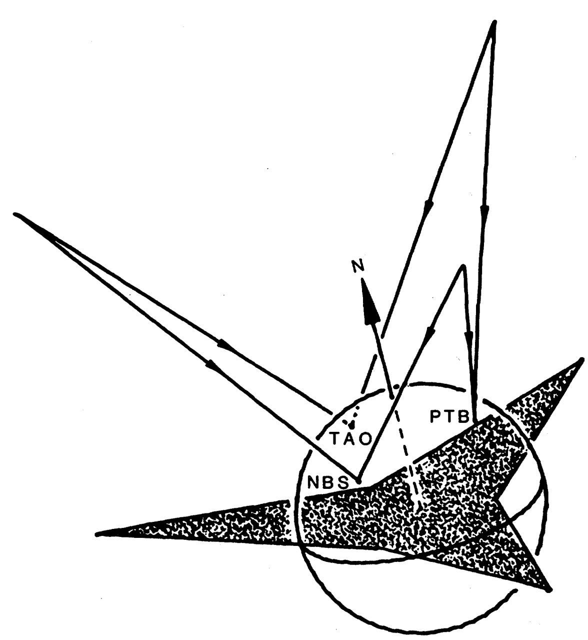

Fig. 1. Diagram showing the paths of electromagnetic signals from the three satellites, observed in common view, to the three earth timing centers. The magnitude of the Sagnac effect is proportional to the shaded area enclosed by the projections of the electromagnetic signal paths on the equatorial plane of the earth.

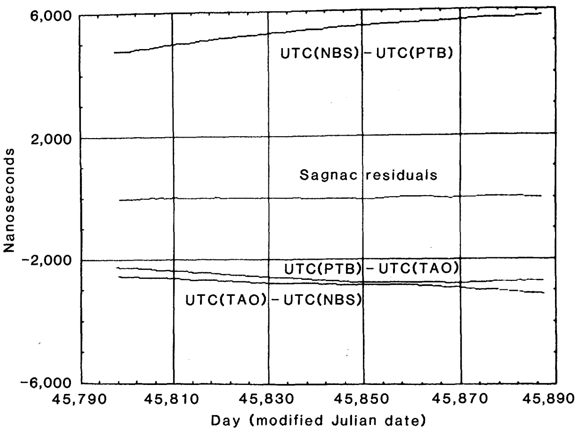

Fig. 2. Measured time differences between master clocks at NBS, PTB, and TAO, as determined with the GPS common-view measurement technique. Also plotted is the Sagnac residual, which is the sum of the three pairs of time differences. The mean value of the Sagnac residual over the 90-day observing period was 5 nsec. UTC, universal time coordinated.

subtracted from the measured time difference (east clock minus west clock) in order to synchronize the clocks so they will measure coordinate time on the earth.

In the experiment reported here signals from GPS satellite vehicles 3, 4, 6, and 8 were utilized in simultaneous common view between three pairs of earth timing centers to accomplish the circumnavigation. The centers were the National Bureau of Standards (NBS) in Boulder, Colorado; Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, West Germany; and Tokyo Astronomical Observatory (TAO) (*8*). A typical geometrical configuration of ground stations and satellites, with the corresponding projected area, is illustrated in Fig. 1. The size of the Sagnac effect calculated from the expression above varies from about 240 to 350 nsec depending on the location of the satellites used in a circumnavigation carried out at a particular moment.

Enough data were collected to perform 90 independent circumnavigations. The data are plotted in Fig. 2, showing the time difference measured between each of the three sites via the GPS signals. The receivers used in this experiment automatically subtract the Sagnac correction, hence if the three pairs of measured time differences are added the result should vanish. The sum of the three pairs is also plotted in Fig. 2 as the Sagnac residual; this residual is obviously very near zero. The actual mean value of the Sagnac residual over the 90 days of observation was 5 nsec, which is less than 2 percent of the magnitude of the calculated total Sagnac effect.

Even though the atomic clocks used in the experiment were among the world's best, they are perturbed by natural random processes. The net time dispersion for this experiment attributable to these perturbations on the three clocks is about 2.5 nsec. The remainder of the non-null result is explained by uncertainties in the propagation delays and in the ephemerides of the GPS satellites.

It is also of interest to observe the clock rate measurement consistency, as the sum of the three pairs of clock rate differences should also be zero. The residual in this case was two parts in 10^{15}. This measurement residual is one order of magnitude smaller than the inaccuracy of the best atomic clocks in the world. This leads to the very important conclusion that, with integration time of 1 week or longer, the GPS common-view measurement technique allows one, for the first time in the history of atomic clocks, to have access to the most accurate clocks in the world at any other site without being limited by measurement uncertainties (*2*).

In conclusion, the theoretical predictions have been well verified by observation. Although the Sagnac effect has been observed frequently in laboratory settings, this is, to our knowledge, the first time it has been observed on such a large scale.

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References and Notes

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Catastrophic Anoxia in the Chesapeake Bay in 1984

Abstract. *In 1984, four climatic sequences combined to produce what may be a major anoxic catastrophe in the northern Chesapeake Bay, sufficient to severely threaten the major benthic species. These sequences are (i) the highest late-winter streamflow on record from the Susquehanna River watershed; (ii) streamflows from the Susquehanna River for the consecutive months of June, July, and August that are higher by 2 standard deviations than the respective monthly mean values measured over the last 34 years; (iii) a stationary high in August off the Atlantic Coast; and (iv) an absence of strong storm events in summer. An empirical equation is proposed for the prediction of the monthly trend of dissolved oxygen decrease in terms of a temperature-dependent subpycnoclinal respiration and a modified estuarine Richardson number. As of 23 August 1984, the summer pycnocline of the northern bay had eroded upward from its historically recorded depth below 10 meters to an abnormally shallow 5 meters, with higher stratification than in earlier years. Dissolved oxygen concentrations directly below the pycnocline decreased to zero during June, 2 months earlier than for previous wet years. At present, oxygen-deficient waters containing significant concentrations of hydrogen sulfide have penetrated into Eastern Bay and the Choptank and Potomac rivers. Because most remaining shellfish-spawning and seed-bed areas in these tributaries are located at depths between 4 and 8 meters, the continued absence of major destratifying events will prolong the present anoxic trend and may result in high benthic mortalities.*

Low-oxygen concentrations in deep waters in many stratified lakes, fjords, and estuaries occur when the oxygen utilization rates exceed the oxygen replenishment rates across a pycnocline brought about by turbulent vertical mixing. This phenomenon occurs annually in summer in the Chesapeake Bay (*1–3*). In 1976 an extensive anoxic catastrophe occurred in the New York Bight, leading