deeper parts of the Straits. An analysis of the profiling data shows that the root-mean-square of the variations in transport is 3.1 Sv due to strength changes and 0.7 Sv due to velocity structure changes. Occasionally, such as during 1982 Julian day 164–169, the intensity and structure changes are nearly equal. Even during this event the cable tracked the transport. One expects that the cable voltage will be better correlated with conductivity-weighted transport than with the simple volume transport. In fact, there is no significant difference, and we chose to express the cable voltage in terms of volume transport.
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Sea Level Variation as an Indicator of Florida Current Volume Transport: Comparisons with Direct Measurements

Abstract. *Sea level measurements from tide gauges at Miami, Florida, and Cat Cay, Bahamas, and bottom pressure measurements from a water depth of 50 meters off Jupiter, Florida, and a water depth of 10 meters off Memory Rock, Bahamas, were correlated with 81 concurrent direct volume transport observations in the Straits of Florida. Daily-averaged sea level from either gauge on the Bahamian side of the Straits was poorly correlated with transport. Bottom pressure off Jupiter had a linear coefficient of determination of* $r^2 = 0.93$*, and Miami sea level, when adjusted for weather effects, had* $r^2 = 0.74$*; the standard errors of estimating transports were* $\pm 1.2 \times 10^6$ *and* $\pm 1.9 \times 10^6$ *cubic meters per second, respectively. A linear multivariate regression, which combined bottom pressure, weather, and the submarine cable observations between Jupiter and the Bahamas, had* $r^2 = 0.94$ *with a standard error of estimating transport of* $\pm 1.1 \times 10^6$ *cubic meters per second. These results suggest that a combination of easily obtained observations is sufficient to adequately monitor the daily volume transport fluctuations of the Florida Current.*

Monthly changes in the Florida Current volume transport have been related to departures in monthly mean sea level from the historically averaged monthly mean sea level at certain coastal stations (*1*). In the Straits of Florida, monthly mean sea level departures from the historical mean at Miami or at Key West were much larger than at Cat Cay, Bahamas. It was reasoned that changes in the mean sea level at Cat Cay reflect changes in the western Sargasso Sea, whereas at Miami and Key West sea level changes reflect variations in the surface discharge through the Straits (*2*). Techniques of spectrum analysis have also been applied to hourly sea level records and to bottom pressure records in the Straits (*3*), but correlations with volume transport were generally inconclusive because of a lack of accurate transport measurements. Recent advances in navigation and current profiling (*4*) provide transport measurements of sufficient accuracy with which to reassess the role of sea level as an indicator of Florida Current variability.

Sea level slope is an indirect but generally accepted first-order measure of ocean surface current velocity in extratropical regions. The basis for this measure is the relationship

$$f[V(0) - V_a(0)] = g\, dh/dx$$

in which $V(0)$ is the south-north surface velocity, $V_a(0)$ is the ageostrophic surface velocity and includes temporal and frictional effects, g is gravity, f is the Coriolis parameter, dh is the differential height of the sea above an equipotential surface where the vertical coordinate $z = 0$, and dx is the differential east-west dimension. The average surface current across a section is

$$\langle V(0)\rangle = \Delta hg/fL + \langle V_a(0)\rangle$$

where L is the width of the section at the surface and Δh is the difference in the sea level height. The distribution of northward flow in the Straits may be written as

$$V(x,z) = \langle V(z)\rangle + V'(x,z)$$

where

$$\iint V'(x,z)dxdz = 0$$

Then the true volume transport (T) may be expressed as

$$T = \int_0^H \int_a^b V(x,z)dxdz = \Delta hg/fL \left[\int_0^H (b-a)G(z)dz\right] + \int_0^H (b-a)\langle V_a(z)\rangle\, dz \qquad (1)$$

where $(b - a)$ arises from application of the mean value theorem to the cross-stream integral from $x = a$ to $x = b$, H is the maximum depth of the section over which the stream-wide transport per unit depth is summed, and $G(z)$ is defined by $\langle V(z)\rangle = \langle V(0)\rangle\, G(z)$. Equation 1 shows that a linear regression of transport on sea level difference is a statistical determination of the equivalent barotropic depth of the current and of the ageostrophic effects in whatever detail is warranted.

Figure 1 shows time series from April 1982 to August 1983 of monthly means of Cat Cay sea level (CCSL) minus Miami sea level (MISL), Key West sea level (KWSL) minus MISL, inverted MISL, volume transport estimated from the electromagnetic cable (*5*), and volume transport estimated by moored current meters (*6*). Each of the monthly means shows the same basic pattern, but each also shows significant differences. Monthly mean differences between the cable and the moored current meters are

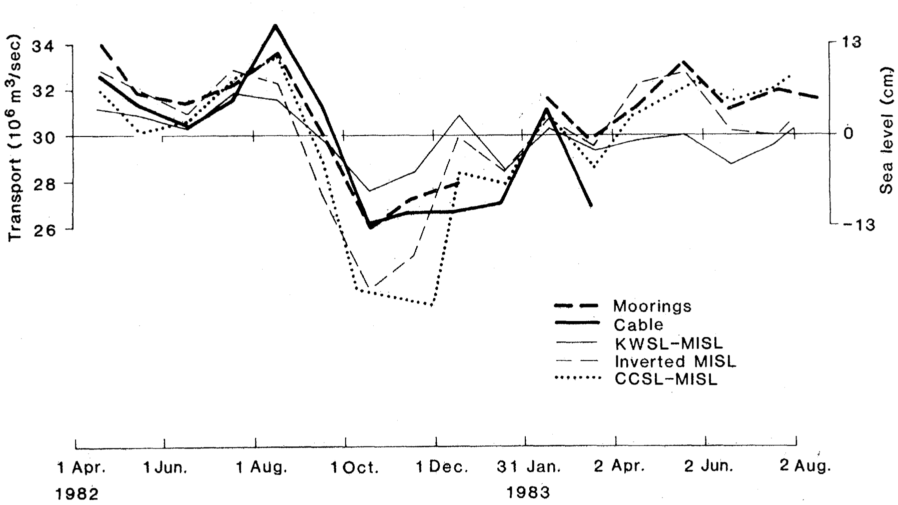


Fig. 1. Plot versus time of the mean monthly cable transports (*5*), mean monthly moored current meter transports (*6*), Key West minus Miami sea levels, Cat Cay minus Miami sea levels, and inverted Miami sea levels.

often quite large; in (6) it is reported that the standard deviation between cable and current meter daily means is $\pm 1.90 \times 10^6$ m^3/sec. Differences in our ability to use sea level measurements to infer transport are most striking during the autumn; this result suggests that the processing of sea level values must go beyond the calculating of simple monthly means (7) if sea level is to be an adequate monitor of the Florida Current.

A tide gauge has operated continuously at Miami since 1931 and at Key West since 1913, and intermittently at Cat Cay or at Bimini. A tide gauge was reestablished at Cat Cay in April 1982 for Subtropical Atlantic Climate Studies (STACS) sea level studies, which include bottom pressure gauges at sites A, B, and C [figure 1 of (8)]. Sea level data were planned for collection at these sites for the time periods shown in table 1 of (8). Data are generally available at the coastal stations, but bottom pressure records are much less continuous because of limited instrument availability or instrument failure after deployment. Weather records from Miami International Airport were used because no coastal data were available.

Most of the spectral energy in Straits of Florida sea level records is due to daily and semidaily tides (3). These were removed from all records with a 40-hour low-pass filter. Figure 2a shows the filtered records from Miami, Cat Cay, gauge site A, and gauge site B [see the cable observations (5)]; also shown is Cat Cay minus Miami. In each record substantial daily to monthly variability can be seen. Visually, correlations between Miami sea level, pressure at gauge site A, and the Jupiter–Settlement Point cable data (5) appear high but with some obvious differences. The records from the east side of the Straits appear uncorrelated with each other and with the results from Miami, gauge site A, and the cable, in agreement with the results of other studies (1–3).

Earlier investigations (2, 3) indicate that coherence between sea level, local weather, and transport fluctuations occur with essentially zero phase. This implies that an unlagged relation exists between several variables and sea level. For the immediate goal of establishing a monitoring strategy, we have carried out multiple linear regression analyses, using the records on hand, including the records of barometric pressure and wind velocity from Miami International Airport. In these correlations we use as the dependent variable the 81 daily-averaged discrete transports (4) observed between April 1982 and August 1983 coincident with sea level and bottom pressure records. It is recognized at the outset that time series analysis for determining the matrix of coherence and phase between data sets is ultimately necessary.

All filtered sea level or bottom pressure records were averaged into 24-hour values and analyzed by standard statistical techniques. Bottom pressure records required additional processing prior to analysis to remove instrument drift, which was noticeable in the early part of each record (Fig. 2a); the cause of the drift is unknown. First the correlation matrix was calculated to determine the linear correlation coefficient (r). These correlations were then used as a guide in computing the coefficient of determination (r^2) between selected variables in a multivariate analysis. Highlights of the correlations of the corrected, filtered, and averaged variables are summarized in Tables 1 and 2. The use of wind speed rather than wind stress is consistent with the approach of Wunsch and his colleagues (3) but not with that of Mooers and Brooks (9). Use of a quadratic wind speed was tested, and we found no significant differences when the quadratic was compared to linear wind speed.

Table 1 is the correlation matrix based on the data from April 1982 through August 1983. The cross-Straits pressure difference near the Pegasus section, that is, pressure gauge B minus pressure gauge A (PG:B − PG:A), PG:A alone, and the Florida-Bahamas electromagnetic cable (CABLE), have the best linear coefficients with volume transport. PG:B − PG:A has $r = +0.97$ with $n = 29$; PG:A alone has $r = -0.96$ with $n = 41$ (see Fig. 2b); CABLE has $r = +0.96$ with $n = 81$. MISL is moderately well correlated with transport ($r = -0.81$, $n = 81$), but CCSL and PG:B are not only poorly correlated with transport but have opposite signs to each other. Sea level differences across the Straits as estimated by CCSL − MISL and by PG:B − PG:A are both positively correlated with transport. PG:B − PG:A has a much better correlation with transport than CCSL − MISL ($n = 73$, $r = +0.67$), but the sample sizes are much different. Along-Straits sea level difference as estimated by KWSL − MISL, with $n = 81$ and $r = +0.46$, appears the least correlated with transport of all the traditional (1) indicators. There seems to be little structure in the correlations of weather with sea level except that the east component of the wind and the barometric pressure have $r = +0.60$.

Table 1. Linear correlation matrix. Linear correlation coefficients (r) of the investigated variables (lower left of the unit diagonal) and number of observations (n) in each analysis (upper right of the unit diagonal). Abbreviation of terms listed top to bottom in the left-hand column are given left to right in the top row, respectively; 1 Sv = 10^6 m^3/sec.

	MISL (cm)	CCSL (cm)	CCSL − MISL (cm)	KWSL (cm)	KWSL − MISL (cm)	CABLE (Sv)	NWIND (m/sec)	EWIND (m/sec)	BARO (mbar)	PG:A (mbar)	PG:B (mbar)	PG:B − PG:A (mbar)	XPORT (Sv)
Miami sea level	1	73	73	81	81	81	81	81	81	41	36	29	81
Cat Cay sea level	+0.54	1	73	73	73	73	73	73	73	33	28	21	73
Cat Cay minus Miami	−0.85	−0.02	1	73	73	73	73	73	73	33	28	21	73
Key West sea level	+0.86	+0.50	−0.65	1	81	81	81	81	81	41	36	29	81
Key West minus Miami	−0.62	−0.08	+0.49	−0.21	1	81	81	81	81	41	36	29	81
Florida-Bahamas cable	−0.83	−0.23	+0.68	−0.75	+0.51	1	81	81	81	41	36	29	81
North wind component	+0.51	+0.07	−0.42	+0.34	−0.60	−0.52	1	81	81	41	36	29	81
East wind component	−0.01	−0.05	+0.03	+0.14	+0.24	+0.09	−0.02	1	81	41	36	29	81
Barometric pressure	−0.30	−0.19	+0.42	−0.12	+0.36	+0.06	−0.06	+0.60	1	41	36	29	81
Pressure gauge A	+0.92	+0.75	−0.91	+0.82	−0.78	−0.96	+0.62	+0.02	−0.14	1	29	29	41
Pressure gauge B	−0.50	−0.35	+0.08	−0.59	+0.18	+0.48	+0.01	−0.45	−0.41	−0.45	1	29	36
Gauge B minus gauge A	−0.83	−0.55	+0.92	−0.63	+0.90	+0.95	−0.71	−0.15	−0.19	−0.99	+0.73	1	29
Volume transport	−0.81	−0.17	+0.67	−0.75	+0.46	+0.96	−0.47	+0.01	+0.03	−0.96	+0.52	+0.97	1

Table 2. Abbreviated volume transport multivariate analysis. Multivariate linear regressions of volume transport (dependent variable) versus listed independent variables except regression 8 which uses PG:A as the dependent variable. All coefficients of determination (r^2) are reported at the 95 percent confidence level based on the variance-ratio F-test (10). Regression coefficients (A_n in Eq. 2) follow each independent variable in parentheses. The units of the standard error (S.E.) of estimate are sverdrups (except regression, for which the unit is centimeters). Abbreviations are given in Table 1.

Variables in regression	r^2	S.E.	n
1. PG:A (−0.30)	0.93	1.2	41
2. MISL (−0.12), CCSL − MISL (+0.16), BARO (−0.47), EWIND (+0.43)	0.57	1.7	73
3. CCSL − MISL (+0.28), BARO (−0.40), EWIND (+0.39)	0.52	1.8	73
4. MISL (−0.31), BARO (−0.62), EWIND (+0.50)	0.74	1.9	81
5. KWSL (−0.32), NWIND (−0.45), EWIND (+0.49), BARO (−0.34)	0.65	2.2	81
6. CABLE (+1.01), EWIND (−0.16)	0.92	1.1	81
7. CABLE (+0.47), PG:A (−0.16)	0.94	1.1	41
8. MISL (+0.83), BARO (+1.23), EWIND (−0.27)	0.84	4.4	302

Upper and lower limits at 95 percent confidence on the linear correlations of selected variables versus volume transport from Table 1 are as follows:

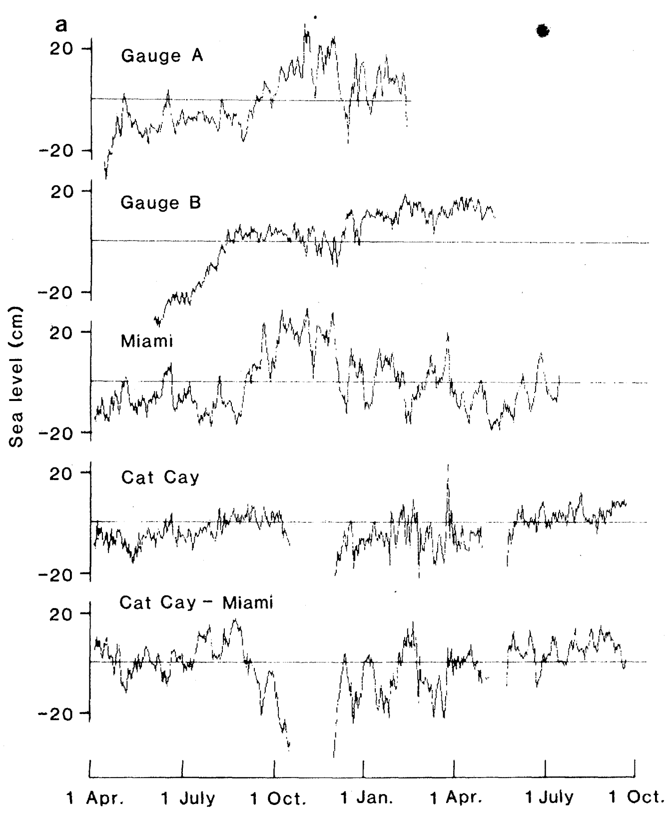
CABLE	$+0.94 < r < +0.97$
PG:B − PG:A	$+0.94 < r < +0.99$
CCSL − MISL	$+0.52 < r < +0.78$
PG:A	$-0.93 < r < -0.98$
MISL	$-0.72 < r < -0.87$
KWSL	$-0.64 < r < -0.83$
KWSL − MISL	$+0.28 < r < +0.62$

Correlation of CABLE, PG:B − PG:A, and PG:A alone with Pegasus transport are statistically identical, but the cable correlation is based on twice as much data. The remaining indicators listed above are all within the same 95 percent confidence limit except MISL, which is statistically a better transport indicator than KWSL − MISL. There appears to be a sharp distinction in the correlations between the data at the latitude of the Pegasus section and at the latitude of Miami (120 km to the south). When considering these results, it is important to interpret them as obtained from band-averaged covariances. The band averaging includes all frequencies contained in each of the signals analyzed.

Our multivariate linear regression uses Pegasus estimated volume transport (XPORT) as the dependent variable and least-squares fits an equation of the form

$$\text{XPORT} = A_0 + A_1 X_1 + A_2 X_2 + \ldots + A_n X_n \qquad (2)$$

to n independent variables (X's) by determining the constant A's. In the multivariate regression experiments summarized in Table 2, the F-test (10) was used to test the hypothesis whether including the next independent variable multiplier (A_n) in the forward stepwise selection was statistically significant; only terms



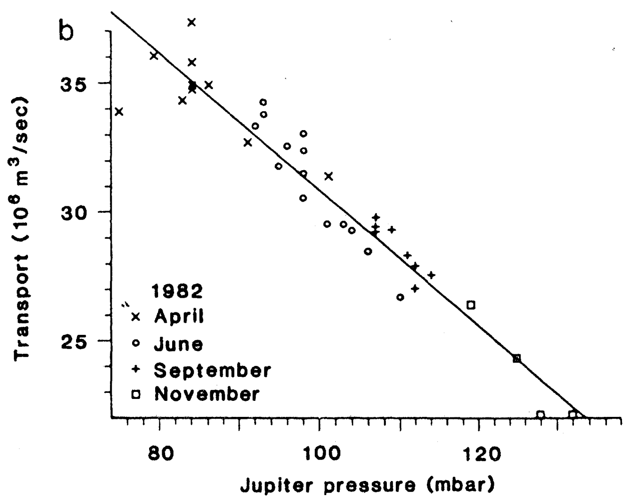


Fig. 2. (a) Time series of 40-hour low-pass–filtered sea level and bottom pressure observations available at this writing (8). (b) Linear correlation between the Jupiter bottom pressure (PG:A) and volume transport [see figure 2 of (5)]. Note the drift in bottom pressure measurements at the beginning of each record; these were removed by least-squares curve-fitting prior to correlation with Pegasus (4) volume transports.

with greater than 95 percent confidence are included in the regression. Many combinations were tried, but for brevity only the important results are reported.

Regression 1 in Table 2 shows that, of the combinations geographically nearest the Pegasus section (excluding CABLE), PG:A with $r^2 = 0.93$, is statistically most significant. The standard error in estimating XPORT from PG:A only is $\pm 1.2 \times 10^6$ m^3/sec based on the use of all $n = 41$ available measurements. PG:B − PG:A also had $r^2 = 0.93$, but with only $n = 29$ data points; the standard error for PG:B − PG:A was $\pm 1.1 \times 10^6$ m^3/sec. When PG:B − PG:A and PG:A alone were analyzed together, PG:A was statistically more significant than PG:B − PG:A; this was caused by a slightly higher correlation of PG:A with transport than of PG:B − PG:A with transport when only $n = 29$ points were included in the multivariate procedure (see Table 1).

Next, calculations to test combinations of sea level gauges and weather (east wind component, EWIND; north wind component, NWIND; barometric pressure, BARO) at the latitude of Miami were performed (Table 2, regressions 2, 3, and 4). As with the pressure gauges, the cross-Straits difference is not better than the west-side-only gauge in reducing the residual variance. Also, correlation is improved when BARO and EWIND are used together with sea level records, but NWIND is not statistically significant. The use of MISL alone in estimating Pegasus volume transport (regression 4) is improved by the inclusion of BARO and EWIND, but the change in the standard error is less than $\pm 0.2 \times 10^6$ m^3/sec when compared with the inclusion of CCSL. KWSL in combination with MISL and weather does not add to variance reduction; KWSL alone (regression) in combination with weather is not as good as weather-adjusted MISL. Of the surface gauges and combinations thereof, it appears that MISL when adjusted for weather effects has as good a correlation with XPORT as any other combination.

As Tables 1 and 2 show, the differences CCSL − MISL or PG:B − PG:A are not as well or are no better correlated with mass transport than PG:A alone. One reason for this may be fluctuations in transport through the Northwest Providence Channel. Net volume transports through the Channel are between 1.5 and 2.5×10^6 m^3/sec to the west, with an eastward flow on the south side and a westward flow on the north side (*11*). Another reason, revealed by synthetic aperture radar imagery from SEASAT, is that there is evidence of vortices in the Northwest Providence Channel (*12*), further suggesting a complicated flow regime. A third reason comes from gauge A temperature measurements, which suggest that these differences could also be due in part to density changes.

Differences in local bottom topography add to the complexity. The Miami sea level gauge is at an entrance to Biscayne Bay and is on a continental shelf approximately 5 km wide, whereas the Cat Cay gauge is very close to the Bahamas Bank shelf break. Moreover, the prevailing easterly winds make Miami a windward shore and Cat Cay a lee shore. Thus sea level on the east side of the Straits may be responding more to local currents (but not Miami weather; r^2 is 0.04 between CCSL and the combination of NWIND, BARO, and EWIND) than to high-frequency transport fluctuations.

A study of the difference between Pegasus transport and predictions based on MISL and weather (Table 2, regression 4) revealed a residual near the fortnightly frequency. We obtained the temporal variability of the fortnightly energy by performing a least-squares analysis for the available data (about 1½ years). There appears to be no seasonal dependence, and the appearance of energy in the fortnightly band occurs randomly. At the present early stage of analysis and for lack of more data, we can offer no explanation for this behavior and are surprised by the variability in the fortnightly band.

Other results (*5*) suggest that the cable from Jupiter to Settlement Point is the best monitoring tool for the Florida Current. Since there is a need for continuity in a monitoring strategy, and since PG:A seems to give nearly as good a result, it is of interest to explore a multivariate combination of tide gauges and the cable. Results of using Eq. 2 are summarized in Table 2 for combinations of weather, pressure gauges, and the cable (regressions 6 and 7). In comparing these r^2 values with those in (*5*), it should be noted that no weighted values of volume transport were used herein.

The calculations show that CABLE is more significant in variance reduction than any other variable, but that a combination of CABLE, PG:A, and EWIND is statistically significant at 95 percent confidence. When CABLE is used in combination with MISL, CCSL, and weather, the evidence indicates that inclusion of data from the latitude of Miami is not significant but that CABLE and EWIND offer a slight improvement over CABLE only. Using MISL and weather (regression 8) to replace lost PG:A data in a monitoring strategy introduces a standard error of about ±4 cm, which is approximately $\pm 1 \times 10^6$ m^3/sec in transport error. It would thus seem important for monitoring to have sea level measurements in close proximity to the cable measurements and to have simultaneous weather observations.

These preliminary results suggest that sea level on the west side of the Straits of Florida is a viable means of estimating transport in the Florida Current. The standard error of estimate, based on the use of a combination of electromagnetic cable, sea level, and meteorological data, is $\pm 1 \times 10^6$ m^3/sec and is probably equal to that derived from the use of Pegasus (*4*). One would like to improve this value by at least a factor of 2 for climate monitoring, and this may be possible by the inclusion of other measurements described elsewhere in these STACS reports (*4–7*).

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Notes and References

1. See C. O'D. Iselin, *Preliminary Report on Long-Period Variations in the Transport of the Gulf Stream* (Papers in Physical Oceanography and Meteorology, Massachusetts Institute of Technology–Woods Hole Oceanographic Institute, MIT Press, Cambridge, Mass., 1940), vol. 8 (No. 1), and references therein.
2. H. Stommel, *The Gulf Stream* (Univ. of California Press, Berkeley, 1966).
3. C. Wunsch, D. V. Hansen, B. D. Zetler, *Deep-Sea Res.* **16**, 447 (1969); C. Wunsch and M. Wimbush, *J. Mar. Res.* **35**, 75 (1976).
4. R. L. Molinari, W. D. Wilson, K. Leaman, *Science* **227**, 295 (1985).
5. J. C. Larsen and T. B. Sanford, *ibid.*, p. 302.
6. T. N. Lee, F. A. Schott, R. Zantopp, *ibid.*, p. 298.
7. F. Schott and R. Zantopp, *ibid.*, p. 308.
8. R. L. Molinari *et al.*, *ibid.*, p. 292.
9. C. N. K. Mooers and D. A. Brooks, *Deep-Sea Res.* **24**, 399 (1977).
10. A. L. Edwards, *An Introduction to Linear Regression and Correlation* (Freeman, San Francisco, 1976), pp. 156–158 and 203–205.
11. W. S. Richardson and J. R. Finlen, *Deep-Sea Res.* **14**, 361 (1967); T. N. Lee, *Bull. Mar. Sci.* **27** (No. 4), 802 (1977).
12. L.-L. Fu and B. Holt, *SEASAT Views Oceans and Sea Ice with Synthetic-Aperture Radar* (JPL Publication 81-120) (1982), pp. 82–83.
13. We thank N. J. Bravo and R. Zantopp for their help in data processing and D. Martin for his efforts in installing and maintaining the Cat Cay tide gauge.

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