Book Reviews

Singular Spaces

Isolated Singular Points on Complete Intersections. E. J. N. LOOIJENGA. Cambridge University Press, New York, 1984. xii, 200 pp. Paper, $22.95. London Mathematical Society Lecture Note Series, 77.

In mathematics the word "singular" is often used as an antonym for "smooth." Most mathematicians prefer to study smooth geometric objects, those that are locally indistinguishable from affine space over the real or the complex numbers, since, for instance, one can integrate and differentiate on them. However, both in science and in mathematics one is often forced to consider singular spaces. A standard technique for dealing with them is to "deform" them, to perturb slightly the defining equations of the space so it becomes smooth. One then studies the resulting one-parameter family obtained by letting the smooth space degenerate back to the singular one. (In the theory of singularities of mappings this is known as "unfolding" the singularity.)

Although the study of singularities in mathematics goes back very far, especially in algebraic geometry, the field only emerged in its modern form in the 1960's. It was shaped by the work of Whitney, Thom, Mather, and Arnold on singularities of mappings; by the remarkable discovery of Brieskorn that "exotic spheres" (that is, spaces that are topologically but not differentiably spheres) arise naturally from very simple algebraic singularities and by the ensuing seminal work of Milnor on the topology of singularities and the work of Kodaira-Spencer and Grothendieck on "moduli," or parameter spaces.

The singular spaces studied in Looijenga's book are those given as the common zeroes of a certain number, say k, of polynomials in complex affine space of dimension, say, n. Looijenga assumes that the singular spaces are "complete intersections," which means that their dimension is $n - k$. (The simplest example of a complete intersection is a hypersurface, which is defined by a single equation.) This assumption is natural for both practical and mathematical reasons: a complete intersection is the only case for which there is a mature deformation theory, and the assumption that singular spaces are complete intersections allows one to view the singularity as the zero set of a mapping from n space to k space and to apply the theory of singularities of mappings mentioned above.

John Milnor's 1968 book *Singular Points of Complex Hypersurfaces* (Princeton University Press) had a decisive influence on the field, providing both the main technical tool (now called the Milnor fibration) and a clear exposition of the whole subject. It remains the best introduction to the field, but it is now somewhat out of date. Since 1968 there have been expository articles on the subject, most notably by Brieskorn, Arnold, and Teissier, but no systematic account of it.

Looijenga's book provides such an account. It is a virtually self-contained, clearly written introduction to the field. It also presents in detail many interesting examples. It is very much a book for the professional mathematician: it is intended for those with at least two years of graduate training, and the intrinsic mathematical beauty of the subject is the only reason to read the book. The examples are all mathematical. There is no description (and, indeed, it would be inappropriate in such a book) of how singularities arise in science, such as is given in V. I. Arnold's beautiful *Catastrophe Theory* (Springer-Verlag, 1984; translated from the Russian edition). It would have been appropriate to include some figures, however. The lack of illustration is the one drawback of the book.

When one compares the books by Milnor and Looijenga, one is struck by how much the subject has changed in 15 years. There is the generalization from hypersurfaces to complete intersections, of course. More important, Looijenga considers in detail the "versal deformation space" parametrizing all possible ways of deforming the singularity, rather than just a one-parameter deformation. This greatly enriches the theory. The discriminant, that is the set of points in the versal deformation space where the fiber is singular, is a hypersurface. The study of its structure and the topology of its embedding into the versal deformation has proved to be a very fruitful line of inquiry in recent years. Looijenga also describes the Gauss-Manin connection, which he himself has used so successfully to construct period maps. There is a good bibliography.

The technical complexity of the book is quite daunting, and the interested reader should definitely read Milnor's book first. Not only is it more elementary, it is one of the jewels of the mathematical literature. Suffice it to say that Looijenga's book is a worthy companion to it.

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Stellar Convection

The Solar Granulation. R. J. BRAY, R. E. LOUGHHEAD, and C. J. DURRANT. Second edition. Cambridge University Press, New York, 1984. xvi, 256 pp., illus. $54.50. Cambridge Astrophysics Series.

The only star sufficiently close to the earth to permit a detailed investigation of its surface is the sun. The solar surface provides a natural laboratory for studying processes occurring in a hot, highly stratified plasma. It permits direct observational verification of theories of astrophysical convection.

Energy generated in the interior of the sun is transported outward by radiation, conduction, waves, and convection. Just below the visible atmosphere of the sun the bulk of the energy flux is transported by convection. Within the solar atmosphere, which is stable against convection, the mode of energy transport rapidly switches to radiation. However, convective elements overshoot into the stable atmosphere and, since they contain excess heat, give it a granular appearance. Waves generated in the solar convection zone and by the penetration of convective elements into the stable atmosphere probably heat the overlying atmospheric layers. A thorough understanding of the solar granulation is necessary in order to understand stellar atmospheres and their activity.

The solar granulation presents us with two difficult challenges. The first is to derive the properties of the granules from observations. The second is to explain these properties by means of theoretical models of astrophysical convection. Following a brief historical summary of early attempts to observe and explain the granulation, *The Solar Granulation* presents in-depth discussions of these two problems.

The authors provide a very compre-