Surprise Proof of an Old Conjecture

An American mathematician claimed to have resolved a famous conjecture, but he had to go to Russia to get a hearing

A mathematics conjecture that has stumped the best investigators for nearly 70 years has now been solved by Louis de Branges, who is on the fringe of the active research community. The conjecture, called the Bieberbach conjecture, was considered so difficult to prove that some eminent mathematicians believed it to be false. The method that de Branges of Purdue University used was thought such a long shot that it would have about a one in a million chance of succeeding. "It is a remarkable achievement, if I do say so myself," de Branges remarks.

De Branges has reason to gloat this time because he has been rejected by the mathematics community for 30 years, starting, he says, with his publication of an incorrect proof. "It has been a very difficult career. I have not been funded and I have been seriously punished," he remarks. Other mathematicians agree that he was not held in high regard. "De Branges is controversial, to say the least," says Felix Browder of the University of Chicago. "He has claimed in the past to have solved other problems but his proofs were wrong. Everyone has been very suspicious of him."

The American math community, in fact, so mistrusted de Branges's claims that when de Branges sent his manuscript purporting to have proved the Bieberbach conjecture to more than a dozen U.S. mathematicians, none read it. It was only in the Soviet Union that he finally got a hearing.

The Bieberbach conjecture was proposed in 1916 by a German, Ludwig Bieberbach, who is at least as well known in the mathematics community for being a notorious Nazi as for being a serious mathematician. But the conjecture he made about analytic functions has proved so difficult and so intriguing that it is routinely discussed in graduate mathematics courses and a number of researchers have devoted their careers to trying to resolve it. Analytic functions of complex variables are infinite polynomials—power series—that converge to the actual value of a function. For example, the power series $1 + z + z^2/2! + z^3/3! + \ldots$ converges to e^z. Such analytic functions form the basis of parts of calculus and differential equations and are essential for practical problems such as solving differential equations and describing the airflow over airplane wings.

Bieberbach's conjecture deals with the size of the coefficients of analytic functions. If an analytic function of the form $z + a_2z^2 + a_3z^3 + \ldots$ never assumes any value more than once on the unit disk then the absolute value of the kth coefficient, a_k, is never more than k for all k, the conjecture says.

The conjecture sounded straightforward, but until now all that mathematicians could do was to peck away at it. Bieberbach himself verified that it was true for the second coefficient, a_2. The German mathematician Charles Loewner showed in 1923 that it was true for the third coefficient, a_3. Then two Stanford mathematicians found that it was true for the fourth coefficient and, in 1968, two mathematicians verified independently that it was true for the sixth coefficient. In 1978, the fifth coefficient fell.

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In the meantime, mathematicians got closer and closer to the Bieberbach estimate for all the coefficients. In 1925, the English mathematician, Dudley Littlewood showed that the absolute value of the kth coefficient is never more than e times k. More recently, the Russian mathematician I. M. Milin of the University of Leningrad showed it is never more than 1.24 times k. Carl FitzGerald of the University of California in San Diego showed that it is never more than 1.08 times k and his student David Horowitz got the estimate down to $1.07k$.

What de Branges did was to prove a stronger conjecture that was proposed by Milin in 1971 and that implies the Bieberbach conjecture. Surprisingly, de Branges' method would not have allowed him to solve the Bieberbach conjecture directly. De Branges' proof, says FitzGerald, is "very clever" and, he remarks, "It's amazing that it works." De Branges had to introduce certain weighting functions and juggle them so that at different times terms in complicated equations were weighted different amounts. The method sounds so unlikely, says FitzGerald, that most mathematicians would be very pessimistic that such an idea could succeed even if it were explained very carefully to them.

De Branges says he worked on the Bieberbach conjecture for 7 years before he had any success. "It's been a long dry stretch," he remarks. He finally succeeded last March but could find no one in the U.S. mathematics community who was willing to read his ponderous manuscript of more than 350 pages. This was not too surprising, says one mathematician who has read several erroneous proofs of the conjecture by highly regarded researchers. This mathematician and several others did begin to read de Branges's proof but stopped when they started finding mistakes. The mistakes, as it turned out, did not ultimately affect the proof.

Fortunately for de Branges, he was scheduled to visit the Soviet Union from April to June as part of an exchange agreement between the U.S. and Soviet Academies of Sciences, and he arranged to lecture on his results to Milin and his colleagues at the University of Leningrad. Even though the Russians agreed to hear him, they were not optimistic that he had actually proved the conjecture, de Branges says. "It was the general expectation that some subtle error would be found."

The Soviets proved to be a patient audience. They sat through a series of five lectures by de Branges, each lasting from 5 until 9 or later in the evening, breaking only for tea. Milin and his colleague E. G. Emel'ianov then confirmed that the proof was correct. In June, de Branges worked with the seminar leader, G. V. Kuz'mina to, as he puts it, "consolidate the findings of the seminar." and, finally, de Branges submitted a 12-

page preprint of his proof to L. D. Faddeev, the director of the Steklov Mathematics Institute and the editor of the leading Soviet mathematics journal.

The Soviets then sent the preprint to mathematicians throughout the world. "I've received three unsolicited copies, none signed," says FitzGerald. FitzGerald and Christian Pommerenke of the Technical University of Berlin have now further simplified the proof and have circulated their version in the mathematics community. They did this, FitzGerald says, "mostly to tell people that this is for real."

It still is not entirely clear when or where the proof will be published. De Branges wanted very much to publish in a Soviet journal but was discouraged, he says, by friends who argued that the Soviets discriminate against Jewish mathematicians and also told him that a publication in a Soviet journal would not have the credibility of an American publication. So he decided to submit his proof to an American journal but has not yet determined which one.

The importance of the conjecture is mainly that it has proved so difficult and that so much useful mathematics was developed as researchers tried to resolve it. Mathematicians agree that it is too soon to say whether de Branges's methods or the very fact that he resolved the Bieberbach conjecture will have siginificance for mathematics in general. But the lack of any immediate practical applications does not diminish the importance of the result in the mathematics community. "Until this work was done, the solution was not within sight and some mathematicians, including myself, were not entirely convinced it was true," says Enrico Bombieri, a researcher at the Institute for Advanced Study who has worked on the problem. "This piece of work is better than any mathematician starting out has any right to expect he could ever do," says FitzGerald. "It is a great achievement," says Browder.

The mathematicians heaping praise on the work are careful also to praise de Branges. His age—52—would alone argue against this sort of achievement, at least according to the conventional wisdom that says mathematicians do their best work when they are young. Moreover, de Branges was handicapped by his reputation which has kept him from the acceptance or even the hearing that he has long thought his due. "It is very much to his credit that he worked so hard on this," FitzGerald says. "Anyone in the field would have told him, 'Don't waste your time.' "

—GINA KOLATA

Impacts of Another Kind

During the hot Washington summer of 1980 Storrs Olson and David Steadman, of the National Museum of Natural History, flew to the island of Antigua, Lesser Antilles, in company with Gregory Pregill, of the Natural History Museum, San Diego. They were in search of a good source of the island's recent fossil record. Within a day of their arrival the three biologists found one: it was a sediment-packed limestone fissure, which radiocarbon dating later showed to be some 4300 years old in the lower section and going up to 2560 years in the upper section. The timespan was perfect because it brackets the point at which the island's first settlers arrived, almost 4000 years ago, and reveals the effects of their entry.

Olson has for some time been interested in the impact of human settlement on ecological communities, particularly on birds. With Helen James he has documented through the fossil record the collapse by 50 percent of the Hawaiian bird population following Polynesian habitation 1600 years ago (*1*). A further reduction of 15 percent in more recent times looks small by comparison, though, being more "visible," it has until now attracted more attention. The Hawaiian example is proving to be rather typical of the impact of human settlement on virgin communities, especially island communities, which are especially vulnerable to extinction. Antigua, it turns out, is no exception.

The fossil accumulation in the limestone fissure that Olson, Steadman, and Pregill examined is in part the remains of owl prey and so is biased toward rather small vertebrates. Comparison of species present before and after initial settlement shows a 33 percent extinction of the original biota, including lizards, snakes, birds, bats, and rodents (*2*). A further 10 percent reduction occurred in historic times. Because of the incomplete nature of this fossil sample, the real extinction profile will be much steeper. Predation and habitat destruction are responsible for this trail of destruction.

Against the background of popular excitement about the possibility of asteroid or cometary impact as a cause of mass extinctions throughout the history of life, there is a growing interest in the less publicized but better documented species loss during the Quaternary, 2 million years ago to the present. The periodic glaciations throughout that time without doubt delivered the death knell to many species, but the hand of man begins to be evident toward the end of the record, particularly from 10,000 years onward. The relative contributions of the two agencies—climate change and human settlement—is a matter of lively debate and is the subject of an impressive collection of papers to be published this month (*3*). Humans, it seems, bear a rather greater weight of responsibility for recent extinctions than has generally been assumed.

The data that Olson and his colleagues collected from Antigua show that although the biota of the Lesser and Greater Antilles are today rather different, this was not always so. And herein lurks a warning to ecologists who seek to infer biological rules of community structure by studying and comparing modern populations, say Olson and his colleagues. "If Antigua is at all representative, then the endemic or localized distributions that characterize many insular species may actually be more a consequence of recent habitat degradation than such factors as niche partitioning and competition, which are now popularly assumed to regulate the kind and even number of species on islands under natural conditions."

Ecological communities that are undisturbed, save by the elements of nature, inevitably have a considerable stochastic component to their makeup. The intervention of human influence, with all its arbitrary impact, must render communities even less amenable to secure structural analysis. It would, however, be premature to throw out all of current theory, opines Oxford University ecologist Peter Boag (*4*)—ROGER LEWIN

References and Notes

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