The Dark Night-Sky Riddle: A "Paradox" That Resisted Solution

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Summary. The riddle of a dark night sky, now known as "Olbers's paradox," can be traced back to Thomas Digges in 1576. Since the time of Edmund Halley (1721) the riddle of a dark night sky in an infinite universe uniformly populated with stars has been regarded as a paradox. Constant emphasis on the paradoxical aspect of the problem of darkness at night, however, leads to a one-sided interpretation of the riddle. Calling the phenomenon a "paradox" distorts the historical perspective, and consequently we incorrectly attribute the origin of the riddle to Edmund Halley. Also it distorts the cosmological perspective and quite probably has greatly delayed the solution of the riddle.

In recent decades the dark night-sky riddle has become widely known as "Olbers's paradox." This popular title, introduced by Hermann Bondi (*1*) in 1952, persists even though many writers (*2–8*) have shown that Willhelm Olbers did not originate the riddle. In this article I show that the word "paradox" is also an unfortunate choice; it causes us to interpret too narrowly the scientific meaning of the riddle and to misjudge the historical evidence.

Olbers in 1823 discussed the darkness of the night sky in a universe uniformly sown with luminous stars and proposed that the most distant stars remain invisible owing to interstellar absorption of starlight (*9*). In 1744 Jean-Philippe Loys de Chéseaux had said much the same (*10*), and Jaki has discussed the circumstances relating to the remarkable similarity of the proposals made by Chéseaux and Olbers (*11*). The Chéseaux-Olbers solution of the dark night-sky riddle fails because the interstellar medium heats up, as shown by John Herschel (*12*), and emits as much radiation as it absorbs. Olbers made no reference to the work by Chéseaux; he referred to Edmund Halley's (*13*) papers, which Chéseaux, though undoubtedly influenced by them (*5, 14*), had failed to acknowledge. In two short but important papers Halley discussed in 1721 the riddle of a dark night sky in an infinite universe; in the second paper he referred to the riddle as a "Metaphysical Paradox," and in the first he attributed the riddle to an undisclosed source by stating:

> Another argument I have heard urged, that if the number of Fixt stars were more than finite, the whole superficies of their apparent Sphere would be luminous.

I have argued elsewhere (*15*) that Johannes Kepler was probably the first to realize that a dark night sky is in direct conflict with the idea of an infinite universe filled with luminous stars. In *Conversation with the Sidereal Messenger* (*16*), Kepler wrote in 1610,

> If this is true, and if they are suns having the same nature as our Sun, why do not these suns collectively outdistance our Sun in brilliance? Why do they all together transmit so dim a light to the most accessible places?

The historical evidence indicates that "Kepler's paradox" rather than "Halley's paradox" might be a more fitting title. Kepler, however, saw nothing paradoxical in a dark night sky; he believed in a finite bounded universe, and darkness at night confirmed his belief.

Alternative Interpretations

Unquestionably the riddle of darkness in an unbounded homogeneous universe of luminous stars raises cosmological issues of extraordinary subtlety. We observe the heavens studded with a finite number of visible stars, and we notice how they are separated by empty gaps of darkness. Why—in a universe that stretches away apparently without limit and contains possibly an unlimited number of luminous stars—do we observe dark gaps? When observing these dark gaps, what do we look at? From the outset we have a choice of alternative interpretations.

The first interpretation takes for granted the idea that the dark gaps are actually filled with distant stars. This idea is supported by the argument that a line of sight in any direction must always intercept the surface of a star, no matter how distant the star. If we suppose that most stars resemble the Sun, then the sky at every point should blaze as bright as the Sun's disk. This startling contradiction between theory and observation justifies the term "paradox."

This interpretation, frequently accepted without question by astronomers and historians who pay attention to the subject, assumes that in an infinite star-populated universe the observed dark gaps are completely filled with a continuous background of invisible stars. Rays of light emitted by the background of stars hurry toward us, and yet for some puzzling reason never reach Earth. Guided by this interpretation, historians have critically examined the various resolutions of the paradox proposed by astronomers in the preceding four centuries. Halley receives recognition as a pioneer, but earlier investigators such as Thomas Digges, William Gilbert, Johannes Kepler, and Otto von Guericke are begrudged recognition because they failed to stress the paradoxical aspect of the riddle of darkness. "Paradox" becomes the operative word, and it is the paradox of a dark night sky that must be resolved.

The second interpretation adopts the apparently simpleminded view that the dark gaps are mostly empty and not filled with a background of invisible stars. Of course, larger and better telescopes reveal more and fainter stars and also numerous extragalactic systems of stars, but however far we look out into the depths of space we always see the most distant stars immersed in pools of darkness. According to this second interpretation, the stars do not cover the sky, and the problem is reduced to explaining why the observed gaps remain unfilled in a star-populated universe of unlimited extent. Because no startling contradiction now exists between observation and expectation, "paradox" is no longer the operative word; we have a puzzle but not a paradox.

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The first interpretation assumes that the stars actually cover the entire sky in contradiction of the evidence; hence the riddle ranks as a paradox, and we must explain why most stars remain unseen. The second interpretation assumes that the stars do not cover the entire sky in agreement with the evidence; hence the riddle falls short of being a paradox, and we must explain why stars are insufficient to fill the dark gaps and form an intensely luminous background. Constant use of the word "paradox" encourages us to overlook the possibility of the unparadoxical second interpretation.

All paradoxes are riddles, but not all riddles are paradoxes. Life abounds with paradox, yet "What is life?" is a riddle, not a paradox. Time flows, we say, and when asked, "How can time flow?", we are beset by a riddle that amounts to a paradox. Riddles are puzzles, problems, or paradoxes, which we attempt to unriddle. Paradoxes generally contain contradictory elements and consist of propositions contrary to known facts or received opinions. Calling a thing a riddle (or, as some would say, a mystery) does not exclude it from being also a paradox. Thus the title "dark night-sky riddle" allows for either of the aforementioned interpretations. Admittedly we lose the sensational appeal of "paradox," but this is a small price to pay for an uncommitted mind. A title such as "Olbers's paradox" or "Halley's paradox" or "dark night-sky paradox" (all of which I have used myself) has the disadvantage of stressing an aspect of the riddle that may be unwarranted.

Of those who adopted the paradoxical first interpretation, Halley gave a geometrical argument (*6, 17*) and concluded that the beams from distant stars "are not sufficient to move our Sense"; Chéseaux and Olbers appealed to interstellar absorption of starlight; and Bondi (*1*), working within the framework of the expanding universe, proposed that the remote stars are invisible because of their large red shifts. These and many other writers assumed that stars cover the sky at every point and tried to explain why the light emitted by most stars remains unseen.

Of those who adopted the unparadoxical second interpretation, Kepler assumed that we look out between the stars and in effect see a dark enclosing wall; John Herschel (*18*), Richard Proctor (*19*), Fournier d'Albe (*20*), and Carl Charlier (*21*) considered a hierarchical universe arranged in larger and larger systems in such a manner that distant stars remain insufficient to cover the sky; and Edgar Allan Poe (*22*) suggested that we look out in space and back in time and see the nothingness that existed before the birth of stars. These and other authors took the contrary view and assumed that the entire sky is not covered by stars.

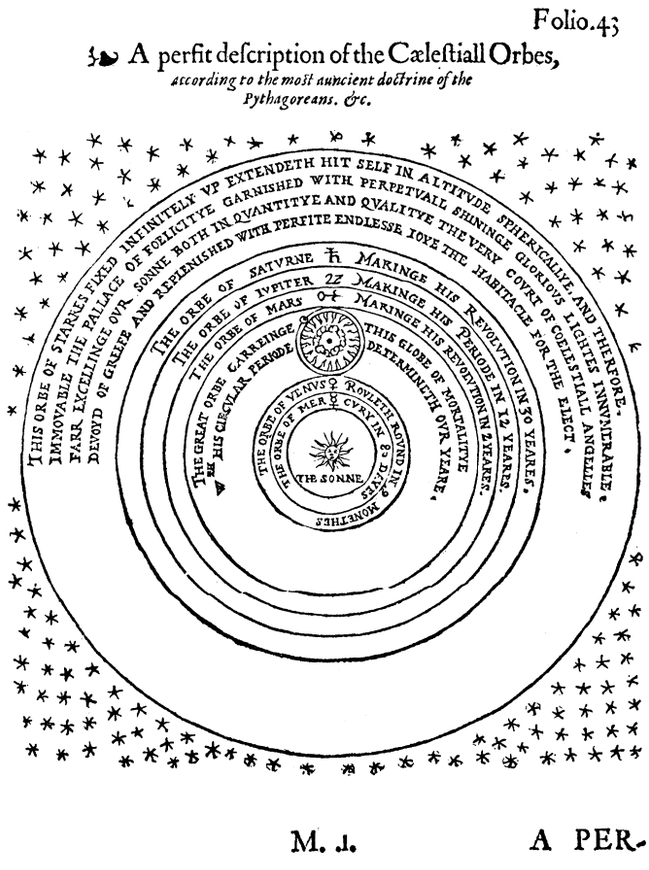


Fig. 1. Thomas Digges's representation of the infinite universe in his *Perfit Description* (*23*). Because of *The Nature of Things* by Lucretius (first century B.C.), discovered in 1417, and because of *Learned Ignorance*, written by Nicholas of Cusa in 1440, the idea of an infinite universe was in the air in the 16th century. Digges, who influenced Giordano Bruno and William Gilbert (see Fig. 2), was the first astronomer to champion openly the idea of an infinite universe. He dismantled the sphere of affixed stars in the Ptolemaic and Copernican systems and dispersed the stars throughout infinite space. He was the first to realize that in an infinite universe the darkness of the night sky needs explanation. The legend in the diagram reads, "This orbe of starres fixed infinitely up extendeth hit self in altitude spherically, and therefore * immovable the pallace of foelicitye garnished with perpetuall shininge glorious lightes innumerable * farr excellinge our sonne both in quantitye and qualitye the very court of coelestiall angelles * devoyd of greefe and replenished with perfite endlesse joye the habitacle for the elect."

Thomas Digges

Calling the puzzle a "paradox" tends to distort the historical picture. Halley referred to the paradoxical aspect of the riddle, and we may justly say that he originated the paradox of a dark night sky. Kepler, and other astronomers before Halley, failed to see anything paradoxical in the riddle. With our attention fixed on the first interpretation, we feel strongly tempted to criticize these earlier astronomers for failing to contribute anything significant to the cause of "Olbers's paradox." Notice how our attitude changes when we abandon the paradox template and regard the problem as an unqualified riddle. We must ask, Who was the first person to realize that the gaps of darkness between visible stars require an explanation? Without doubt, from this more general viewpoint, Kepler preceded Halley, and Digges preceded Kepler.

Thomas Digges, the foremost astronomer in England, revised in 1576 his father's book *Prognostication Everlastinge*. In an appended work (*23*) entitled "A Perfit Description of the Caelestiall Orbes," Digges explained the Copernican system to a wide audience and introduced a major modification. He wrote,

> Especially of that fixed Orbe garnished with lightes innumerable and reachinge up in *Sphaericall altitude* without ende. Of which lightes Celestiall it is to bee thoughte that we onely behoulde sutch as are in the inferioure partes of the same Orbe, and as they are hygher, so seeme they of lesse and lesser quantity, even tyll our sighte beinge not able farder to reache or conceyve, the greatest part rest by reason of their wonderfull distance invisible unto us.

By grafting endless space on to the Copernican system and dispersing the sphere of affixed stars of the Ptolemaic system, Digges pioneered in 16th-century astronomy the idea of an infinite universe filled with countless stars, a universe "fixed infinitely up" and "gar-

nished with perpetuall shininge glorious lightes innumerable" (Figs. 1 and 2). Copernicus revived the heliocentric theory of Aristarchus, and Digges revived the infinite universe of Democritus. It would be unfair to deny Digges the distinction of taking this extremely important step in astronomy on the grounds that he referred to the star-filled depths of space as the "pallace of foelicitye" and the "court of coelestiall angelles devoyd of greefe and replenished with perfite endlesse joye." Possibly he felt it imperative to make this concession to the theological convictions of his audience. If we deny Digges the innovation of an infinite universe, we must with equal injustice deny the more mystical Kepler his discovery of the three laws of planetary motion.

In Digges's *Perfit Description*, published 33 years after the death of Copernicus, we see the beginning of the dark night-sky riddle. Conceived in the Copernican Revolution and born when the infinite universe entered Western European astronomy, the riddle had yet to mature into the burning question, Why is the sky dark at night? This step was taken 34 years later by Kepler (*16*) in response to Galileo's discoveries with the recently developed telescope. The riddle has emerged as a realization that the invisibility of distant stars demands an explanation. The response given by Digges and accepted by many astronomers who followed was that most stars could not be seen because "the greatest part rest by reason of their wonderfull distance invisible unto us."

What could be more natural, given the rudimentary state of optical science in the 16th century, than to suppose that the most distant stars were too faint to be seen? At its birth the riddle received what seemed a perfectly sensible solution. Yet in the spirit of the first interpretation, Digges contributed nothing of significance to "Olbers's paradox" because he found nothing paradoxical in the darkness of the night sky. But in the spirit of the second interpretation, unencumbered by paradox, Digges originated the riddle of a dark night sky because he was the first person to realize that the dark gaps between visible stars need an explanation.

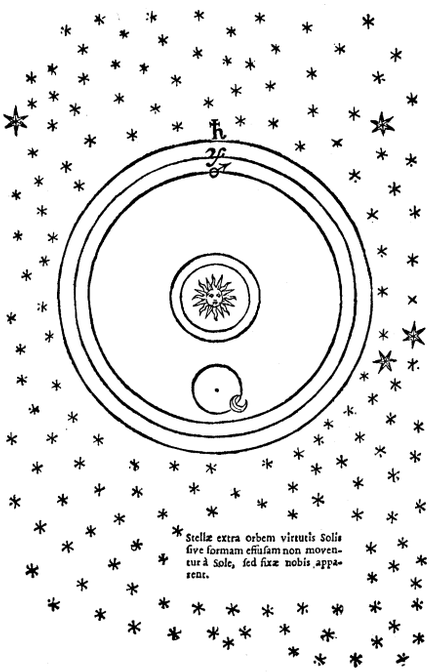
Did Halley Mislead Chéseaux?

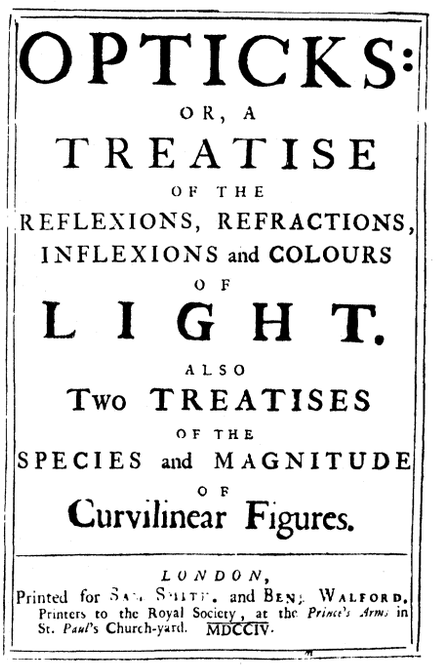
Using the word "paradox" also tends to distort the cosmological picture. When we suppose the riddle to be paradoxical, we take for granted that stars cover the entire sky. The problem then consists of explaining why this immense multitude of stars remains unseen. Chéseaux avoided the fault in Halley's argument; he understood that stars increase in number with distance in a way that compensates for the decrease in their individual apparent brightness. Unfortunately he saw the riddle in terms of the first interpretation, possibly because Halley had recently stressed its paradoxical aspect. Ole Roemer (*24*) had shown in 1676 that light travels at finite speed (Fig. 3), and this result had been confirmed in 1729 by James Bradley's (*25*) discovery of the aberration of light. Chéseaux knew that light propagates at finite speed. Furthermore, he had estimated roughly the average distance between stars in the solar neighborhood and could (and in effect did) calculate the number of stars needed to cover the entire sky. Without doubt, in conformity with biblical authority, he believed in a created universe of finite age (*26*). He had available enough information to show that stars capable of transmitting light to Earth since the day of creation were far too few to cover the sky.

A simple and convincing explanation of the darkness of the night sky lay at hand. Instead, Chéseaux explored the idea of absorption and embarked on a complicated calculation in radiative transfer. Perhaps if Halley had asked the simple question, Why do dark gaps exist between the stars? instead of the paradoxical question, Why cannot we see the stars that fill the dark gaps? (*27*), Chéseaux might have solved the dark night-sky riddle.

The Expanding Universe

In an expanding universe the radiation received from receding extragalactic sources is enfeebled by red shift. This is the solution of the riddle first proposed

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OR, A
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REFLEXIONS, REFRACTIONS,
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Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.
LONDON,
Printed for Sam. Smith and Ben. Walford,
Printers to the Royal Society, at the Prince's Arms in
St. Paul's Church-yard. MDCCIV.

Fig. 2 (left). William Gilbert's representation of the infinite universe in his posthumous *New Philosophy* (Amsterdam, 1651). His cosmological ideas were similar to those of Digges and Bruno; he rejected the geocentric system and championed the infinite universe introduced by Digges. In his great work *On the Magnet*, published in 1600, Gilbert wrote, "It is evident that all the heavenly bodies, set as if in destined places, are there formed unto spheres, that they tend to their own centres, and that round them there is a confluence of all their parts" (*34*). Fig. 3 (right). Title page of Newton's *Opticks*, published in 1704. The heading of proposition XI, book 2, part III, reads, "Light is propagated from luminous Bodies in time, and spends about seven or eight Minutes of an Hour in passing from the Sun to the Earth." The first sentence of the proposition states, "This was observed by Roemer, and then by others, by means of the Eclipses of the Satellites of Jupiter." The only thing needed to solve Digges's riddle within the context of Judaic-Christian-Islamic cosmology of a created universe of finite age was the discovery of the finite speed of light by Roemer in 1676. When Halley read his two papers to the Royal Society in 1721, Newton as president was in the chair (*6*). Why did Newton remain silent when the answer to the dark night-sky riddle lay at hand? Why did Halley not realize that he had the information and ideas needed to solve the riddle? Why, among the hundreds of astronomers in the last three centuries who have commented on the riddle and who believed in a universe of finite age, did no one realize how simple the solution was? This is now the only remaining puzzling aspect of the dark night-sky riddle.

by Bondi (*1*) that immediately springs to mind. Much of the subtlety of "Olbers's paradox" comes from the fact that its proponents had in mind the steady-state theory and were, if not historically, technically correct. In the de Sitter metric of the steady-state model the surface of the Hubble sphere acts as an event horizon (*28*). (The Hubble sphere is defined as the region of space around an observer in which the recession velocity is less than or equal to the velocity of light; the radius of this sphere is about 10^{10} light-years.) In the steady-state universe, which has an infinite age, the observer's backward light cone asymptotically approaches the surface of the Hubble sphere, and the world lines of an infinite number of stars and galaxies intersect the backward light cone. Stars therefore cover the entire sky and remain invisible owing to their extreme red shift. In this particular cosmological model the first interpretation of the dark night sky is correct.

But steady-state theorists and members of their audience believed that the red-shift solution had general validity and could be applied to all expanding cosmological models, including those with big bangs having particle horizons instead of event horizons. We may say, roughly speaking, that particle horizons exist in cosmological models of finite age (*28*). For several years, after Bondi had drawn attention to the subject, many investigators stated that the darkness of the night sky provides proof that the universe is expanding; some even claimed that expansion is the necessary and sufficient condition for darkness. Most writers who discussed the red-shift resolution of Olbers's paradox deemed mathematical confirmation quite unnecessary. Calculations in any case were awkward for evolving models and required integrations over the backward light cone in curved expanding space, and the results were generally difficult to interpret (*29*).

To find the right answer we must first ask the right question. When we ask a paradoxical question, we must not be surprised if we receive a paradoxical answer. Undoubtedly something about the red-shift solution is paradoxically odd. It implies that in a static universe the night sky is intensely bright. But a static universe of the kind imagined by Olbers and earlier workers, as we now realize, does not contain enough energy to create a bright night sky (*30*). If all matter in the universe were suddenly annihilated and the released energy converted into thermal radiation, the night sky would still be dark. The energy density of the radiation would be too low by a factor 10^{-13}. Why then must we appeal to the red shift of an expanding universe to resolve the so-called paradox when in fact a bright night sky is already impossible in a static universe? Come to that, why must we appeal to hierarchy, or any of the many variants of a static homogeneous universe (*5*), when already a bright night sky is impossible?

Something else about the red-shift solution, and about all other solutions in the category of the first interpretation, strikes one as rather odd. Simple estimates (*7*) show that the number of stars required to cover the entire sky has the enormous value of 10^{60}. But the number of stars in the observable region of an evolving universe (a region roughly equal in size to the Hubble sphere) is of order 10^{20}. When we observe the night sky, we look out in space 10^{10} light-years and back in time 10^{10} years to the early universe. The stars in the observable region of the universe, independently of whether they are huddled together in galaxies or uniformly scattered, are insufficient to fill more than 10^{-13} of the solid angle of the sky. Hence the dark gaps cannot be entirely filled with invisible stars. Interestingly enough, much the same situation occurs in a static model of the universe: The stars have typical luminous lifetimes of 10^{10} years, and, on looking out beyond a distance of 10^{10} light-years (or a few times this distance for several stellar generations), we look back to a dark era before the origin of stars. Again luminous stars cover only 10^{-13} of the sky.

Calculations made with powerful thermodynamic methods (*30, 31*) show that the extragalactic red shift of stellar light in most cosmological models is quite unimportant for the solution of the riddle. Far more important is the fact that stars cannot shine long enough to fill the universe with radiation equal in intensity to that at the surface of stars (*7, 15, 28*). Even in the standard model of an infinite homogeneous static universe of finite age, of the kind in which the riddle was conceived, the light emitted by stars falls a long way short of creating a bright sky, simply because the stars exhaust their energy reserves long before the universe fills with starlight. The time needed to fill the universe with starlight in equilibrium with the stars is roughly 10^{23} years, which greatly exceeds the luminous lifetime of stars. With the new methods (*32*) it is easy to design theoretical bright-sky as well as dark-sky static, nonstatic, and even steady-state cosmological models.

Conclusions

The sky happens to be dark in an evolving universe simply because stellar disks fail to cover the entire sky. The first interpretation, which treats the riddle as a paradox, is therefore wrong and the second interpretation correct. Since the time of Halley the riddle has usually been stated in the form of a paradox, and this may explain why earlier astronomers, who were unable to regard it as a paradox, have been denied credit for its discovery and development. I have shown that probably Thomas Digges was the first to realize that the dark gaps between the visible stars call for an explanation, and therefore he should receive credit as the originator of the riddle.

The habit of stating the riddle in paradoxical form may have greatly delayed the discovery of the explanation of why we live in a dark-sky universe. Both Halley and Chéseaux had sufficient knowledge to give a finite-velocity-of-light solution that would have been obvious and acceptable to everyone.

Of the various assumptions attributed to Olbers by authors who have discussed "Olbers's paradox," the one of crucial importance, and never mentioned, is his assumption that we can afford to ignore the speed of light. Olbers, repeating Chéseaux's argument, estimated with reasonable accuracy the average distance between stars in the neighborhood of the Sun. All he had to do was multiply the speed of light by the age of the universe—using either the Mosaic chronology or one of many estimates by earlier scientists, such as the 100,000 years from Immanuel Kant's (*33*) cosmogony of 1755—and in a sphere of this radius he would have found insufficient stars to cover the whole sky.

The region accessible to observation in a universe of finite age is always of finite size. The failure to realize this important truth by many scientists in the 18th, 19th, and 20th centuries constitutes the only puzzling feature that survives in the dark night-sky riddle.

Telescopes reveal numerous stars in the Galaxy that are invisible to the unaided eye; telescopes also reveal numerous galaxies stretching away to the horizon of the observable universe. The stars accessible to observation are found to be insufficient to cover all points of the sky. Through the gaps between stars we look back to the beginning of the universe. Poe (*22*) in 1848 correctly solved the riddle in a static model of the universe by supposing that "the distance of the in-

visible background so immense that no ray has yet been able to reach us at all." This finite-velocity-of-light solution needs only slight modification when adapted to an expanding universe originating from a singular state. A similar remark applies to the suggestion made by the British scientist Fournier d'Albe (*20*), who stated in 1907:

> If the world were created 100,000 years ago, then no light from bodies more than 100,000 light-years away from us could possibly have reached us up to the present; but light from stars further and further away would be continually arriving at the earth's surface, and thus our vision into space, confined at present by the Milky Way, would be expanding at the rate of 186,000 miles per second.

In the modern expanding and evolving universe we look through the gaps and "see" the big bang. The red-shift of starlight, which is of only minor importance in solving the riddle, now assumes paramount importance: We see the high-temperature big bang red-shifted into the feeble 3-degree afterglow that fills the universe.

The riddle of a dark night sky unfolds as an extraordinary story in the annals of science. One of the most remarkable features of the riddle is the mischief played by the seductive word "paradox," which has misled the astronomer into an unwarranted interpretation of the phenomena and the historian into too narrow an appreciation of the issues involved.

References and Notes

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2. O. Struve, "The constitution of diffuse matter in interstellar space," *J. Wash. Acad. Sci.* **31**, 217 (1941); "Some thoughts on Olbers' Paradox," *Sky Telescope* **25**, 140 (1963).
3. G. A. Tammann, "Jean-Philippe Loys de Chéseaux and his discovery of the so-called Olbers' paradox," *Scientia (Milan)* **60**, 22 (1966).
4. S. L. Jaki, "Olbers', Halley's, or whose paradox?" *Am. J. Phys.* **35**, 200 (1967).
5. _____, *The Paradox of Olbers' Paradox* (Herder & Herder, New York, 1969). Contains much bibliographical material.
6. M. Hoskins ["Dark skies and fixed stars," *J. Br. Astron. Assoc.* **83**, 4 (1973)] criticizes the use of the word "paradox."
7. E. R. Harrison, "Why is the sky dark at night?" *Phys. Today* **28**, 69 (February 1974).
8. D. D. Clayton, *The Dark Night Sky* (Quadrangle, New York, 1975).
9. H. W. M. Olbers, "Ueber die Durchtigkeit des Weltraumes," *Astronomische Jarhbuch für das Jahr 1826*, J. E. Bode, Ed. (C. F. E. Späthen, Berlin, 1823), p. 10; reproduced in (*5*) as appendix 3; translated: "On the transparency of space," *Edinburgh New Philos. J.* **1**, 141 (1826).
10. J.-P. Loys de Chéseaux, *Traité de la Comète* (M. M. Bousequet, Lausanne, 1744), p. 223; reproduced in (*5*) as appendix 2. Chéseaux elaborated on Halley's idea of spherical shells of equal thickness and found, unlike Halley, that all shells give equal increments of light. Olbers followed this procedure and came to the same conclusion; also he used the novel argument that any line of sight must ultimately intercept the surface of a distant star and thus demonstrated that clustering of a finite number of levels in a multilevel universe cannot avert a bright sky.
11. S. L. Jaki, "New light on Olbers's dependence on Chéseaux," *J. Hist. Astron.* **1**, 53 (1970).
12. J. F. W. Herschel, "Humboldt's *Kosmos*," *Edinburgh Rev.* **87**, 170 (1848); reproduced in *Essays* (Longman, Brown, Green, Longmans and Roberts, London, 1857), p. 257. Strictly speaking, the statement that absorption fails to prevent a bright sky is inadequate; we must show that the absorbing matter heats up in less than the lifetime of the universe or the lifetime of the luminous stars, whichever is the smaller. For example, Herschel's criticism fails in the case of Fournier d'Albe's (*20*) proposal that for every luminous star there are 10^{12} nonluminous stars.
13. E. Halley, "Of the infinity of the fix'd stars," *Philos. Trans.* **31**, 22 (1720–1721); "Of the number, order, and light of the fix'd stars," *ibid.*, p. 24; both papers are reproduced in (*5*) as appendix 1. According to the *Journal Book* of the Royal Society, these papers were read in March 1721, and this is the year to which they should be assigned (*6*).
14. Possibly Chéseaux assumed that the educated reader was familiar with the literature on the subject, particularly with the papers by Halley, who had only recently died in 1742. Halley's two papers (*13*) on the infinity of the universe and the riddle of darkness were undoubtedly well known in astronomical circles; they had been reproduced, with other selected papers by different authors from 1719 to 1733, in a special edition of the *Philosophical Transactions* in six volumes; see vol. 6, part I (Brotherton, London, 1734), p. 147.
15. E. R. Harrison, "The dark night sky paradox," *Am. J. Phys.* **45**, 119 (1977).
16. E. Rosen, *Kepler's Conversation with the Sidereal Messenger* (Johnson, New York, 1965). See also A. Koyré, *From the Closed World to the Infinite Universe* (Harper, New York, 1958).
17. G. J. Whitrow, "Why is the sky dark at night?" *Hist. Sci.* **10**, 128 (1971).
18. J. F. W. Herschel, "Humboldt's *Kosmos*," *Edinburgh Rev.* **87**, 170 (1848); reproduced in *Essays* (Longman, Brown, Green, Longmans and Roberts, London, 1857), p. 285.
19. R. A. Proctor, *Other Worlds than Ours* (Appleton, New York, 1871), p. 286.
20. E. E. Fournier d'Albe, *Two New Worlds* (Longmans, Green, New York, 1907). This author makes several proposals, including hierarchy in a "multi-universe," absorption of light by nonluminous stars, and a finite-velocity-of-light solution similar to that made by Poe (*22*).
21. C. V. L. Charlier, "Ist die Welt endlich oder unendlich in Raum and Zeit?" *Arch. syst. Philos. (Berlin)* **2**, 477 (1896); "Wie eine unendliche Welt aufgebaut sein kann," *Ark. Mat. Astron. Fys.* **4**, No. 24 (1908). Charlier showed that the night sky remains uncovered by stars if, at each level, $R_i/R_{i-1} > \sqrt{N_i}$, where R_i is the radius of clusters of the ith level, containing N_i subclusters of radius R_{i-1}. He showed that, when hierarchy satisfies this condition, it resolves also the "gravity paradox," which concerned Hugo Seeliger ["Ueber das Newton'sche Gravitationsgesetz," *Astron. Nachr.* **137**, No. 3273 (1895)]. According to this so-called paradox, the gravity potential is everywhere infinite in a universe of unlimited extent and finite density. Advocates of this paradox omit to mention that without proper boundary conditions the Newtonian gravity potential is undefined. General relativity gives the correct theoretical treatment, and this paradox can be decently buried.
22. E. A. Poe, *Eureka: A Prose Poem* (G. Putnam, New York, 1848). Reprinted in *The Science Fiction of Edgar Allan Poe*, H. Beaver, Ed. (Penguin, Harmondsworth, Middlesex, England, 1976). Perhaps other solutions similar to Poe's treatment, in addition to that by Fournier d'Albe (*20*), exist in the literature, and, though interesting, have been viewed as irrelevent because their authors failed to conform to the paradox convention.
23. T. Digges, "A perfit description of the caelestiall orbes" in *Prognostication Everlastinge* (London, 1576); reproduced by F. R. Johnson and S. V. Larkey, "Thomas Digges, the Copernican system, and the idea of the infinity of the universe in 1576," *Huntington Libr. Bul.* (Harvard Univ. Press, Cambridge, Mass., 1934), No. 5, p. 69. See also F. R. Johnson, "Thomas Digges and the infinity of the universe," *Astronomical Thought in Renaissance England* (Johns Hopkins Press, Baltimore, 1937), chap. 4; reproduced in M. K. Munitz, *Theories of the Universe* (Free Press, New York, 1957).
24. I. B. Cohen, "Roemer and the first determination of the velocity of light (1676)," *Isis* **31**, 327 (1940). Roemer studied the phase shift in the period of Io's Jovian orbit owing to the Doppler shift produced by Earth's motion. In effect, he measured the astronomical unit (the Sun-Earth distance) in light-travel time and found it to be 11 minutes. Halley [in *Philos. Trans.* **18** (No. 214), 237 (1694)] reviewed the eclipse observations of Io and obtained a value of 8.5 minutes—only a few seconds more than the modern value. Halley and Newton knew from the investigations by James Gregory that the separating distance between neighboring stars is of order 10^6 astronomical units—roughly 10 light-years. [See M. Hoskin, "The English background to the cosmology of Wright and Herschel," in *Cosmology, History, and Theology*, W. Yourgrau and A. D. Breck, Eds. (Plenum, New York, 1977.)] Neither Halley nor Newton seemed to realize that the age of the universe, when divided by the light-travel time from the Sun to Earth, would give the maximum possible visible distance measured in astronomical units. By using the 6000 years from biblical records for the age of the universe, derived by James Ussher, in *The Annals of the World Deduced from the Origin of Time* (1658), or by Newton, in *The Chronology of Ancient Kingdoms Ammended* (1710), or by any of the numerous reputable scholars such as Dante and Kepler, or by using from whatever source any cosmic age less than 10^{17} years, Halley in 1721 could have shown quite easily that the visible stars were too few to cover the sky. The riddle is why he thought the subject was a paradox.
25. J. Bradley, "A new discovered motion of the fix'd stars," *Philos. Trans.* (No. 406) (1729), p. 637. A reluctance (by Cartesians but not Newtonians) to accept Roemer's discovery of the finite speed of light vanished in 1729 when James Bradley discovered the aberration of light [see G. Sarton, "Discovery of the aberration of light," *Isis* **16**, 233 (1931)].
26. F. C. Haber, *The Age of the World: From Moses to Darwin* (Johns Hopkins Press, Baltimore, 1959).
27. Halley in his first paper (*13*) states, "so that, tho' it were true, that some such Stars are in such a place, yet their Beams, aided by any help yet known, are not sufficient to move our Sense."
28. W. Rindler, "Visual horizons in world-models," *Mon. Not. R. Astron. Soc.* **116**, 662 (1956). See also E. R. Harrison, *Cosmology: The Science of the Universe* (Cambridge Univ. Press, New York, 1981); "Cosmological horizons," *Phys. Today*, in press.
29. G. J. Whitrow and B. D. Yallop, "The background radiation in homogeneous isotropic world-models I," *Mon. Not. R. Astron. Soc.* **127**, 301 (1964); "The background radiation in homogeneous isotropic world-models II," *ibid.* **130**, 31 (1965).
30. E. R. Harrison, "Olbers' paradox," *Nature (London)* **204**, 271 (1964); "Olbers' paradox and the background radiation density in an isotropic homogeneous universe," *Mon. Not. R. Astron. Soc.* **132**, 1 (1965).
31. W. Davidson, "The cosmological implications of the recent counts of radio sources. II an evolutionary model," *Mon. Not. R. Astron. Soc.* **124**, 79 (1962); "Local thermodynamics and the universe," *Nature (London)* **206**, 249 (1965).
32. E. R. Harrison, "Radiation in isotropic and homogeneous models of the universe," *Vistas Astron.* **20**, 341 (1977). Contains a moderately complete bibliography on radiation in various cosmological models.
33. G. J. Whitrow, Ed., *Kant's Cosmogony*, translated by W. Hastie (Johnson, New York, 1970).
34. W. Gilbert, *On the Loadstone*, translated by S. P. Thompson (Chiswick, London, 1900).

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