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Localization, Interactions, and the Metal-Insulator Transition

R. C. Dynes and P. A. Lee

Summary. Recent advances in our understanding of electronic conduction have pointed up deficiencies in traditional thinking. For a metal at a sufficiently low temperature, it is known both theoretically and experimentally that the conventional picture in terms of the Boltzmann theory breaks down. Improved understanding of both electron localization and the effects of electron-electron interactions in a disordered medium has led to experimentally verifiable predictions. These effects have an important influence on the nature of the metal-insulator transition.

Any material can be characterized as a metal or an insulator according to the following definition: upon extrapolation to absolute zero temperature, if the conductivity remains finite, it is a metal, and if the conductivity goes to zero, it is an insulator. The zero-temperature conductivity of a metal can be understood in terms of the scattering of the conduction electrons by impurities. Furthermore, as the temperature is raised, the traditional picture in terms of Boltzmann transport theory states that thermal excitation of various inelastic processes enhances the scattering rate, so that the conductivity decreases. Phase space arguments show that the conductivity can be described by

$$\sigma = \sigma_0 - AT^n \qquad (1)$$

where T is temperature and the power $n = 2$ if the scattering is due to electron-electron collision and is generally larger than 2 if other scattering processes such as electron-phonon scattering dominate.

Recent theoretical developments, together with experiments on a variety of systems, have shown that almost all aspects of Eq. 1 are wrong as far as the asymptotic low-temperature behavior is concerned. The power n is found to be much less than 2 (instead it is typically 1/2) and the sign of A is often negative, so that the conductivity of a metal increases with increasing temperature. The new understanding of the metal also has important implications for the understanding of the insulator-to-metal transition, which is the process by which an insulator is transformed into a metal by changing some material parameter such as the concentration of dopants in a semiconductor. The recent advances are based on improved understanding of two aspects of the problem: Anderson localization and the effects of electron-electron interactions in a disordered medium.

Anderson Localization

The concept of Anderson (*1*) localization deals with the nature of a one-electron wave function in a disordered medium. If the disorder is weak the wave function is extended; that is, it is like a plane wave except that its phase becomes randomized on a length scale defined as the mean free path l. In 1958, Anderson showed that if the disorder is strong the wave function may change its nature completely and become localized; that is, the wave function envelope decays exponentially from a center. The decay length ζ is the localization length and may become much longer than the mean free path. When the wave function at the Fermi energy becomes localized, we have an insulator. If the disorder is gradually reduced, the localization length increases until at some point the wave function becomes extended and an insulator-to-metal transition occurs.

Mott (*2*) proposed that at this transition, the conductivity jumps to a finite value σ_{min}. His reasoning was based on an extrapolation to the strong disorder region of the usual Boltzmann formula for the conductivity of a metal

$$\sigma = \frac{e^2}{3\pi^2\hbar} k_F^2 l \qquad (2)$$

where e is the electron charge, $\hbar$ is Planck's constant divided by 2π, and k_F^{-1} is the de Broglie wavelength of the electron. It is reasonable to suppose that localization sets in when the mean free path l becomes of order of k_F^{-1}, because if l becomes any shorter, the phase is so random that a plane wave description no longer makes sense. Putting the so-called Ioffe-Regel criterion (*3*) ($k_F l \approx 1$) into Eq. 2, we obtain Mott's estimate of σ_{min} (up to a numerical factor)

$$\sigma_{min} \approx \frac{e^2}{3\pi^2\hbar} k_F \qquad (3)$$

It is interesting to observe that $e^2/\hbar = (2.44 \times 10^{-4})\ (\text{ohm})^{-1}$ has the dimensions of a conductance, so that Eq. 3 can be interpreted as the condition that a microscopic sample of size k_F^{-3} has a conductance of $e^2/\hbar$. This point of view is even more transparent in two dimensions (2D), where conductivity has the same dimension as conductance and Eq. 3 becomes

$$\sigma_{min}^{(2D)} \approx \frac{e^2}{2\pi\hbar} \qquad (4)$$

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These observations led Licciardello and Thouless (*4*) to propose that the conductance $G(L)$ of an L^d sample, expressed in dimensionless units as $g(L) = G(L)/(e^2/\hbar)$, is the relevant variable which determines the behavior of a $(2L)^d$ block built up from L^d blocks in d-dimensional space. This scaling idea was formulated by Abrahams *et al.* (*5*) by introducing a scaling equation

$$\frac{d \ln g}{d \ln L} = \beta(g) \tag{5}$$

The point of this equation is that the right-hand side is a function of g only, independent of L or any other material parameter. Analogous scaling equations are well known in the theory of critical phenomena: in the Ising model a very similar equation exists for the dimensionless variable J/kT, where J is the nearest-neighbor coupling and k is the Boltzmann constant. The mapping of the Anderson localization problem to a phase transition problem has since been put on a firm footing by Wegner (*6*). According to the scaling theory, all the necessary information is contained in the function $\beta(g)$. Abrahams *et al.* (*5*) considered the limiting cases of large and small g. For small g we expect the wave functions to be localized; g must decrease rapidly with increasing L and $\beta(g)$ is large and negative. For large g ($k_F l \gg 1$) we expect to find a metal in the zeroth-order approximations, so that the conductivity is independent of L. Recalling that the conductance is related to the conductivity σ by $G(L) = \sigma L^{d-2}$, we find that $\beta(g) \to (d - 2)$ for large g. Interpolating smoothly between these limits, Abrahams *et al.* produced a picture of $\beta(g)$ as shown in Fig. 1. There are a number of interesting consequences. In one dimension β is always negative, so that g always scales toward $g = 0$ (the localized limit) as L increases. This is in agreement with the known result that all states are localized in one dimension (*7*). In three dimensions β crosses zero at a certain value $g = g^*$. This means that at some critical amount of disorder, the conductance (not the conductivity) is independent of sample size, a peculiar situation indeed. The conductivity of a large sample will then become arbitrarily small, since $\sigma = G/L$. In a slightly less disordered sample, g is slightly greater than g^* initially, but gradually increases as L increases. Beyond a length scale $L = \zeta$, we reach the flat part of the β function ($\beta \to 1$), and the conductivity beyond that length scale becomes constant and is given by

$$\sigma = \frac{e^2}{\hbar} \frac{g^*}{\zeta} \tag{6}$$

As in critical phenomena, the length ζ diverges near g^* as

$$\zeta \sim (g - g^*)^{-\nu} \tag{7}$$

Combining Eqs. 6 and 7 shows that the conductivity goes to zero like a power law near the metal-insulator transition. This result was first obtained by Wegner (*8*) from general scaling arguments and is in disagreement with Mott's concept of σ_{min}.

While the existence of σ_{min} cannot be reconciled with the scaling theory in three dimensions, it is possible in principle in two dimensions. This would require a β function that vanishes for $g > g^*$, as shown by the dashed lines in Fig. 1. However, this is ruled out by Abrahams *et al.* (*5*), who performed a perturbation calculation for large g and showed that

$$\sigma = \sigma_0 - \frac{1}{2\pi^2} \frac{e^2}{\hbar} \ln L \tag{8}$$

where σ_0 is the Boltzmann conductivity given by Eq. 1. If a one-parameter scaling theory exists, Eqs. 8 and 5 imply that $\beta \to -g^{-1}$ in the large g limit. This, together with the assumption that $\beta(g)$ is monotonic, ensures that β is negative for all g. Consequently, even if g is initially very large (very weak disorder), as L increases g scales from right to left in Fig. 1 until it eventually becomes exponentially small (localized). The length scale at which this happens (the localization length) may be exponentially large,

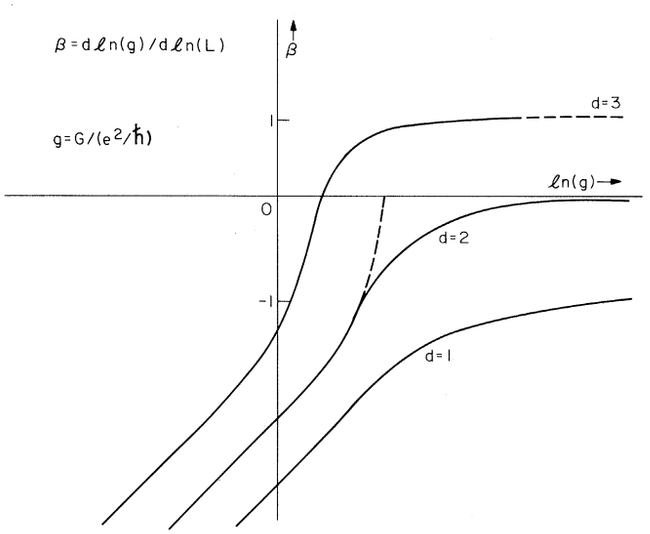
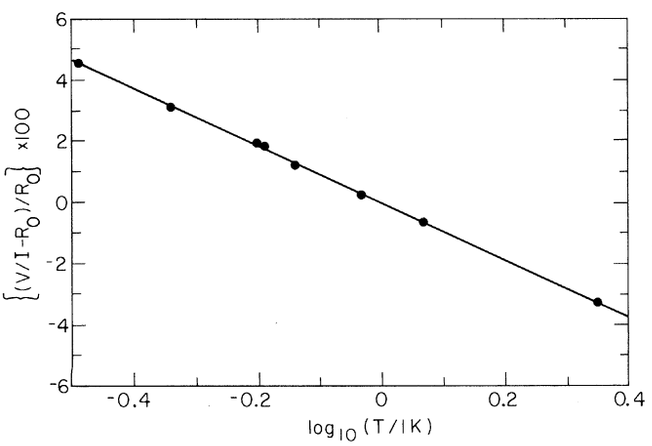


Fig. 1. Variation of the normalized conductance g with length scale L as a function of the conductance [from Abrahams *et al.* (*5*)]. The dimensionality is d, and the dashed line is that expected in two dimensions from the Mott argument. [Courtesy of *Physical Review Letters*]



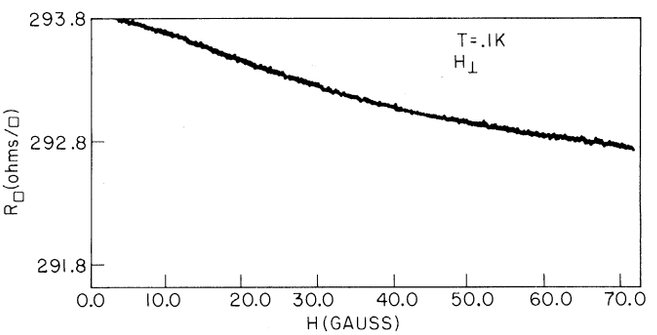


Fig. 2 (left). Resistance of thin Au-Pd films as a function of log T [from Dolan and Osheroff (*16*)]. Fig. 3 (right). Magnetoresistance in perpendicular H field of a two-dimensional electron gas on the surface of a MOSFET (*23*). The solid line through the data is a fit to the theory. [Courtesy of *Physical Review Letters*]

but nevertheless all states are in principle localized in two dimensions. While the original experiment was based on lowest order perturbation theory and an assumption about the existence of the scaling theory, subsequent mapping onto a field theory model has put this picture on a much firmer footing (*6*).

The theory so far has been developed for finite samples at a temperature of absolute zero. To make contact with experiments we need a theory for a finite temperature and a macroscopic sample size. The following physical argument by Thouless (*9*) provides the link. At finite T, an electron undergoes an inelastic collision in a time τ_{in} and loses phase memory. Thus the distance that the electron has diffused during this time

$$L_T = (D\tau_{in})^{1/2} \tag{9}$$

where D is the electron diffusivity, serves as an effective length scale to be used in Eq. 8. In general, $\tau_{in} \propto T^{-p}$, where the power p depends on the mechanism for inelastic scattering. Substitution into Eq. 8 produces the following prediction for the temperature dependence of the conductivity:

$$\sigma(T) = \sigma_0 - \frac{e^2}{\hbar}\frac{p}{4\pi^2} \ln T \tag{10}$$

For an infinite sample, varying T is equivalent to moving along the trajectory of Fig. 1 for $d = 2$. Lowering T results in moving right to left, so that the logarithmic temperature dependence crosses over continuously into the exponentially localized region. Clearly, a rising resistivity with decreasing temperature which eventually reaches infinity is beyond the scope of the Boltzmann transport theory.

Interaction Effects

So far we have considered a single electron moving in a disordered potential. In a real metal, the electrons interact through a screened Coulomb potential. The classical theory of interacting electrons, the Landau-Fermi liquid theory, states that as long as we consider low-lying excitations near the Fermi level, called quasi-particle excitations, the properties of an interacting system are not altogether different from those of a noninteracting one, except that various constants such as effective mass are renormalized (*10*). However, the Fermi liquid theory has been studied in the past mainly for systems without disorder. Thus it came as quite a surprise when Altshuler and Aronov (*11*) pointed out that, even in weakly disordered systems, interaction has a profound effect on the physical properties, leading to various singular corrections at low temperatures. For example, the low-temperature conductivity in three dimensions (3D) is predicted to take the form

$$\sigma_{3D}(T) = \sigma_0 \left[1 + \frac{\alpha}{k_F l}\left(\frac{kT\tau}{\hbar}\right)^{1/2}\right] \tag{11}$$

Here τ is the elastic scattering time for the electron. It is this time which determines the electronic mean free path l. Subsequent work has shown that the constant α can be either positive or negative in sign. Again, Eq. 11 is in strong contrast with the Boltzmann prediction discussed in the introduction. The tunneling density of states N_{3D} is predicted to show a cusp for energy E near the Fermi energy E_F:

$$N_{3D}(E) = N_0 \left\{1 + \frac{\alpha'}{k_F l}\left[(E - E_F)\frac{\tau}{\hbar}\right]^{1/2}\right\} \tag{12}$$

Even more singular behaviors are predicted for two-dimensional systems (*12*):

$$\sigma_{2D}(T) = \sigma_0 + \alpha_2 \ln(T\tau) \tag{13}$$

and

$$N_{2D}(E) = N_0 \left[1 + \frac{\alpha'_2}{k_F l} \ln |(E - E_F)\tau|\right] \tag{14}$$

Note that the logarithmic behavior with temperature is of the same form as the prediction of the localization theory, except that in Eq. 13 the constant α_2 depends on the screening parameter of the material.

Crudely speaking, interaction effects are much enhanced in disordered systems because electrons diffuse rather than freely propagating as plane waves. As a result, an electron stays much longer in a given region in space, where it can interact with other electrons. In the usual Fermi liquid theory, electrons occupy plane wave states that are uncorrelated with each other except for the effects of interactions. In the presence of disorder, the wave functions, while extended, experience the same disordered potential. It is then not surprising that wave functions that are nearby in energy are also correlated in space. It turns out that the requirement of density diffusion alone introduces a sufficient correlation between electrons that interactions between them lead to the singular corrections shown in Eqs. 11 and 14 (*13*).

Thus we see that localization and interaction effects both play important roles in determining the low-temperature properties of metals. It turns out that their relative contributions to the conductivity correction can be separated by the effect of a magnetic field. The interference effect that leads to the logarithmic size dependence of Eq. 8 is a subtle one. Any time reversal symmetry-break-

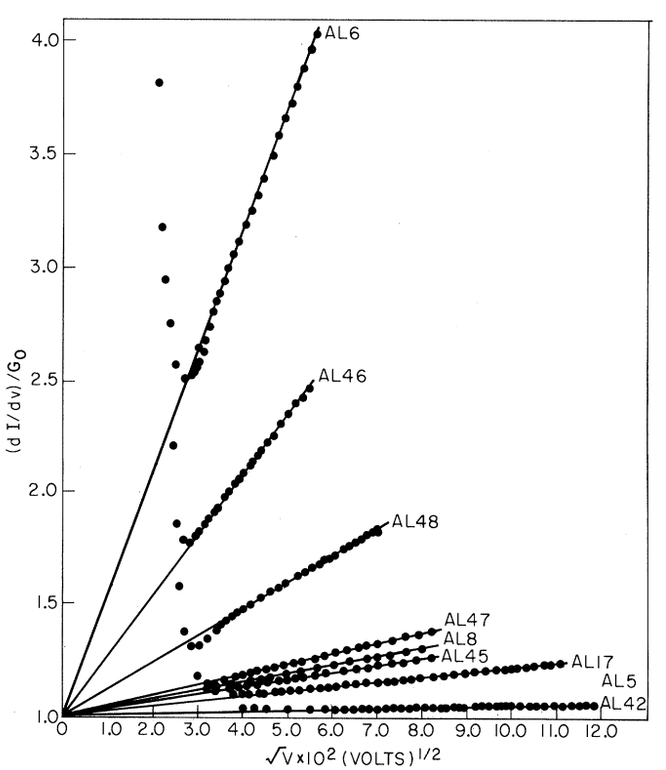


Fig. 4. Tunnel junction conductance $(dI/dV)/G_0$ normalized to the zero-voltage conductance G_0 versus $\sqrt{V}$ for various granular Al samples (*27*). A straight line indicates the $\sqrt{E}$ dependence of the density of states. Lines with steeper slope correspond to samples with increasing resistivity. Note that G_0 goes to zero with increasing resistivity. The deviation at low voltage is due to the superconducting energy gap.

ing terms will destroy this logarithmic dependence. A magnetic field H normal to the plane of the film destroys the time reversal symmetry and introduces an additional length scale, the Landau orbit radius $L_H = (\hbar c/eH)^{1/2}$, where c is the velocity of light. In the scaling process, when the length scale L reaches L_H, the time reversal symmetry-breaking is felt, so that beyond that scale, the conductivity is no longer dependent on L. Thus, instead of Eq. 8, we have (*14*)

$$\sigma(H) = \sigma_0 - \frac{1}{2\pi^2}\frac{e^2}{\hbar}\ln L_H \qquad (15)$$

The changeover from the ln T behavior of Eq. 10 occurs when $L_T = L_H$. At low temperature, L_T can be sizable, of the order of several thousand angstroms, so that the crossover field can be very small, of the order of tens of gauss. Such sensitivity to a magnetic field is highly unusual and does not occur for the interaction-derived ln T correction of Eq. 13.

Experimental Observations

From Fig. 1 it is clear that for $d = 2$ the resistivity is dimensionless, and it is expected that the various phenomena will not depend in a direct way on the material studied but will depend rather generally on the universal parameter g. The Mott argument in two dimensions predicts a transition at a resistance $R = 10$ kilohms per square. It is seen that the scaling extension of these ideas predicts a transition in this general region, but now from logarithmic to exponential behavior. Such a transition has been observed in several systems with widely different physical characteristics. As outlined in the previous section, the L dependence can be related to the T dependence, and an onset to an exponential temperature dependence to conduction at ~ 10 kilohms per square has been seen in such different two-dimensional systems as very thin "metal" films (*15–17*), the inversion layer of a silicon-metal-oxide semiconductor field-effect transistor (MOSFET) (*18, 19*), and semimetal indium oxide films (*20*). These systems with extremely different electron densities display remarkably similar behavior (the interelectron spacing in the metal films is ~ 1 to 2 Å, while in the MOSFET it is ~ 100 Å.

An example of such a logarithmic dependence on temperature for a thin Au-Pd alloy film (*16*) is illustrated in Fig. 2, where it is seen that at low T (below 1 K) R increases logarithmically with decreasing T. At higher R a smooth and continuous transition to exponential behavior results.

This logarithmic temperature dependence is that anticipated from both Eq. 10 (localization effects) and Eq. 13 (interaction effects), and in order to separate the relative contributions the magnetoresistance was measured (*21–23*). As outlined earlier, with an applied magnetic field perpendicular to the conducting sheet a new length is introduced into the problem (*14*), L_H, the Landau orbit radius. With increasing H, L_H decreases in such a fashion as to destroy the localization effect (or reduce the resistance). Interaction effects, on the other hand, do not depend as strongly on this orbital term, and so low-field magnetoresistance measurements allow a determination of the relative contributions of the two logarithmic corrections to the conductance. Figure 3 shows an example of a magnetoresistance measurement (*23*) in a perpendicular magnetic field. The logarithmic sensitivity at relatively low fields reveals the strength of the localization effects, and the solid line through the data represents a fit to the theory. Experimentally, from measurements such as these, it is observed that different systems (different metal films and semiconductors) show varying relative strengths of the localization and interaction contributions to the logarithmic dependence of conductance on H, T, and presumably L. The details of these differences remain to be explored. Hall effect measurements (*24*) have also indicated that both effects contribute to the transport.

Electron tunneling measurements are capable of probing a density of states about the Fermi energy E_F. From Eq. 14 it is predicted that interaction effects

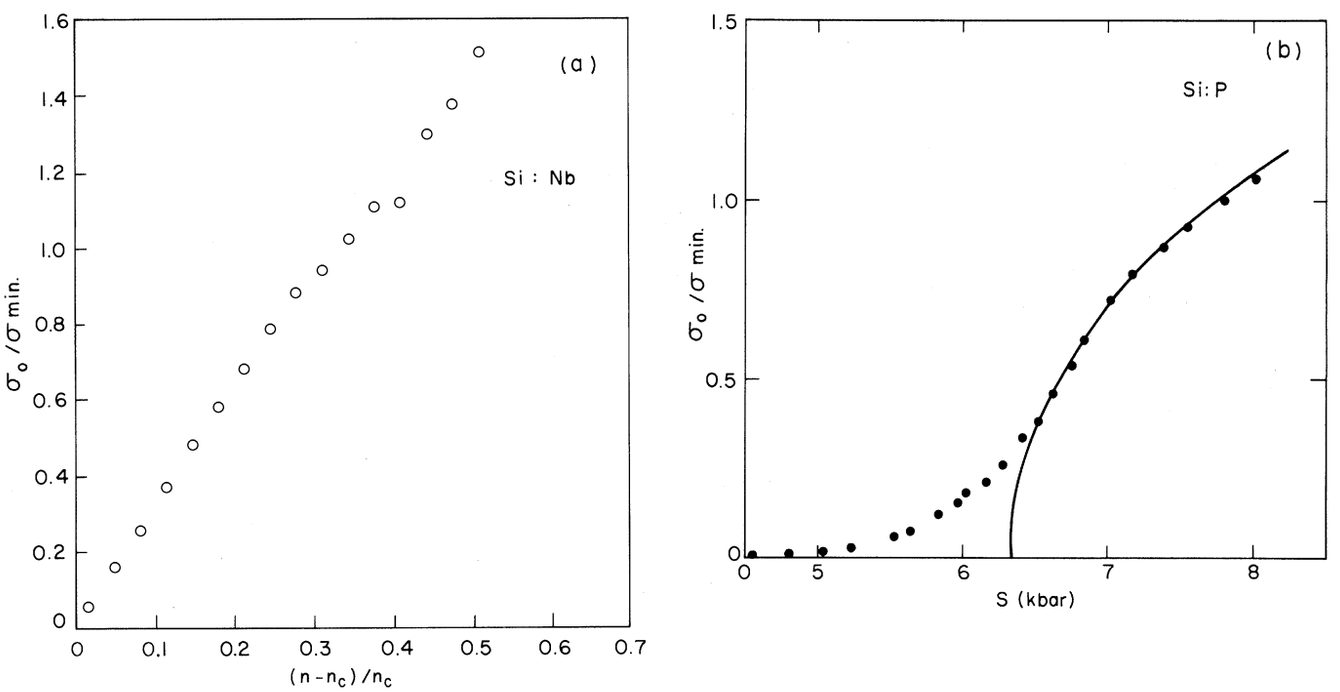


Fig. 5. (a) Low-temperature conductivity (σ_0/σ_{min}) of the amorphous alloy Si:Nb as a function of deviation from critical concentration $(n - n_c)/n_c$. Here n_c is 11.8 percent Nb in Si. (b) Low-temperature conductivity of phosphorus-doped silicon as a function of uniaxial stress S (*33*). The solid line is a fit to $\sigma(0) \propto (S - S_c)^{\nu}$ with $\nu = 0.49$.

rather drastically alter this density of states symmetrically about E_F with a strength that scales like the resistance. Such effects have been observed (*25*) with a strength increasing as the resistance per square approaches 10 kilohms, in qualitative agreement with the predictions of Eq. 14.

From the conductance, magnetic field, and tunneling measurements, it is now clear that both localization and interaction effects play an important role in the nature of electron transport in the limit $k_F l \rightarrow 1$ in two dimensions. It is also clear that the qualitative aspects of the trajectory illustrated for $d = 2$ in Fig. 1 are borne out. Specifically, the experiments show that there is no extended state for any value of conductance. Rather, in the region where Mott anticipated an abrupt localization-delocalization transition, there is a smooth and continuous transition from exponential to logarithmic dependence on L (and T) as anticipated by Fig. 1, and the trajectory asymptotically approaches the limit $\beta = 0$. This single-particle description, however, is quantitatively incorrect as interaction effects become strong (with the same logarithmic dependence). A proper description of transport in this limit must include both effects. These experimental techniques have, in some cases, allowed a quantitative determination of the relative contribution of these two effects, and they point the way to further investigations in other two-dimensional systems.

Many of the techniques used in two dimensions have also been applied to three-dimensional systems. It is clear from Fig. 1 that in the case of three dimensions, unlike that of two dimensions, a metal-insulator transition occurs, and one of the important issues is the nature of that transition—whether it is continuous or discontinuous. As in two dimensions, it is expected that both localization and interaction effects will have an important influence on the nature of the transport in this regime.

From Eq. 12, it is seen that the tunneling density of states is also strongly altered in three dimensions because of interaction effects. In this case a cusp in $N(E)$ which depends on $E^{1/2}$ is predicted again with a strength increasing with decreasing $k_F l$. Electron tunneling measurements have been performed on several systems (*25–28*) to probe this density of states, and the results are in reasonably good agreement with the predictions. Some typical plots of the tunneling conductance (which is approximately the tunneling density of states) for the granular aluminum system (*27*) are shown in Fig. 4. Applied voltage is equivalent to energy in this case, and so these can be interpreted as plots of $N(E)$ versus $E^{1/2}$. The data can be fitted by the expression

$$N(E) = N(0)[1 + (E/\Delta)^{1/2}] \quad (16)$$

with $N(0)$ going to zero as the resistivity of the film increases. The data in Fig. 4 have been normalized to the zero-field tunneling conductance and should be a measure of $N(E)/N(0)$. Insofar as these plots are straight lines (except at lower voltages, where the superconducting energy gap alters the dependence) they demonstrate the validity of the $E^{1/2}$ predication of Eq. 12. In the case shown in Fig. 4 increasing slope is correlated with increasing resistivity. The slopes give a measurement of the energy scale Δ defined in Eq. 16. The data shown here correspond to tunneling into granular aluminum samples with conductivities all less than σ_{min}, and they imply (i) that the metal-insulator transition does not occur abruptly at σ_{min} but proceeds continuously to $\sigma = 0$, and (ii) at the metal-insulator transition $N(E)$ approaches zero continuously through this square root cusp in $N(E)$. This and other tunneling measurements clearly illustrate the influence of interaction effects near the transition and again imply that the single-particle picture must be altered to incorporate these many-body contributions. A phenomenological scaling theory that incorporates both localization and interaction effects has been proposed by McMillan (*29*), and, more recently, a scaling theory was constructed by examining higher order terms in the perturbation theory (*30*).

Transport measurements have been performed in a variety of systems near the metal-insulator transition. Granular aluminum (*31*), doped semiconductors (*32*), and amorphous alloys (*28*) have all been studied, and although some differences between the various systems exist, there are some striking similarities. Conductivity as a function of temperature has been studied in several systems and results qualitatively in agreement with Eq. 11 have been obtained. From all these various measurements, it appears that as a function of some appropriate critical variable (concentration, for example) σ_0 decreases continuously to zero at the transition. Results from two separate studies of σ_0 are illustrated in Fig. 5. The asymptotic σ_0 is plotted in Fig. 5a as a function of concentration deviation from the critical concentration for the amorphous alloy Nb:Si (*28*) and in Fig. 5b as a function of stress for the crystalline semiconductor phosphorus-doped silicon (*33*). Although there are clear differences in the dependences, it is seen that σ_0 decreases continuously to zero at the critical value. It appears in the alloy system that σ_0 varies linearly with n, while in the doped semiconductor the

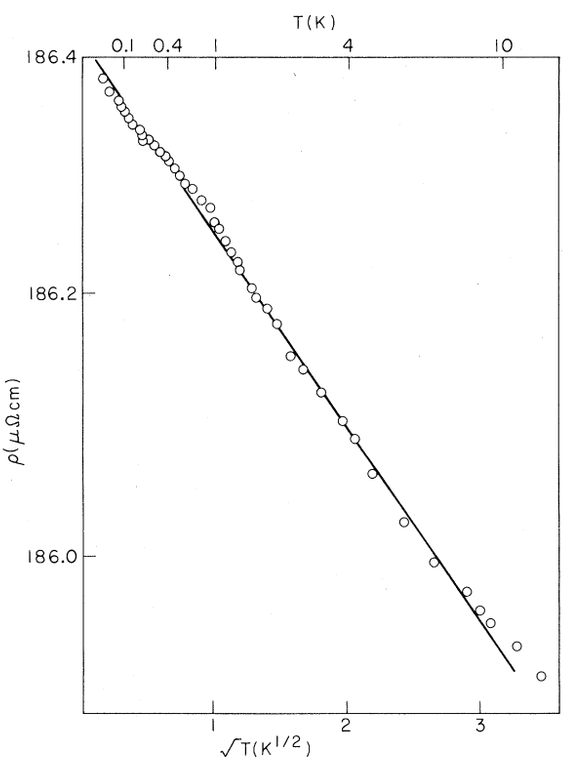


Fig. 6. Resistivity as a function of $\sqrt{T}$ for the metallic glass $Fe_{40}Ni_{40}P_{14}B_6$ (*35*).

variation with stress is best fit by a square root dependence. These details are not understood at present and require further investigation. These results and others do, however, show that no abrupt transition occurs at σ_{min} and the transition appears continuous.

The temperature-dependent portion of the conductivity in these and other systems is seen to behave qualitatively as predicted in Eq. 11. Resistivity measurements on the metallic side of phosphorus-doped silicon show $T^{1/2}$ behavior over more than two decades in temperature (*34*). Another example of this dependence in a quite different system is illustrated in Fig. 6 (*33*), where it is seen that the conductivity in amorphous alloys varies as $T^{1/2}$.

Near the metal-insulator transition, other systems have shown a temperature dependence varying from $T^{1/3}$ to $T^{2/3}$, but the central result—a weak temperature dependence asymptotically approaching a temperature-independent σ_0 at low temperatures—appears general.

Magnetoresistance measurements in three-dimensional systems illustrate that, in addition to these interaction effects, localization effects are operative. Orbital contributions similar to those discussed in the two-dimensional case are observed in several systems and indicate that these effects must also be included in any quantitative description of electron transport in this highly disordered regime.

Conclusions

Much theoretical and experimental progress has been made on the subject of electron transport in highly disordered systems. Many questions still remain unanswered. Experiments have shown clearly that in several different systems, in both two and three dimensions, the scaling ideas of electron localization are qualitatively correct but the picture is too simple. Strong interaction effects between the electrons have been shown to influence the transport as much as these single-particle localization effects. In two dimensions, the electrons are apparently localized at all values of the disorder and the transition is one from logarithmic to exponential dependence on the length scale. This transition is strongly affected by both localization and many-body effects.

In three dimensions, it appears that the transition is continuous rather than abrupt. Again, the transport is strongly affected by both contributions, and although there are differences between the various systems studied, the similarities give us a qualitative description of transport in the disordered state and the approach to the metal-insulator transition.

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