Book Reviews

Singular Spaces

Isolated Singular Points on Complete Intersections. E. J. N. LOOIJENGA. Cambridge University Press, New York, 1984. xii, 200 pp. Paper, $22.95. London Mathematical Society Lecture Note Series, 77.

In mathematics the word "singular" is often used as an antonym for "smooth." Most mathematicians prefer to study smooth geometric objects, those that are locally indistinguishable from affine space over the real or the complex numbers, since, for instance, one can integrate and differentiate on them. However, both in science and in mathematics one is often forced to consider singular spaces. A standard technique for dealing with them is to "deform" them, to perturb slightly the defining equations of the space so it becomes smooth. One then studies the resulting one-parameter family obtained by letting the smooth space degenerate back to the singular one. (In the theory of singularities of mappings this is known as "unfolding" the singularity.)

Although the study of singularities in mathematics goes back very far, especially in algebraic geometry, the field only emerged in its modern form in the 1960's. It was shaped by the work of Whitney, Thom, Mather, and Arnold on singularities of mappings; by the remarkable discovery of Brieskorn that "exotic spheres" (that is, spaces that are topologically but not differentiably spheres) arise naturally from very simple algebraic singularities and by the ensuing seminal work of Milnor on the topology of singularities and the work of Kodaira-Spencer and Grothendieck on "moduli," or parameter spaces.

The singular spaces studied in Looijenga's book are those given as the common zeroes of a certain number, say k, of polynomials in complex affine space of dimension, say, n. Looijenga assumes that the singular spaces are "complete intersections," which means that their dimension is $n - k$. (The simplest example of a complete intersection is a hypersurface, which is defined by a single equation.) This assumption is natural for both practical and mathematical reasons: a complete intersection is the only case for which there is a mature deformation theory, and the assumption that singular spaces are complete intersections allows one to view the singularity as the zero set of a mapping from n space to k space and to apply the theory of singularities of mappings mentioned above.

John Milnor's 1968 book *Singular Points of Complex Hypersurfaces* (Princeton University Press) had a decisive influence on the field, providing both the main technical tool (now called the Milnor fibration) and a clear exposition of the whole subject. It remains the best introduction to the field, but it is now somewhat out of date. Since 1968 there have been expository articles on the subject, most notably by Brieskorn, Arnold, and Teissier, but no systematic account of it.

Looijenga's book provides such an account. It is a virtually self-contained, clearly written introduction to the field. It also presents in detail many interesting examples. It is very much a book for the professional mathematician: it is intended for those with at least two years of graduate training, and the intrinsic mathematical beauty of the subject is the only reason to read the book. The examples are all mathematical. There is no description (and, indeed, it would be inappropriate in such a book) of how singularities arise in science, such as is given in V. I. Arnold's beautiful *Catastrophe Theory* (Springer-Verlag, 1984; translated from the Russian edition). It would have been appropriate to include some figures, however. The lack of illustration is the one drawback of the book.

When one compares the books by Milnor and Looijenga, one is struck by how much the subject has changed in 15 years. There is the generalization from hypersurfaces to complete intersections, of course. More important, Looijenga considers in detail the "versal deformation space" parametrizing all possible ways of deforming the singularity, rather than just a one-parameter deformation. This greatly enriches the theory. The discriminant, that is the set of points in the versal deformation space where the fiber is singular, is a hypersurface. The study of its structure and the topology of its embedding into the versal deformation has proved to be a very fruitful line of inquiry in recent years. Looijenga also describes the Gauss-Manin connection, which he himself has used so successfully to construct period maps. There is a good bibliography.

The technical complexity of the book is quite daunting, and the interested reader should definitely read Milnor's book first. Not only is it more elementary, it is one of the jewels of the mathematical literature. Suffice it to say that Looijenga's book is a worthy companion to it.

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Stellar Convection

The Solar Granulation. R. J. BRAY, R. E. LOUGHHEAD, and C. J. DURRANT. Second edition. Cambridge University Press, New York, 1984. xvi, 256 pp., illus. $54.50. Cambridge Astrophysics Series.

The only star sufficiently close to the earth to permit a detailed investigation of its surface is the sun. The solar surface provides a natural laboratory for studying processes occurring in a hot, highly stratified plasma. It permits direct observational verification of theories of astrophysical convection.

Energy generated in the interior of the sun is transported outward by radiation, conduction, waves, and convection. Just below the visible atmosphere of the sun the bulk of the energy flux is transported by convection. Within the solar atmosphere, which is stable against convection, the mode of energy transport rapidly switches to radiation. However, convective elements overshoot into the stable atmosphere and, since they contain excess heat, give it a granular appearance. Waves generated in the solar convection zone and by the penetration of convective elements into the stable atmosphere probably heat the overlying atmospheric layers. A thorough understanding of the solar granulation is necessary in order to understand stellar atmospheres and their activity.

The solar granulation presents us with two difficult challenges. The first is to derive the properties of the granules from observations. The second is to explain these properties by means of theoretical models of astrophysical convection. Following a brief historical summary of early attempts to observe and explain the granulation, *The Solar Granulation* presents in-depth discussions of these two problems.

The authors provide a very comprehensive view of the progress made in observing the solar granulation between 1967, when the first edition of this book was published, and 1982, the most recent year for which developments are considered. Observations of the solar granulation are not definitive for several reasons. A major reason is that the spatial scale associated with the granules lies near the resolution limit imposed by instrumentation and by turbulence in the earth's atmosphere. Attempts to overcome limitations of resolution are thoroughly discussed. Most information about velocities, temperatures, pressures, and densities within the granulation comes from analyzing spectral lines formed in the solar atmosphere. Unfortunately, line profile diagnosis of the highly inhomogeneous solar atmosphere is not straightforward. No uniqueness is associated with physical quantities obtained by inverting line profile observations. Particular profiles can be synthesized by adjusting either temperature or velocity distributions. A related problem is introduced by the simultaneous existence of several dynamical processes in the solar atmosphere. Granular effects must be separated from those caused by oscillations and waves. However, these processes overlap in both their temporal and their spatial behavior. A figure in the book showing the height dependence of the velocity fluctuations is a good example of discrepancies introduced by using different methods to separate these motions and to interpret the observed line profile shifts. The book presents a lively account of these problems and is careful to point out where controversies exist.

The Solar Granulation provides a basic introduction to the theory of convection, including the two most widely used approximations, the anelastic and the Boussinesq, and describes problems that are encountered when the convecting medium becomes highly turbulent. Theories of astrophysical convection are described, and the problems encountered when radiative heat exchange between convecting elements becomes important are discussed. Two models of convection that directly attempt to explain the granulation are treated in more detail. Although the authors attempt to make this treatment of convective theory accessible to nonspecialists, it would be difficult to follow without previous experience in fluid dynamics and prior knowledge of the literature. An attempt to distinguish between the well-ordered appearance of the granulation and the more chaotic character of smaller-scale turbulence is confusing. The more accepted idea that the granules are best described as the energy-containing eddy of a turbulent convective spectrum containing a wide range of sizes is not presented. Several of the more developed schools of thought on stellar convection (such as that of Unno and his colleagues in Japan) are not well represented. However, the authors do provide a comprehensive picture of the great difficulties encountered in developing a convective model and of the shortcomings of current models in explaining the solar granulation.

The book concludes by discussing how some properties of the granulation can be interpreted using granular models and by describing current efforts to use knowledge of the solar granulation to deduce information about similar processes on stars. Researchers in this rapidly expanding field will find this latter information very useful. The book should serve as an excellent introduction to those wishing to study the dynamical properties of stellar atmospheres and as a comprehensive guide to research on the solar granulation. It is a must for any complete astronomy library.

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A Diverse Group of Wetlands

European Mires. PETER D. MOORE, Ed. Academic Press, New York, 1984. x, 367 pp., illus. $75

Mires include all peat-forming ecosystems, and therefore this book deals with a very diverse group of wetlands. It is the second major book on peatlands published in about a year. The first book, *Mires: Swamp, Bog, Fen and Moor*, edited by A. J. P. Gore (Elsevier), was in two volumes: one dealing with conditions and processes and the other describing mires in different parts of the world, especially with respect to regional variation. *European Mires* overlaps to some extent with parts of the second of these volumes, but the differences are more important. In general, it provides much more detail on the floristic composition of mires than the volumes edited by Gore, but much less on processes. The nine chapters by 15 contributors deal primarily with the phytosociology of mires (80 percent) and with stratigraphy and palynology (17 percent), with little space devoted to other topics such as chemistry and hydrology.

A paper on the classification of Finnish mires by Eurola, Hicks, and Kaakinen is by far the longest contribution, almost a third of the book, and certainly the most informative. A 23-page introduction, based mainly on Eurola's earlier work, is devoted mostly to the environmental factors that cause the differences in mire vegetation. The introduction contains many insightful diagrams. In several cases an effort is required to grasp the contents of the diagrams, but all are packed with information. Finnish mire ecology has retained its own identity in spite of a strong Swedish influence, primarily that of Du Rietz, Granlund, Osvald, and Sjörs. For readers familiar with the Scandinavian peatland literature, most of it in German, Finnish, and Swedish, this section is a valuable summary of current thought in Finnish mire ecology that includes some new ideas; for others it will provide interesting insights into the environmental control of mire vegetation. The main part of this chapter consists of a key to a description of the Finnish mire types. A glossary of terms and an appendix with the names of Finnish mire types and their German and English equivalents are useful additions.

A paper on Estonian bogs by Masing is perhaps most important for its clear illustration of the value of complementary approaches and different scales in describing the vegetation of peat bogs. Masing also shows that the concept of synusiae (one-layered communities, based on Lipmaa's ideas) can be used to advantage in studies of productivity as well as in describing the vegetation.

Rybníček gives an overview of the vegetation of the mires of the central European mountains, from the Alps to the Carpathians. The continental and oceanic influences in this vegetation are small, and affinities are mostly with the boreal and subarctic mires. The other papers supply details on the mires of other parts of Europe.

In a book on European mires, one would expect a better geographical balance. About 44 percent of the book deals with the British Isles, 30 percent with Finland, 15 percent with central Europe, and 8 percent with the Baltic States. There is no information on the mires of European Russia or on those of the northwest European plain from France to East Germany. Not much is left of the latter, it is true, but a compilation of what is known is much needed, and this volume would have been the appropriate place for it. The range of peatlands discussed varies also from paper to paper. Masing's paper is restricted to the bogs of Estonia, whereas in a chapter on wet-