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Dynamics of Globular Clusters

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Globular clusters are nearly spherical stellar systems associated with many galaxies and generally containing from 10^5 to 3×10^6 stars. Figure 1 is a photograph of Messier 19 (No. 19 in a catalog of some hundred diffuse objects compiled by Charles Messier late in the 18th century), a conspicuous such cluster in our own Galaxy, at a distance of some 3000 parsecs (1 parsec = 3.26 light years). stars move as mass points under their mutual gravitational attraction, with random velocities of some tens of kilometers per second. The large value of N, the total number of stars, can be expected to average out any large statistical fluctuations, and the way a spherical cluster evolves with time, as a result solely of Newton's laws of motion, appears deceptively simple.

Summary. In their attempt to reach kinetic equilibrium, through gravitational encounters between separate stars, globular clusters are driven to destruction, with their cores collapsing and their outer regions expanding. The effects of core collapse, which apparently produces x-ray sources, are not yet fully understood, but white dwarfs and neutron stars, probably in binary systems, are thought to be involved, and possibly black holes as well.

Typically the relatively dense central core of a globular cluster, with a radius of about 1 parsec, contains some 10^4 stars, while the outer regions of the cluster extend with much diminished density out to distances of roughly 25 to 100 parsecs. Studies of the stellar spectra indicate that these systems within our own Galaxy were formed early in the life of the Universe, about 10^{10} years ago, not very long after the initial Big Bang.

The dynamical evolution of these beautifully symmetrical, very ancient systems has provided astrophysicists with an intriguing and challenging problem, which so far is only partly solved. Even in the cores, the average distance between neighboring stars is generally more than 10^4 times the radii of even the giant stars and direct collisions between stars are extremely rare. Thus the cluster

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Although the general principles underlying this evolution have been known for some time, it is only within the last decade that theoretical analyses, supported by high-speed computers, have provided a detailed understanding of the later evolutionary phases. As we shall see below, these involve actual collapse of the central core and lead to the occurrence of new physical processes not important at the earlier stages. During all this activity the outer regions of the cluster gradually expand. The evolution of the cluster in the post-collapse phase is an active research field. The x-ray sources observed in the cores of some of the more centrally condensed clusters may well result from processes occurring during and after the core collapse.

In this article, as in several general surveys (*1*, *2*), first the physical principles affecting the early evolution of the cluster and the detailed evolutionary models based on these principles are outlined. Then additional physical processes that become important during the collapse phase, such as formation of binary systems, both by tidal capture in a close two-body encounter and by direct three-body encounters, are discussed.

Physical Principles

In discussing stellar motions in a globular cluster, we first separate the gravitational potential energy, $\phi(\mathbf{r}, t)$ into the sum of two terms. The first is a smoothed, spherically symmetric potential obtained by averaging $\phi(\mathbf{r}, t)$ over a time interval including several orbital periods of the stars. A star moving at 10 kilometers per second goes 1 parsec in 10^5 years, and the time required to travel back and forth across a cluster is generally less than 10^6 years, which in turn, is a small fraction of the evolution time. The average of $\phi(\mathbf{r}, t)$, over roughly 10^6 years we denote by $\phi_A(r)$, assumed to be spherically symmetric. This smoothed potential will change slowly as the cluster evolves. In a zero-order approximation each star moves in this spherical potential, with constant energy E and angular momentum $\mathbf{J}$, both measured per unit mass.

On this approximation no evolution occurs. A basic constraint on a cluster in this approximation is that the average smoothed stellar density, $\rho_A(r)$, must be consistent with Poisson's equation

$$\nabla^2 \phi_A(r) = 4\pi\rho_A(r) \qquad (1)$$

where $\rho_A(r)$ is averaged over the same time interval used in determining $\phi_A(r)$. Many equilibrium solutions are possible.

The difference in potential $\phi(\mathbf{r}, t) - \phi_A(r)$ results from the granularity of the gravitational field. It is generally assumed that this granularity can be represented in a first approximation by two-body encounters between stars, and that the effects of such encounters in altering E and $\mathbf{J}$ of each star can be computed as though the two stars involved were moving in a hyperbolic path relative to each other, unaffected by other stars. The effects of such encounters have been computed in detail (*3*, *4*). The results can be used to follow the way in which the distribution of stars among different orbits is changed and thus how the cluster evolves. It should be emphasized that, for the cluster as a whole, the effects produced by stellar encounters occur very slowly in comparison with the time for a star to move across the cluster. Essentially the mean free path is many orders of magnitude greater than the dimensions of the cluster, and thousands of cluster crossings are required for appreciable evolution.

One important feature of two-body gravitational encounters is that the cumulative effect of many distant encounters, each of which produces only a small change in stellar velocities, tends to outweigh the less frequent close encounters, in which the stars are deflected by some 90° or more. Thus the velocity of a star is subject to a diffusion process, and similar diffusion occurs in E and $\mathbf{J}$. Changes in the velocity distribution of stars are governed by the integrodifferential Fokker-Planck equation (5).

While the details of these dynamical interactions are somewhat complex, the general physical tendency is clear. Encounters between stars will tend to increase entropy, evolving the stellar system toward a state of higher probability. The distribution of stars can be described by the density $f(\mathbf{r}, \mathbf{v})$ in phase space; $f(\mathbf{r}, \mathbf{v})$, multiplied by the phase space volume element $dxdydzdv_xdv_ydv_z$ is the number of stars within this volume element centered at $\mathbf{r}$, $\mathbf{v}$. In the local state of highest probability, toward which the cluster evolves, the phase space density $f(\mathbf{r}, \mathbf{v})$, which we designate simply by f, is given by

$$f = Ke^{-\beta E} \tag{2}$$

where E is the energy of each star per unit mass, β is inversely proportional to the average energy, and K is a normalization constant. For stars within a small region of space the potential energy is constant, and only the kinetic energy, $mv^2/2$, need be considered; Eq. 2 then gives f_M, the usual Maxwellian distribution function

$$f_M = \frac{n}{v_m{}^3}\left(\frac{3}{2\pi}\right)^{3/2} e^{-3v^2/2v_m{}^2} \tag{3}$$

where v_m is the root-mean-square (rms) velocity, and n is the number of stars per unit volume of physical space. Multiplication of f_M by $4\pi v^2 dv$ gives the number of stars per unit volume whose total velocity lies between v and $v + dv$.

It is this tendency toward a more probable state, as in thermodynamic equilibrium, which leads the cluster straight to catastrophe. The volume of accessible phase space per energy increment is greatest for stars which are at the greatest distances from the cluster center, especially those which escape the cluster entirely and have an entire galaxy to roam around in. On the other hand, f in Eq. 2 is maximized if some of the cluster stars are very close together, giving a large negative potential energy E. Thus velocity perturbations lead to an expansion of some regions of the cluster and contraction of others. Analysis of the various ways in which these processes occur in a spherical star cluster provides a challenging task, whose status is summarized in this article.

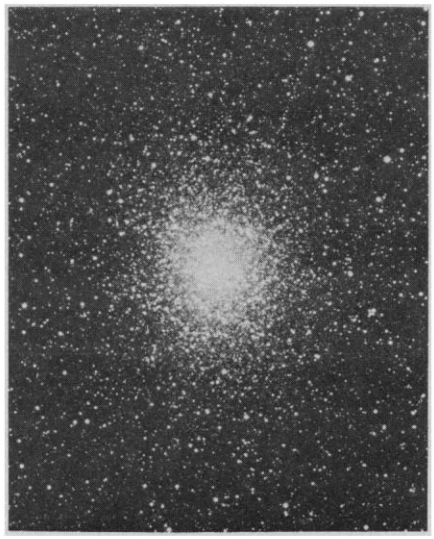

Fig. 1. Photograph of globular cluster M19 (NGC 6273) with the 3.9-m Anglo-Australian telescope.

Catastrophes with Simple Models

The simultaneous processes of expansion and contraction to which star clusters are subject can be understood physically from very simple models. While precise numerical results can be obtained only from the realistic, detailed calculations discussed later in this article, these simple models are helpful in understanding and interpreting the more complex calculations. Three of these simple models, each of which leads the cluster to catastrophe in a different way, are presented below.

In the first model, discussed some 40 years ago (6, 7), the cluster is regarded as a uniform sphere, whose density ρ and rms velocity v_m are constant. The total mass is M, and all stars are taken to have the same mass, m. We make use of the virial theorem, which states that for an isolated system of self-gravitating mass points in equilibrium

$$2T = Mv_m{}^2 = -W \tag{4}$$

where T is the total kinetic energy and W is the total gravitational energy. Thus the average kinetic energy per star is half the corresponding average gravitational binding energy. However, the average change of potential energy involved in removing one star initially from the cluster is twice the average potential binding energy of all the stars; this may be seen if one computes the energy required to disassemble the entire cluster—the energy required per star declines steadily as the remaining mass decreases, with the initial value twice the average value. It follows that the average energy for escape of the first few stars is not twice but four times the average kinetic energy; if we denote the escape velocity by v_{esc}, we obtain the general result for any isolated stellar system

$$<v_{esc}{}^2> = 4v_m{}^2 \equiv 4<v^2> \tag{5}$$

where the brackets denote average values over all the stars.

The Maxwellian distribution in Eq. 3 can be used to compute the fraction, ξ_e, of stars for which $v^2 > 4v_m{}^2$, giving $\xi_e = 7.4 \times 10^{-3}$. Encounters between stars will tend to establish a Maxwellian velocity distribution during some time interval, which is called the time of relaxation and is denoted by t_r. If we assume that for velocities exceeding $4v_m{}^2$, f approaches its Maxwellian value in the time t_r and that all particles escape if their kinetic energy exceeds four times the average, we obtain

$$\frac{1}{M}\frac{dM}{dt} = -\frac{\xi_e}{t_r} \tag{6}$$

For the relaxation time we adopt the value (8)

$$t_r = \frac{v_m{}^3}{15.4G^2m^2n\ln(0.4N)} \tag{7}$$

where v_m is again the rms stellar velocity, m the stellar mass, n the density of stars per unit volume, and $N = M/m$, the total number of stars in the system. The general form of Eq. 7 follows from the fact that the cross section for a 90° deflection in the relative orbit is of order $\pi(Gm/v^2)^2$; the numerical constant and the logarithmic term are obtained from the detailed theory of stellar encounters. For the uniform sphere considered here t_r is independent of position in the cluster.

The assumption that the fraction of stars escaping during the time t_r is given so directly by f_M is, of course, a simplification. A solution of the Fokker-Planck equation for a system of stars in a hypothetical square-well spherical potential (constant inside the cluster and zero outside) gives (9) Eq. 6 with the constant

ξ_e now equal to 8.5×10^{-3}. The stars which diffuse to values exceeding v_{esc} leave the cluster with very little excess energy. As a result, the total energy of the cluster, proportional to M^2/R, where R is the cluster radius, remains constant as M decreases. Hence $v_m{}^2$ varies as $1/M$, n varies as M^{-5}, and Eqs. 6 and 7 may be integrated approximately to yield (*10*)

$$M(t) = M(0)\left(1 - \frac{7}{2}\frac{\xi_e t}{t_r(0)}\right)^{2/7} \quad (8)$$

where $M(0)$ and $t_r(0)$ are the initial values of M and t_r. Evidently, evaporation of stars produces a collapse of the cluster, with cluster mass M and radius R approaching zero together after a time interval equal to $2t_r(0)/7\xi_e$.

Equation 8 is applicable not only to this idealized homogeneous cluster but also to any cluster of constant total energy, E_T, which undergoes homologous contraction; that is, a cluster in which the smoothed density is a function of $r/r_c(t)$, where $r_c(t)$ is some characteristic time-dependent cluster dimension, either the outer radius of a uniform cluster or the radius of a compact central core.

For homologous contraction the structure of the system, including the spatial variation of ρ and v_m, remains constant except for time-dependent scale factors. If the evaporating stars carry away appreciable energy, diminishing E_T, and if ζ is the ratio of the fractional loss of energy to the fractional loss of mass, then

$$\zeta = \frac{dE_T}{E_T dt}\bigg/\frac{dM}{M dt} \quad (9)$$

and Eq. 8 is replaced (*10*) by

$$1 - (3.5 - 1.5\zeta)\frac{\xi_e t}{t_r(0)} = \left[\frac{M(t)}{M(0)}\right]^{3.5 - 1.5\zeta} \quad (10)$$

In addition, r_c becomes proportional to $M^{2-\zeta}$, and v_m to $M^{(\zeta - 1)/2}$.

We turn now to a second model, in which the cluster is replaced by an isothermal sphere, whose equilibrium structure has been extensively studied. Since in such a sphere $\rho(r)$ varies asymptotically as $1/r^2$, the mass is infinite if the radius is infinite. To give a model with finite mass, the sphere is truncated at some radius R with a hypothetical rigid, confining shell. For large R, the phenomena of interest occur well inside this confining surface, which does not much affect the results. Since the central regions of a cluster are in fact nearly isothermal, this model is much more realistic than the first.

This model is subject to the remarkable "gravothermal" instability (*11*), associated with the negative specific heat of self-gravitating stellar systems. According to the virial theorem in Eq. 4, the total energy $T + W$ is, of course, negative and equal to $-T$. Thus if the total energy is increased (becomes less strongly negative), T will decrease. For example, if a small satellite loses energy as it orbits around the Earth (from frictional retardation by the Earth's atmosphere), it spirals inward, accelerating its motion, so that the centrifugal force remains nearly in balance with the gravitational force.

Since Eq. 4 applies to isolated systems confined by their self-gravitational attraction, it is not strictly valid for a system confined by a rigid wall. Nevertheless, results based on this equation provide a good first approximation for the compact core of a bounded isothermal sphere, in view of the dominant self-attraction of this core.

Consequently, the core of an isothermal sphere can contract, heat up, and release energy, which flows to the outer regions. The outer regions, being less bound gravitationally, will tend to have a positive specific heat; but if the sphere is sufficiently condensed at the center, the core temperature will increase faster than the temperature of the outer regions, and the temperature gradient will increase, accelerating the core collapse. The rate of the collapse will be limited only by the rate at which heat can flow outward. Analysis shows that if the velocity distribution is nearly isotropic, the gravothermal instability can occur (*11, 12*) if the density at the center exceeds the density at the assumed bounding shell by a factor of 709.

A detailed time-dependent solution for such a collapsing sphere has been found (*13*) on the assumption that the contraction is homologous, as defined above. The result does not apply exactly to actual clusters, since the mean free path is assumed to be short, and the velocity distribution is consequently nearly isotropic. In fact, stars on radial orbits, which pass frequently through the heated collapsing core, will have a higher kinetic energy than stars in outer circular orbits, which are less immediately affected by the process of collapse. In the short-mean-free-path approximation, Eq. 10 is valid, with M replaced by M_c, the mass in the core, which remains essentially isothermal. The value of ζ is found to be 0.74, giving v_m varying very slowly with the core density ρ_c (as $\rho_c{}^{0.047}$). The density distribution outside the core differs slightly from that of an isothermal sphere in equilibrium, with ρ varying asymptotically as $r^{-2.21}$ instead of r^{-2}. While the inner regions have an inward velocity, the outer regions move outward, with the velocity vanishing at the radius where $\rho(r)/\rho(0) = 0.0071$. As we shall see below, the properties of this theoretical model are in general agreement with those obtained from more detailed, more realistic models.

A third simplified model considers effects associated with stars of two different masses, which tend toward equipartition of energy as a result of mutual encounters. In this model, the system of heavier stars must inevitably collapse if their relative number exceeds a small limiting value. We omit the detailed analysis but derive this result from simplified physical arguments. First we assume that $\rho_2(0)$, the smoothed density of heavy stars at the cluster center, is small compared to $\rho_1(0)$, the corresponding density for the lighter stars. We can then assume that the gravitational potential is entirely produced by the lighter stars. We take this potential, $\phi(r)$, to be zero at $r = 0$. Then in equilibrium, the radial distance attained by stars of each type is determined by the condition that the mean potential energy is proportional to the mean kinetic energy. If $\rho_1(r)$ is constant with r, the gravitational potential varies as r^2 according to Eq. 1, and we may write

$$\frac{v_{2m}{}^2}{v_{1m}{}^2} = k\frac{r_{2m}{}^2}{r_{1m}{}^2} \quad (11)$$

where $v_{2m}{}^2$ and $v_{1m}{}^2$ are the mean square velocities for stars of the two types, and $r_{2m}{}^2$ and $r_{1m}{}^2$ are the mean square distances from the center. The numerical constant k is needed because in determining $r_{1m}{}^2$ one cannot neglect the decrease of ρ_1 with increasing r, and the effect of this change on $\phi(r)$. This effect is negligible for the heavier stars, provided the mass m_2 of such a star appreciably exceeds m_1.

In equipartition, $v_{2m}{}^2/v_{1m}{}^2$ equals m_1/m_2. Evidently as m_2/m_1 becomes larger, the equilibrium condition (Eq. 11) requires that $r_{2m}{}^2/r_{1m}{}^2$ decrease; as the velocities of the heavier stars become smaller, because of equipartition, the radial distance out to which they can rise, against the gravitational attraction of the lighter stars, decreases in proportion.

However, equilibrium becomes impossible if the ratio $\rho_2(0)/\rho_1(0)$ becomes too great, since in this circumstance the self-attraction of the heavier stars becomes appreciable, and the value of $v_{2m}{}^2$ required for equilibrium consequently increases as r_{2m} decreases. Thus if the total mass M_2 of the heavier stars is sufficiently large compared to M_1, the total mass of the lighter stars, there is no equilibrium distribution of heavy stars in which v_{2m} is much less than v_{1m}. From Eq. 11, plus the assumed equipartition of kinetic energies, we find that

$$\frac{\rho_2(0)}{\rho_1(0)} = \kappa k^{3/2} \frac{M_2}{M_1} \left(\frac{m_2}{m_1}\right)^{3/2} \qquad (12)$$

where κ is another numerical constant relating $\rho(0)$ to M/r_m^3. Determination of the critical value of $\rho_2(0)/\rho_1(0)$, below which equilibrium is possible, and of the constants κ and k shows (*14*) that $(M_2/M_1)\,(m_2/m_1)^{3/2}$ must be less than 0.16 for equilibrium. For higher values, the loss of kinetic energy to the lighter stars will lead to continuing contraction of a dense system of the heavier stars, which, as we have seen above, will heat up as they lose energy, another example of the negative specific heat of a self-gravitating system.

In the realistic models described below, the three effects shown here separately all occur together, each contributing to the cluster collapse.

Detailed Models of Globular Clusters

To follow the dynamical evolution of a spherical cluster a number of detailed numerical calculations have been made. While the procedures have varied, all have considered the motion of point-mass stars in the smoothed potential, ϕ_A, given by Eq. 1, with perturbations of these motions by two-body encounters. We discuss first the analyses of systems that are (i) isolated from other gravitating masses and (ii) composed of stars all of the same mass. While these two assumptions are unrealistic, they simplify the problem and provide a clear indication of the physical processes involved.

Two different approaches have been followed. In the first, the orbits of stars in the smoothed potential field are considered, and the changes in energy, E, and angular momentum, J, resulting from stellar encounters are considered. This approach has been adopted in Monte Carlo computations (*15, 16*), with a number of representative stars followed through time, with frequent small changes in E and J computed in accordance with the appropriate probability distributions. The Fokker-Planck equation, transformed to give the diffusion of stars in E, J space, has also been solved numerically (*17*). In the second approach (*8, 18*), the motions of 1000 representative stars in the potential field $\phi_A(r)$ are followed by numerical integration; frequent small changes in velocity, produced by two-body stellar encounters, are obtained with the usual Monte Carlo techniques. In both approaches, changes of the smoothed potential with changing density are, of course, taken into account.

The results obtained by these different methods are in close agreement. The various models show that whatever its initial origin, the spherically symmetrical system develops a core-halo structure, with a nearly isothermal central region surrounded by a halo in which the orbits are mostly radial. The resultant structure is shown in Fig. 2, where the computed values of the smoothed density, ρ, are plotted against radius, r (both in dimensionless units). This particular system began as a homogeneous sphere, shown by the dotted line, with all stars in circular orbits about the cluster center but with random orientation. Evidently for r less than about 50, the density profile in the evolved system is close to that of an isothermal sphere. At larger r, the orbits are more nearly radial, and the density approaches the theoretically anticipated (*19*) relation $\rho \propto r^{-3.5}$ for an isolated cluster, shown by the dashed line.

Before discussing further the results obtained with these numerical models, we introduce the reference relaxation time t_{rh}, which is a convenient measure of a cluster's evolutionary age; the quantity t_{rh} is defined as the value of Eq. 7 when $\rho \equiv mn$ is set equal to the mean density inside the radius r_h, containing half the cluster mass, and v_m is set equal to the rms velocity for the entire cluster. During the evolution of the cluster, t_{rh} usually remains relatively constant. If the virial theorem is used to equate v_m^2 and $0.4\ GM/r_h$ (a reasonably accurate approximation for $-W/M$), we obtain

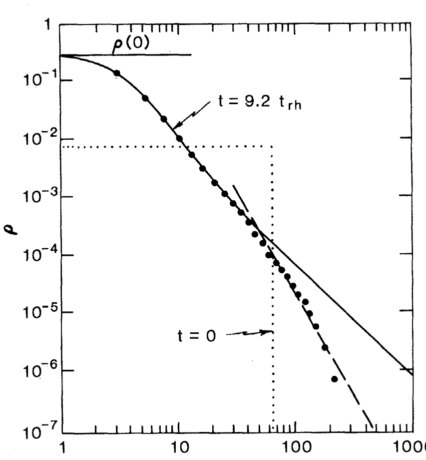

Fig. 2. Structure of an evolving globular cluster. The dotted line shows the initial density, ρ, as a function of radius, r, at $t = 0$, when the sphere is in equilibrium, with all orbits circular. The plotted points (*18*) show $\rho(r)$ at $t = 9.2\ t_{rh}$, where t_{rh} is the reference relaxation time (see text). The solid curve represents the theoretical relationship for an isothermal sphere, and the dashed straight line represents ρ varying as $r^{-3.5}$, the behavior predicted theoretically in the halo, where the orbits are predominantly radial.

$$t_{rh} = \frac{0.060\ M^{1/2} r_h^{3/2}}{m G^{1/2} \log(0.4N)} \qquad (13)$$

If we express r_h in parsecs and m in terms of the solar mass, $M_\odot$, we obtain for t_{rh} in years

$$t_{rh} \frac{0.90 (r_h)^{3/2} N^{1/2}}{(m/M_\odot)^{1/2} \log(0.4N)} \times 10^6 \qquad (14)$$

For most clusters t_{rh} is from 10^8 to 10^{10} years. For comparison, the ratio of the period of a circular orbit at the half radius, r_h, to t_{rh} equals $148 \log(0.4N)/N$ and is less than 1/147 for $N > 10^5$. This small ratio is an example of the general result, referred to earlier, that the mean free path for a star, before encounters strongly modify its velocity, much exceeds the dimensions of a typical cluster.

The system shown in Fig. 2 is rather advanced in its evolution, which has proceeded for a time interval $9.2\ t_{rh}$ since the origin of the uniform system at $t = 0$. As we see below, the collapse of the core to a central singularity occurs at $12.1\ t_{rh}$ for this particular model, only $2.9\ t_{rh}$ after the state shown in Fig. 2.

Another important characteristic of a cluster, in addition to its density profile, is the variation of the phase space density function $f(r, v)$ and, in particular, how this differs from Eq. 2, valid for an isothermal sphere. In the central regions, the velocity distribution is isotropic and f is a function of E only. Values of $f(E)$ near the center of a model cluster (*17*), relatively late in its evolution, are plotted in Fig. 3; both f and E are in dimensionless units. Since the relaxation time much exceeds the orbital period of a cluster star, the phase space density is relatively constant along each orbit in the cluster. Hence these values of $f(E)$ refer to all radial orbits, at any distance from the cluster center. The figure shows that, as expected, for appreciable $-E$, encounters between stars establish the exponential form of $f(E)$ in Eq. 2. However, $f(E)$ must clearly vanish for positive energy, since unbound stars escape rapidly. As shown by the light upper curve in Fig. 3, the computed values of f can be fitted reasonably well with the "lowered Maxwellian" distribution, given by

$$f(E) = K(e^{-\beta E} - e^{-\beta E_0}) \text{ for } E < E_0 \qquad (15)$$

and vanishes otherwise. For an isolated cluster the energy E_0 at which $f(E)$ is taken to vanish is zero.

Last, we discuss the evolutionary changes shown by these numerically derived globular clusters. Not surprisingly, the numerical models demonstrate the three effects found earlier with simpler models—escape of stars, gravothermal instability, and mass segregation; the relative importance of each in the final collapse is demonstrated by these more realistic models. The observed rate of escape from the cluster may be used in Eq. 6, with the reference relaxation time t_{rh} replacing t_r, to determine an effective value of ξ_e. The escape of stars from an isolated cluster results from the energy change of a halo star, with an energy only slightly negative, in its passage through the high-density central region where encounters are important. Thus the period of these halo stars plays an important part in the escape rate. As a result, models for computing the diffusion of orbits in *E, J* space which ignore the values of the orbital periods give no escape at all from an isolated cluster. The models integrating the detailed equations of motion yield "observed" values of ξ_e of about 3×10^{-3}. Although this evaporation process, together with the gradual accumulation of stars in the far halo, is responsible for the initial contraction of the cluster core, as in the simple model, it can apparently not explain the later evolutionary stages of isolated clusters.

The nature of this final evolution is shown in Fig. 4, where the radii containing indicated percentages of the total initial mass are shown plotted against the time (*1*). The initial state of the system was the same uniform sphere in equilibrium whose structure is plotted in Fig. 2. The radii are expressed in units of r_h, the radius initially containing half the mass, while one unit of the dimensionless time indicated equals about 9.1 t_{rh}. For comparison, extrapolation of the results indicates that the core approaches a singularity at a "collapse time," t_{coll} equal to 12.1 t_{rh}. The agreement between the solid line (obtained from the diffusion of orbits in *E* and *J*) and the plotted points (obtained from integration of dynamical trajectories) is excellent. The figure shows that the outer half of the cluster mass expands almost from the very beginning, while the radii containing less than half the mass first contract, then expand; the late expansion for the radii containing 2 and 10 percent of the mass is shown in calculations (*16*) that extend closer to the final collapse.

The general evolutionary behavior of these models is remarkably close to the behavior predicted for the gravothermal instability in a gaseous sphere. In the contraction of the central core, r_c^2 is observed to decrease nearly linearly with $t_{coll} - t$, permitting an accurate determination of the time t_{coll} at which the density becomes infinite; in the theoretical model $r_c^{1.9}$ varies linearly with t_{coll} − t. Similarly, v_m increases as $\rho_c^{0.05}$, in agreement with a theoretical variation as $\rho_c^{0.047}$. Finally the density at the radius where contraction stops and expansion begins is about two orders of magnitude less than the central density, as compared with the theoretical ratio 0.0071 noted above. Exact agreement is not to be expected, in view of the approximations in the instability theory, especially the assumption of a short mean free path and a consequently isotropic velocity distribution. However, there seems little question that the collapse found in the model clusters must be due to the gravothermal instability of an isothermal sphere.

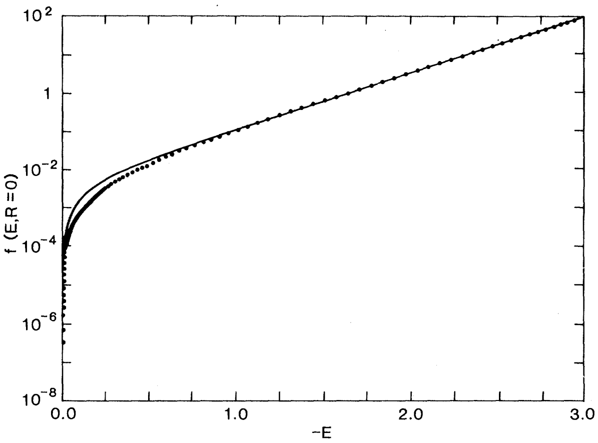

Fig. 3. Phase space density for radial orbits in an evolved cluster. The points represent *f*, the density in phase space for stars at zero distance *R* from the center of a highly evolved cluster (*17*). These values also apply to all radial orbits passing through the central core. The light upper curve represents a lowered Maxwellian, given by Eq. 15 with $E_0 = 0$.

The numerical models also show the expected mass segregation when stars of differing masses are assumed to be present. This segregation occurs relatively rapidly, within a time of 1 to 2 t_{rh}. For example, a two-component model was computed with a stellar mass ratio of 5 to 1, and with 10 percent of the cluster mass in the heavier stars, uniformly distributed initially. After a time interval of only 0.81 t_{rh}, at the cluster center the more massive stars provide 62 percent of the smoothed stellar density, a dramatic increase in the relative densities in the two stellar components. While the initial collapse rate for these models results from the tendency towards equipartition, at a later time this process slows down, since the relative number of lighter stars near the center becomes progressively smaller. Hence it seems likely, though not yet proven, that the final collapse of multicomponent models is due to the gravothermal instability.

Several other effects must be taken into account before a detailed comparison can be made between any of these models and the observations. The most important of these is the gravitational force of the Galaxy. The tidal force produced by the mass in the inner regions of our Galaxy can draw some stars out of globular clusters if their distance from the cluster center exceeds the "tidal cutoff," r_t. While the dynamics of the cluster stars in the presence of such a tidal force form a complex problem, a simple first approximation is that the cluster remains spherical, with vanishing density for $r > r_t$. A theoretical model for such a system can be computed (*20*) if a lowered Maxwellian distribution function, given in Eq. 15, is assumed, with E_0 (per unit stellar mass) set equal to $-GM/r_t$, where *M* is again the cluster mass. Such "King models" have been widely used for comparison with observed cluster data; with two parameters, r_c and r_t, they usually provide a good fit to the observed surface density profiles. The evaporation probability ξ_e per time interval t_{rh} for these tidally truncated models can exceed by more than an order or magnitude the corresponding value for an isolated cluster, with a major effect on the evolution.

Other important physical effects are the gravitational perturbations produced when a cluster crosses the galactic plane; such perturbations heat the cluster, leading to the escape of halo stars and usually (if somewhat paradoxically) a more rapid collapse of the central core (*1*). It has even been suggested that most of the high-velocity subdwarf stars in the Galaxy may have been formed in globular clusters that were subsequently dissipated by the increased exporation rate resulting from this process. This scenario requires (*21*) that most of these early clusters had mean densities one or two orders of magnitude smaller than those observed in present clusters.

Mass loss from individual cluster stars also affects the dynamics of the system, although at the present epoch, when giant stars lose mass only shortly before their death, this effect may be somewhat minor. The presence of a massive black hole at the center would certainly affect the cluster dynamics (*2, 16*), although the x-ray evidence discussed below does not support this possibility. Finally, binary star effects may be important.

Formation and Evolution of Binaries

Binaries are potentially very important in cluster evolution because they can give up energy to passing stars and become more and more tightly bound. The energy available is more than enough to slow down or even reverse the core collapse discussed above. If we regard the two stars in a binary system as mass points, with masses m_A and m_B, the total energy, denoted by $-x$, can be written as

$$x = Gm_Am_B/2a \qquad (16)$$

where a is the semimajor axis of the binary orbit. The factor 1/2 in Eq. 16 results from Eq. 4, according to which the average kinetic energy is half the average negative gravitational energy.

When a single star encounters a binary, the net binding energy, x, may be changed. The result depends critically on whether the mean stellar kinetic energy of translation is large or small compared to x. If the former, the binary is called soft; the velocities of the stars in the binary orbit are less than those of passing stars, and one would expect from equipartition arguments that encounters will impart energy to the binary, decreasing x on the average (*22*). If x exceeds the mean stellar kinetic energy, the binary is called hard; the equipartition argument is now less directly applicable, since a passing star is itself accelerated as it approaches the binary, but x will, in fact, increase on the average (*23, 24*). Thus there is a "watershed" value of x, comparable with the mean kinetic energy of single stars; binaries with greater x will become more tightly bound, on the average, giving up energy to the system.

While the energy absorbed by soft binaries is negligibly small, that given up by hard binaries may strongly influence cluster evolution. In any one encounter, a hard binary may change its binding energy by any amount, from zero to infinity, but on the average the rate of change of x is given by

$$\left\langle \frac{dx}{dt} \right\rangle = \frac{4 \times 3^{1/2} A}{35} \times \frac{G^2 m_s^3 n_s}{v_{sm}} \qquad (17)$$

where m_s is the stellar mass, here assumed the same for all stars, while n_s and

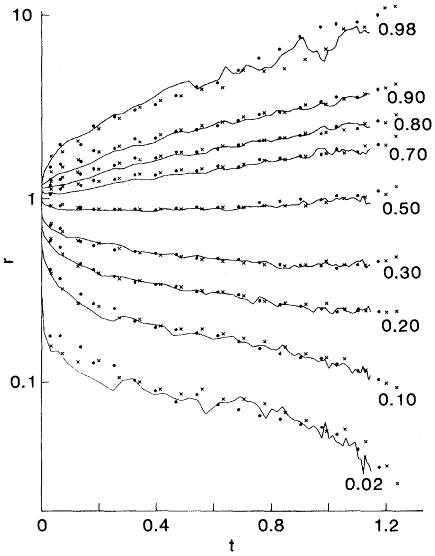

Fig. 4. Time development of an evolving globular cluster. The ordinate shows the values of the radius *r*, containing the fractions of the total mass on the right. The model is the same as the one portrayed in Fig. 2. The solid line represents the results of Monte Carlo computations based on the diffusion of orbits in *E*, *J* space, while the dots and crosses represent results obtained from direct integration of stellar orbits, with diffusion in velocity (*1*).

v_{sm} are the particle density and the random rms velocity of the single stars. The constant A, found by averaging some 10^6 numerical orbits, is between 30 and 35 for x someone to two orders of magnitude greater than $mv_{sm}^2/2$ (*25*). While $\langle dx/dt \rangle$ is nearly independent of x, the increase of x per close encounter averages $0.4x$. As a result, when a binary gradually hardens, it loses energy less and less frequently but in progressively larger increments. When x becomes substantially greater than the energy required to escape from a globular cluster, interaction with a passing star can result in the ejection of both the star and the binary.

Hard binaries, with their important dynamical effects, may appear in globular clusters through three different routes. (i) They may be primordial—that is, present in the initial stellar population from which the cluster formed. (ii) They may be formed by three-body encounters between cluster stars. (iii) They may be produced by dissipative two-body collisions between these stars. Detailed computations of formation rates have been carried out for the second (*24*) and third (*26*) of these processes. For globular clusters these formation rates are negligible under normal conditions but can become important during core collapse.

The effect of primordial binaries on the evolution of a cluster has been analyzed in several models. Computations for a cluster in a square-well potential, the simple model used originally in computation of the evaporation rate, show (*27*) that the energy released from hard binaries will about cancel the contraction produced by evaporation if N, the total number of stars, is in the range 10^4 to 10^5, and if about 35 percent of the mass of the system is in binaries, with all stars of the same mass. However, this result is not supported in a group of Monte Carlo models that consider the detailed density profile of the cluster (*28*). In these models, the binaries, each with twice the mass of a single star, settle toward the center and outnumber single stars in the inner regions, driving the mass segregation instability discussed above. Binary-binary reactions become dominant and have been taken into account approximately. The energy released by binaries is mostly carried away by energetic reaction products, which travel away from the center and give up relatively little of their energy to the central core. As a result, the cores finally collapse, even if 50 percent of the system mass is in primordial binaries. The final stage of this collapse may result from the gravothermal instability shown in systems of single stars.

The details of the evolution shown by these models would certainly have been altered if a distribution of stellar masses had been taken into account. In particular, exchange reactions, in which a heavy single star displaces a lighter star in a binary system, will occur frequently (*29, 30*). Even if only a few binaries are present in the core, these will probably end up containing the heaviest stars, or possibly some even more massive black holes, formed long ago by supernovae.

Unfortunately the relative number of hard primordial binaries in globular clusters is uncertain. The orbital velocity in a hard binary should substantially exceed the random velocities of single stars, and the variable Doppler shift should be easily measurable spectroscopically with modern techniques. In a recent study (*31*) of the globular cluster M3, 33 stars (all of them red giants, the brightest cluster stars) were measured two or more times and showed no variation of radial velocity significantly greater than the measuring error of about 1 km/sec. There is a similar scarcity (*32*) of spectroscopic binaries for other old stars distributed through the galactic disk. Like the globular clusters themselves, these stars were formed early in the life of our Galaxy, and have large velocities, over 100 km/sec relative to the Galaxy.

The chief evidence for the presence of

some binaries in globular clusters is the presence of x-ray sources in several of these systems. In the galactic disk, most strong x-ray sources are believed to be binaries, with gas expelled from a relatively normal star falling on its dense companion, generally a neutron star. The position of eight high-luminosity x-ray sources with respect to the visible cores of globular clusters is shown (*33*) in Fig. 5. The circle represents the radius at which the apparent surface brightness in each core is half its central value. If the visible stars are assumed all to have the same mass (0.8 $M_{\odot}$ for the giant stars in question), the distribution of source positions is consistent (*34*) with a mass of about 1.5 ± 0.5 $M_{\odot}$, supporting the assumption that these are normal x-ray binaries.

In addition to these strong sources, which are believed to be neutron stars, with normal less massive stars as components, weaker x-ray sources, with luminosities less by some three orders of magnitude, are also seen in several clusters (*35*). Some of these are found well outside the cluster cores, suggesting a binary mass less than 1 $M_{\odot}$. The evidence suggests that these sources are binaries in which gas from a normal dwarf star falls onto a lower-mass white dwarf. Thus, some binaries containing neutron stars and some containing white dwarfs seem to be present in a number of globular clusters.

As we shall see later these relatively compact binaries may have been formed during core collapse, especially since the neutron-star binaries are found mostly in the most compact clusters. Thus they provide no evidence for the presence of primordial binaries. While it seems unlikely that primordial binaries will avert the initial collapse of cores, binaries formed early or late in the cluster's history almost certainly play a major role in the final stages of this collapse, a complex process to which we now turn.

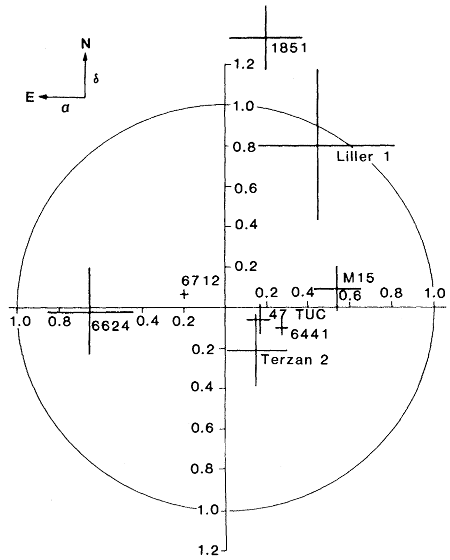

Fig. 5. Location of x-ray sources in eight globular clusters. The circle defines one core radius (see text). The plotted points (*33*) show preliminary Einstein Observatory results for the position of the observed strong sources; the error bars show approximate 1 = σ values. Final positions (*34*) to be published.

To Collapse and Beyond

The final stages of core collapse and the subsequent dynamical fate of the surrounding cluster have been explored only partially. The discussion of this fascinating problem must rely partly on approximate models with homogeneous spherical cores and partly on some provisional, more realistic, calculations.

Interesting guidance is provided by precise dynamical computations for systems with a small value of N (*36*). For a cluster with 250 stars and a distribution of stellar masses, binaries form by three-body encounters in the contracting core, and two of the most massive stars generally end up as a central binary, whose binding energy may exceed half the total binding energy of the cluster. The energy released by the hardening of this massive binary goes into the expansion of the cluster, with escape of many stars. The system gradually dissipates at a slower and slower rate as its density falls. In the final, most probable, accessible state—beyond the range of the computations—the central binary is presumably surrounded by an unbound, expanding aggregation of single stars, binaries, and perhaps some triple systems.

The effect of three-body binary formation on a system with a large N has been computed (*37*) in some detail with a Monte Carlo program, following the diffusion of particle orbits in E and J. Among the modifications made to earlier programs of this type were the following: assumption of a suitable distribution of stellar masses; inclusion of binary formation by three-body encounters (*24*); dynamical integration of encounters between single stars and binaries; inclusion of binary-binary encounters (*28*); and modification of the E, J diffusion calculations to give an approximate value for the escape rate. The results show that binaries are formed in the very central region, harden and then escape because of their recoil energy. The central density reaches a maximum, about 10^5 to 10^6 times its initial value, and then decreases systematically. The total number of binaries also decreases after peak density is reached. The general behavior is qualitatively similar to that of the small N systems. Further studies of such models will be needed to clarify the details of what is happening.

Binaries formed by tidal capture are also important during the collapse phase. The process involved is a close encounter between two stars, with a distance of closest approach between stellar centers equal to roughly three stellar radii. The tides each star produces in the other absorb sufficient energy that the two stars are bound in a highly elliptical orbit. At each successive close approach, further energy is lost, decreasing the semimajor axis of the orbit and the eccentricity, until the orbit becomes circular with a radius roughly twice the original distance of closest approach.

The theory of tidal capture receives some confirmation from the presence of x-ray sources in globular clusters. In particular, the number of weaker sources can be accounted for by this process (*35*) if white dwarfs, thought to be the compact objects in these systems, constitute about 10 percent of the stellar population. The number of binaries containing neutron stars, and constituting the stronger sources, seems also consistent with preliminary calculations based on the tidal capture theory. However, a more conclusive comparison of theory with observation requires more detailed models, taking into account the strong concentration of the relatively massive neutron stars toward the cluster center and the concentration of neutron stars in binaries as a result of exchange reactions with lower-mass binaries.

A binary produced by tidal capture will have an orbital velocity exceeding 100 km/sec. Such a binary is so hard that if it interacts with a single star, both the binary and the star will likely be ejected from the cluster with appreciable energy, and only a small part of the reaction energy will be available to heat the cluster by reducing the mass M of the bound system. On the other hand, tidal dissipation involves a loss of translational kinetic energy by the interacting stars (including the interactions that dissipate only a fraction of this kinetic energy) and therefore cools the system, accelerating the collapse. Detailed calculations (*38, 39*) indicate that this cooling by two-body tidal interactions tends to exceed the heating by three-body binaries, at least during the early stages of collapse.

There are no detailed models that take the spatial structure of the cluster into account and that include the effects of tidal energy loss and tidal capture. Hence the discussion of these effects must be tentative. The newly formed binaries, each with a mass of two stars, will tend to settle to the center of the collapsing cluster. At sufficiently high densities they will interact with each other and with single stars, and will be ejected. This loss of mass from the center of the system tends to produce an expansion of the system, offsetting, perhaps, the tidal cooling. Formation of quadruple systems, with two very hard binaries bound together, will also occur through encounters involving two of these binaries and a third mass. Direct collisions of stars will also occur (*2*), leading either to single stars or to contact binaries, with two stellar nuclei in a common envelope. Some black holes may be formed by coalescence of several stars.

While we do not yet know which processes are most important late in the collapse of a globular cluster, the accelerating collapse must clearly end. Certainly the gravothermal instability cannot produce continued collapse when there are only a handful of stars in the dense central core. The equations used for deriving this instability, either in the short- or long-mean-free-path limit, are not applicable when the number of stars in the core is as small as 10 to 100. After the collapse terminates, whatever processes occur near the center will presumably maintain a quasi-steady state, with a continuing release of energy from binaries leading to a gradual expansion of the cluster as a whole.

The nature of this post-collapse phase has been explored in some half dozen investigations, most of them based on the assumption that some unspecified source of energy is available at the center. Solutions involving homologous expansion have been obtained, again assuming stars all of the same mass. These solutions are very similar to those for collapse, except that conditions are more nearly isothermal (*40*). In one investigation (*41*) large-amplitude oscillations were found under some conditions, with the cluster expanding after collapse to a normal pre-collapse configuration, and then repeating the earlier gravothermal collapse.

The nature of the collapse and post-collapse phases is likely to be relevant for at least some globular clusters. According to the model computations, the time remaining at any instant until the singularity is reached equals about 200 t_{r0} at that instant, where t_{r0} is the central relaxation time. A detailed tabulation of globular cluster properties (*42*) shows that of 41 values of t_{r0}, 7 are less than 10^8 years; 4 of these are less than 3×10^7 years. The distribution of these t_{r0} values among clusters is consistent with the view that a few of these clusters have already collapsed and are now expanding (*43*). However, these are not readily distinguished from clusters which are still collapsing.

In any case clusters not far from collapse should show a density increase far into the cluster. In fact, precise profile measurements (*44*) have recently shown that several compact clusters do show radial density gradients extending into radii of about 1 arcsecond, the limit set by variable refraction through our inhomogeneous atmosphere. In a few years, the Hubble Space Telescope, yielding 0.1-arcsecond images, may give more conclusive data on the inner structure of these systems and should help to unveil the final evolutionary history of globular clusters.

References and Notes

1. L. Spitzer, in *Dynamics of Stellar Systems* (IAU Symposium Number 69), A. Hayli, Ed. (Reidel, Dordrecht, Netherlands, 1975), p. 3.
2. A. P. Lightman and S. L. Shapiro, *Rev. Mod. Phys.* **50**, 437 (1978).
3. S. Chandrasekhar, *Principles of Stellar Dynamics* (Univ. of Chicago Press, Chicago, 1942).
4. ______, *Astrophys. J.* **97**, 255 (1943).
5. M. N. Rosenbluth, W. M. MacDonald, D. L. Judd, *Phys. Rev.* **107**, 1 (1957).
6. V. A. Ambartsumian, *Ann. Leningrad State Univ. (Astron. Ser.), No. 22* (1938); *Tr. Astron. Obs.*, Issue 4, p. 19.
7. L. Spitzer, *Mon. Not. R. Astron. Soc.* **100**, 396 (1940).
8. L. Spitzer and M. H. Hart, *Astrophys. J.* **164**, 399 (1971).
9. L. Spitzer and R. Harm, *ibid.* **127**, 544 (1958).
10. I. R. King, *Astron. J.* **63**, 114 (1958).
11. V. A. Antonov, *Vestn. Leningr. Univ. Mat. Mekh. Astron.* **7**, 135 (1962).
12. D. Lynden-Bell and R. Wood, *Mon. Not. R. Astron. Soc.* **138**, 495 (1968).
13. D. Lynden-Bell and P. P. Eggleton, *ibid.* **191**, 483 (1980).
14. L. Spitzer, *Astrophys. J. Lett.* **158**, L139 (1969).
15. M. Hénon, *Astrophys. Space Sci.* **13**, 284 (1971); *ibid.* **14**, 151 (1971).
16. A. B. Marchant and S. L. Shapiro, *Astrophys. J.* **239**, 685 (1980).
17. H. Cohn, *ibid.* **234**, 1036 (1979).
18. L. Spitzer and T. X. Thuan, *ibid.* **175**, 31 (1972).
19. L. Spitzer and S. L. Shapiro, *ibid.* **173**, 529 (1972).
20. I. R. King, *Astron. J.* **71**, 64 (1966).
21. J. P. Ostriker, L. Spitzer, R. A. Chevalier, *Astrophys. Lett.* **176**, L51 (1972).
22. L. E. Gurevich and B. Yu. Levin, *Astron. Zh.* **27**, 273 (1950); *NASA Technical Translation TT F-11, 541* (1968).
23. D. C. Heggie, in *Dynamics of Stellar Systems* (IAU Symposium Number 69), A. Hayli, Ed. (Reidel, Dordrecht, Netherlands, 1975), p. 73; see also the discussions on pp. 93 and 94.
24. ______, *Mon. Not. R. Astron. Soc.* **173**, 729 (1975).
25. P. Hut, *Astrophys. Lett.* **272**, L29 (1983).
26. W. H. Press and S. A. Teukolsky, *Astrophys. J.* **213**, 183 (1977).
27. J. G. Hills, *Astron. J.* **80**, 1075 (1975).
28. L. Spitzer and R. Mathieu, *Astrophys. J.* **241**, 618 (1980).
29. J. G. Hills, *Astron. J.* **82**, 626 (1977).
30. ______ and L. W. Fullerton, *ibid.* **85**, 1281 (1980).
31. J. E. Gunn and R. F. Griffin, *ibid.* **84**, 752 (1979).
32. H. Abt, *ibid.*, p. 1591.
33. J. E. Grindlay, in *X-Ray Data with the Einstein Satellite*, R. Giacconi, Ed. (Astrophysics and Space Science Library, Reidel, Dordrecht, Netherlands, 1981), vol. 87, pp. 79–109.
34. ______, P. Hertz, J. E. Steiner, S. S. Murray, A. P. Lightman, *Astrophys. Lett.*, in press.
35. P. Hertz and J. E. Grindlay, *ibid.* **267**, L83 (1983).
36. S. J. Aarseth and M. Lecar, *Annu. Rev. Astron. Astrophys.* **13**, 1 (1975).
37. J. S. Stodolkiewicz, *Publ. Astron. Inst. Czechoslovak Acad. Sci.* **56**, 103 (1983).
38. L. M. Ozernoy and V. I. Dokuchaev, *Astron. Astrophys.* **111**, 1 and 16 (1982).
39. S. Inagaki, *Mon. Not. R. Astron. Soc.* **206**, 149 (1984).
40. ______ and D. Lynden-Bell, *ibid.* **205**, 913 (1983).
41. E. Bettwieser and D. Sugimoto, *ibid.* **208**, 493 (1984).
42. C. J. Peterson and I. R. King, *Astron. J.* **80**, 427 (1975).
43. H. Cohn and P. Hut, *Astrophys. Lett.* **277**, L45 (1984).
44. S. Djorgovski and I. R. King, *ibid.*, p. L49.
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