he nearly had the answer. "I was working on the class number problem for 4 or 5 years," he says, "and I thought I was almost there." What Goldfeld found was that he could provide the class number list to anyone who could provide him with a particular mathematical object—an elliptic curve with certain properties.

It sounded easy. Everyone, including Goldfeld, thought they would polish off the problem in short order. "It seemed very simple, but we soon realized it was way beyond our knowledge," says Gross. The mathematical object they were looking for has to do with the theory of elliptic curves, which are expressed by cubic equations of the form $x^3 + y^3 = z^3$.

Mathematicians associate a function, called an L-function, to elliptic curves and these L-functions are conjectured to tell how many rational points the curves have. Mathematicians look for what is known as the order of vanishing of these L-functions at a specific point in the complex plane to determine how many rational points are on the elliptic curves. Goldfeld proposed that if anyone can find an elliptic curve whose L-function has an order of vanishing that is greater than or equal to 3, he can solve the class number problem.

For 6 years, mathematicians worked on the problem. "Not only did it sound easy, but, if our conjectures were right, we thought we knew some elliptic curves that had that order of vanishing," says Mazur. "We thought we had gone from an easier theory, the order of vanishing, to a harder one, the class number problem. But the theory of vanishing turned out to be much harder than anyone expected."

What Gross and Zagier finally did was to link the arithmetic of elliptic curves to the theory of vanishing. They found that special points on elliptic curves have the property that if they are nonzero, the order of vanishing is precisely 1. If they are 0, the order of vanishing is greater than 1. They also know whether the order of vanishing is even or odd. Thus they can find curves with orders of vanishing greater than or equal to 3 by finding curves that have odd orders of vanishing and whose special points have zero values. Gross and Zagier call these special points H, in honor of Heegner who used them in his long-neglected proof that there are only nine *d*'s.

The proof itself, says Gross, consists of a horrendous equation. "To calculate both sides of the equation takes 100 pages. Then you have to pair up terms on each side of the equation and prove the two sides of the equation are the same. It's a mess." The proof, then is by no means elegant. "I still have hope that it will be tremendously simplified," says Stark.

But what truly impresses mathematicians is the idea behind the proof. "The proof is surprisingly indirect. What they are doing is using an elliptic curve to somehow govern the structure of this infinite family of number systems," says Mazur. "The full gamut of these number systems is governed by a single mathematical object." Mathematicians might naïvely think there is no connection between an elliptic curve and these families of number systems. But the recent proof shows instead, Mazur remarks, that "here there is every connection." Gross agrees, adding, "that connection is really quite extraordinary."—**Gina Kolata**

A Step Toward Artificial Chromosomes

In their continuing efforts to relate the behavior of chromosomes during cell division to their structure investigators have turned to the construction of artificial chromosomes. For example, in the 15 September issue of *Nature*, Andrew Murray and Jack Szostak of the Dana-Farber Cancer Institute of Harvard Medical School report that they have made artificial chromosomes that behave in yeast much like the natural ones do, although not perfectly so.

Over the past few years, investigators from several laboratories have made considerable progress in isolating and characterizing sequences from the yeast genome that appear to be needed for chromosome activities. These functional elements include, in addition to the genes themselves, the centromeres, to which the fibers that pull the chromosomes apart during mitosis and meiosis are attached; the autonomously replicating sequences, which may be the origins for chromosome replication; and the telomeres, the chromosome ends that are needed to complete chromosome replication.

Murray and Szostak wanted to know whether these elements are sufficient for chromosome function. "If these are the only elements required for chromosome function it should be possible to assemble them artificially and show that the products act like a real chromosome," Murray explains.

In their first efforts, the Harvard workers constructed artificial chromosomes that were only 10,000 to 15,000 base pairs (bp) in length. For comparison, the smallest yeast chromosome is about 150,000 bp in length. When the short synthetic chromosomes were transferred into yeast cells, they did not behave like the natural chromosomes, Murray says. There were too many copies per cell, and they were lost very quickly when the cells divided.

In a paper that was published earlier this summer,* Ginger Martin Dani and Virginia Zakian of the Hutchinson Cancer Research Center in Seattle, reported similar findings for small linear chromosomes that they had constructed.

Larger artificial chromosomes work better. Murray and Szostak went on to test constructs containing about 50,000 bp. "These show some of the properties of normal chromosomes," Murray says. "They were more stably inherited, passed through meiosis, and were present in only a few copies per cell. But they are still quite a long way from behaving like real chromosomes." In particular, although they are inherited in a more stable fashion than the smaller artificial chromosomes, they are still lost about two orders of magnitude faster than natural chromosomes.

The problems may be caused by incorrect spacing between the different functional elements or because even the longer artificial chromosomes are not long enough. Alternatively, all the structural elements needed for normal chromosome function may not yet have been identified. Nevertheless, when investigators can make synthetic chromosomes that behave generally as the natural ones do, they have come a long way toward understanding how chromosomes work.—**Jean L. Marx**

*G. M. Dani and V. A. Zakian, *Proc. Natl. Acad. Sci. U.S.A.* **80**, 3406 (1983).