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Facing Quantum Mechanical Reality

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Local hidden variables theory is dead. It received its coup de grace by two precision experiments carried out last year in Paris (*1*, *2*). That theory had been the main opponent to standard quantum mechanics. But it had not been given high odds for survival ever since the first discriminatory tests were carried out; they confirmed quantum mechanics (QM) and disproved local hidden variables theories (HVT's). However, these tests were found to be not entirely free of objections. The two Paris experiments met these objections and clinched the matter.

Summary. Two recent precision experiments provide conclusive evidence against any local hidden variables theory and in favor of standard quantum mechanics. Therefore the epistemology and the ontology of quantum mechanics must now be taken more seriously than ever before. The consequences of the standard interpretation of quantum mechanics are summarized in nontechnical language. The implications of the finiteness of Planck's constant ($h > 0$) for the quantum world are as strange as the implications of the finiteness of the speed of light ($c < \infty$) for space and time in relativity theory. Both lead to realities beyond our common experience that cannot be rejected.

Quantum mechanics has had its opponents since its inception in the middle 1920's. These included Albert Einstein (*3*, *4*) as well as Schrödinger, de Broglie, and others. No one denied the success of QM in accounting correctly for the many and diverse phenomena of the atomic and subatomic world. What has been at issue is the interpretation. Is the quantum world really probabilistic? Or is there a deterministic theory hidden underneath it of which QM is some sort of average? Is QM therefore an incomplete theory?

In recent years other questions were raised concerning the interpretation of QM. Are the concepts of QM related to those of Eastern philosophy and of a holistic metaphysics (*5*)? Does QM relate physical reality to the human consciousness (*6*)? These questions as well as the proposals of HVT's (*7*) received wide publicity. As a result considerable confusion has been generated, not only among nonscientists but also among scientists who are not well acquainted with QM.

The overwhelming majority of physicists who are actively engaged in fundamental research involving QM on the atomic and subatomic level have accepted the standard view. This view emerged from the school of Niels Bohr in Copenhagen and the mathematical work of von Neumann (Copenhagen interpretation). The old Copenhagen view was influenced by instrumentalism and by the then dominant view in the philosophy of science, logical positivism. Since none of the founders of QM were professional philosophers, it was not difficult to find philosophical inadequacies.

But QM has existed now for more than half a century and it has been confirmed in innumerable ways. Similarly, its interpretation has become much better understood. Nevertheless, popular presentations have usually stressed the unconventional interpretations (*5*–*8*), with only few notable exceptions (*9*).

The experimental disproof of local HVT now makes it necessary to take standard QM more seriously than ever before. It is therefore desirable to restate the standard view and to touch on its epistemological and ontological assertions. This is attempted in this article in relatively nontechnical language.

I will first review the experimental refutation of local HVT and then briefly consider the prospects of nonlocal HVT. The remainder of the article will be devoted to the implications of QM as accepted in the standard interpretation. How does it modify our classical world view?

Death of Local Hidden Variables Theories

Starting with the assumption that QM is incomplete and presents us with averages that hide the detailed and deterministic description, one is led to postulating the existence of "hidden variables" which do permit such a description. The type of theory that uses hidden variables can furthermore be required to be local in the following sense of classical (non–quantum mechanical) physics: if two particles are spatially separated, then a measurement on one of them does not "in any manner whatsoever" (*3*) perturb the other. The effects of a measurement are localized. Since, according to special relativity, signals cannot propagate faster than the speed of light, suitable separation of the two measurements will ensure the absence of any communication between them. One therefore speaks of "Einstein locality."

A hidden variables theory that adopts this notion is called a local HVT. It can be distinguished from QM by appropriate experiments. A measurable quantity that permits such a distinction is given in an inequality first derived by Bell and generalized by others (*10*). It expresses a correlation S of two particles simultaneously created by a source. The predic-

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tion of S by local HVT differs in general from its prediction by QM.

The derivation of one form of Bell's inequality is particularly simple and shows the main ideas of local HVT (*11*). Consider two electrons, 1 and 2, moving away from a common source and from one another, and having a known total spin angular momentum. Local HVT accepts the result of QM that the spin of an electron along a given direction can have only the value +1 or −1 [in units of $h/(4\pi)$ where h is Planck's constant]. Thus, if $s_1(a)$ is the spin of electron 1 along direction a, then $s_1(a) = \pm 1$. If another direction, b, is chosen instead of a, then $s_1(b) = \pm 1$ also. Similarly, for electron 2, a measurement gives $s_2(a') = \pm 1$ or $s_2(b') = \pm 1$ for alternative direction choice a' or b'.

By the locality assumption, the measurement on 1 does not affect the outcome of a measurement on 2 and vice versa. The two-electron system is completely specified by giving the hidden variables λ so that the outcome of a given measurement on both is certain—for example, $s_1(a) = +1$, $s_2(a') = -1$.

One now observes the identity

$$\left| s_1^\lambda(a)[s_2^\lambda(a') + s_2^\lambda(b')] + s_1^\lambda(b)[s_2^\lambda(a') - s_2^\lambda(b')] \right| = 2 \qquad (1)$$

since $s_2^\lambda(a') + s_2^\lambda(b')$ can take on only the values 2, 0, and −2, while $s_2^\lambda(a') - s_2^\lambda(b')$ has the corresponding values 0, ±2, and 0. This equation holds for any possible choice of λ, that is, for any possible outcome of a measurement on the pair. If λ can vary continuously the probability of finding the electron pair in a state characterized by hidden variables between λ and $\lambda + d\lambda$ is $\rho(\lambda)d\lambda$; the joint probability $P_{12}(a,a')$ for electron 1 along a and electron 2 along a' when λ is not known is then

$$P_{12}(a,a') = \int s_1^\lambda(a) s_2^\lambda(a') \rho(\lambda) d\lambda \qquad (2)$$

It now follows from Eq. 1, since

$$\left| \int f(\lambda) d\lambda \right| \leq \int \left| f(\lambda) \right| d\lambda$$

that

$$\left| P_{12}(a,a') + P_{12}(a,b') + P_{12}(b,a') - P_{12}(b,b') \right| \leq 2 \qquad (3)$$

This is Bell's inequality for this particular system. If we denote the left side of Eq. 3 by S_{HVT} we have

$$S_{\mathrm{HVT}} \leq 2 \qquad (4)$$

The quantum mechanical calculation is based on coherence of the probability amplitudes of the two particles, a notion absent in HVT. For the particular combination of correlations involving two different spin directions for each electron (a and b on 1, and a' and b' on 2) it yields a quantity S_{QM} which is not restricted as in Eq. 4. The quantity S_{QM} can be larger than 2 for suitable choices of the directions.

In a typical experiment of this sort one places one measuring device at a distance L to the left of the source (to measure electron 1 along a, say) and the other at a distance L to the right of the source (to measure electron 2 along a'), and one records the outcome. The average of a large number of such measurements gives $P_{12}(a,a')$.

In the Paris experiments a two-photon system rather than a two-electron system was studied. The photon polarization took the place of the electron spin. The formulas above are equally applicable in that case. In the first experiment (*1*) two correlated photons (in a state of total angular momentum zero) were emitted from an excited calcium atom. The result was

$$S_{\mathrm{exp}} = 2.697 \pm 0.015 \qquad (5)$$

which is in excellent agreement with the quantum mechanical prediction

$$S_{\mathrm{QM}} = 2.70 \qquad (6)$$

and in violation of the local HVT prediction (*4*).

There is, however, one objection to this experiment. The distance of the measuring devices from the source, L, and their distance from one another, $2L$, were not large enough to exclude the possibility of signals passing during the measurements. Full localization was not ensured.

This objection was met by the second experiment (*2*), where L was extended to 6 m and the choice between directions a and b (a' and b') was made random and in a time only half as long as it would take a light ray to reach the source (2×10^{-8} second). No communication between the measurements on 1 and 2 was therefore possible. The result for this different geometric configuration was

$$S_{\mathrm{exp}} = 2.101 \pm 0.020 \qquad (7)$$

in agreement with the QM prediction

$$S_{\mathrm{QM}} = 2.112 \qquad (8)$$

That is five standard deviations larger than the prediction of HVT, which is $S_{\mathrm{HVT}} \leq 2$.

This result represents an experimental refutation of the set of theories of hidden variables that satisfy Einstein locality.

Nonlocal Hidden Variables Theories

The experimental evidence presented above does not bear on hidden variables theories that are not local. Such a theory was first suggested by Bohm (*12*). It was later followed by other such theories by several authors (*13*).

Can one distinguish nonlocal HVT's from QM by experiments? At the time of this writing the answer is, no. These theories are constructed to give results identical to those of QM, which has been so amply confirmed.

For any nonlocal HVT to become a viable scientific alternative to QM it would have to reproduce the results of QM (which it does by construction), but in addition it would have to account for at least one experiment not accounted for by QM. It would thus become testable.

In the absence of experimental distinguishability between a nonlocal HVT and QM, the former becomes a substructure to QM that is scientifically gratuitous. It would be based completely on philosophical rather than empirical grounds. Such philosophical grounds were, however, strong enough for Einstein to reject the belief that QM is a complete theory on the level of interpretation. A small number of reputable physicists continue to hold this view at present (*10*).

But what about the future? It might be suggested that a future scientific revolution may make the deterministic HVT supersede the probabilistic QM. Developments in theoretical physics during the last half-century, however, have been in exactly the opposite direction. Quantum mechanics is not compatible with the special theory of relativity and is valid only for systems in which the massive particles move slowly compared to the speed of light. A generalization of QM has been developed that is consistent with special relativity: quantum field theory. This theory has had great success when applied to electromagnetic interactions and recently also when applied to nuclear and subnuclear interactions (quantum chromodynamics and electroweak theory). And one finds that quantum field theory is even more probabilistic than QM. For example, in QM one can refer measurements to a precise instant of time; in quantum field theory one can only make measurements during finite time intervals (*14*). Thus quantum field theory is one more step farther away from the sharpness of classical physics. This development is not in the direction toward HVT but away from it.

What Quantum Mechanics Tells Us

The world to which quantum mechanics applies is far beyond our common experience. It would be irresponsible for a scientist to extrapolate and to assert that the concepts and laws of our common environment also apply to the quantum world without change. Early in this century Einstein found that the finiteness of the speed of light, c, requires a revision of such well-established space-time concepts as simultaneity and the addition of velocities (in the special theory of relativity). Twenty years later the finiteness of Planck's constant, $h > 0$, required similarly severe revisions in our concepts of what and how we know about the quantum world. In both cases, the biggest obstacles to progress were our preconceived notions (which we apply often, sometimes without being fully aware of it); they have proved to be wrong many times.

Physical theories are approximations characterized by validity limits. But the domains of validity are often very large. The effects of special relativity can be safely neglected even for the high speed of the earth around the sun, 30 kilometers per second; and the finiteness of h can be neglected even for a tiny grain of silver in a photographic emulsion (10^{10} atoms). These matters are deceiving, so that we are surprised when we go beyond these domains and enter into the relativistic or quantum domains, which are governed by laws that seem strange to us. We then find that "relativistic weirdness" or "quantum weirdness" sets in.

The quantum world. This differs from our everyday world (the "classical" world) not only quantitatively (*15*) but also qualitatively. The fundamental particles (electrons, protons, photons, and so on) have no individual characteristics. They cannot be tagged; they are indistinguishable. In fact, it is extremely unlikely that the same electron will ever be seen twice. We only observe a sample of the genus "electron" and, by repeating the same observations often enough, we acquire good enough statistics to say something about an electron in this ensemble. We find, in fact, that this indistinguishability is responsible for statistical properties of sets of electrons that do not exist in the classical world (Fermi-Dirac quantum statistics).

Beyond this, we find that our common language is utterly inadequate for the description of the quantum world and that the mathematical language is much more suitable. For example, classically, "electron" has the connotation of "particle" in contradistinction to "wave," while in QM "electron" is an object that has properties resembling both. It is "just like a particle" or "just like a wave" only in limiting cases depending on the particular experiment. Common language has no words for it.

Finally, we find that certain observables that are known to be continuous in the classical world are discrete in the quantum world. Examples are angular momentum or the energy of (nearly) monochromatic electromagnetic radiation.

How we know the quantum world. Our human limitations to very narrow ranges of classical observables (lengths, time intervals, temperature, speed, and so on) permit us to explore the quantum world only indirectly. We need a classical apparatus to "see" the quantum world. An electron can leave a visible track on a photographic plate because it triggers the development of silver crystals. What we see is not the electron but the line of silver crystals on the photographic plate. And this is a typical situation: only when the quantum world has left a permanent record in our measuring apparatus do we have an observation. The last step, the human perception of the permanent record, takes place entirely within the domain of classical physics. Our consciousness thus may enter this sequence of events with a considerable time delay. It can hardly influence what became of the electron as it left its track.

The apparatus, however, plays a much more important role in measurements of the quantum world than in measurements of the classical world. In the latter one can minimize the effect of the apparatus on the system to an arbitrarily high degree (in principle). Not so for quantum measurements. Here the effect of the measuring instrument on the system plays an important role. Contrary to one's first reaction, this is not a weakness but a strength of QM (*11*): the measuring device is not outside the "domain of discourse" but can itself be a physical system subject to the same laws and theories as the system we observe. The system-apparatus interaction can be like a system-system interaction. The (necessary) fact that the apparatus must be classical can be taken into account separately. This expresses the universality of QM, a positive feature. It states beautifully what is completely ignored in classical physics: through man nature can observe itself.

What we can know precisely. In classical physics all observables of a system can be known at the same time with (in principle) arbitrary precision. Perhaps the most important effect of the finiteness of h is that this is not the case in the quantum world. Each quantum system is completely characterized by a set of observables whose precise values can be known, and this set is a small subset of all the observables that can be measured on the system (*16*). The state of the system is "sharp" in these observables only (*17*). For those acquainted with the language of mathematics, these observables are self-adjoint operators on a Hilbert space that commute with one another (complete commuting set of operators); their values are their eigenvalues and the state is the simultaneous eigenfunction of these operators.

In contradistinction, observables that do not belong to the complete commuting set are not sharp in these states. This means that they can occur with various different values. Which of these values will occur is precisely specified by a probability distribution, very much like the probability for a particular set of eyes in a throw of dice. I will return to such observables below.

An observable that is not a member of the set of commuting observables necessarily does not commute with at least one of this set. An immediate mathematical consequence of such noncommutativity of two observables A and B is the famous Heisenberg uncertainty relation: their simultaneous measurement cannot yield precise values for both. The uncertainties of their values (their dispersions) satisfy $\Delta a \Delta b > h$. The sharper one of them, the less sharp the other will be. Only in the classical limit $h \rightarrow 0$ could both Δa and Δb be arbitrarily small.

That two observables are not simultaneously sharp is not outside our common experience: looking at the fineness of the weave of a rug through a magnifying glass and looking at the whole rug from 10 feet away provide two observations with information that cannot be obtained simultaneously. But both are needed for a full knowledge of the rug. The two pieces of information complement each other.

The dynamics of the state of a quantum system is not probabilistic; the state evolves according to a deterministic equation (the Schrödinger equation) and is uniquely determined by the specification of its initial state. Of course, as it encounters another system and interacts with it changes take place that can be computed only with full cognizance of both systems and of their mutual interaction. Such an interaction will change the system so that when it ceases (as, for example, in the case of two particles colliding and flying apart again) the system will be in an eigenstate of (in general) a different complete commuting set of observables. A measurement also involves such an interaction.

What we cannot know precisely is not a consequence of our inability or imperfection. According to QM it is because the system is in a state in which this information is not present: the observable has a distribution of values in this state. And that distribution is known precisely. One can compute the average value that this observable has as well as the dispersion associated with this distribution $\Delta a = (< A^2 > - < A >^2)^{1/2}$. The key difference between this quantum distribution of probability and a classical distribution of probability lies in the fact that the former is the square of a sum of probability amplitudes (coherent superposition of states) while the latter is a sum of squares of them (incoherent superposition). This means that in the quantum case the probability for two different values of an observable to occur can interfere with one another. Such interference is impossible in the classical case. It is exactly this interference that made for the difference between the predictions of QM and of HVT observed by the two Paris experiments.

This situation, that an observable has a distribution of values, will occur wherever we measure an observable A on a system which is not in one of the eigenstates of A. The interaction between the system and the measuring apparatus will change the state of the system to that particular eigenstate of A whose eigenvalue is recorded by the apparatus.

What goes on in this process is commonly referred to as "the collapse of the wave packet." It is perhaps more often criticized than any other item in the interpretation of QM. Unfortunately, Bohr's statement that what takes place is an "uncontrollable interaction between the object and the measuring instruments" (*18*) contributed to the confusion: to the nonexpert this statement implies a much larger freedom than is actually present in the mathematical formulation of the measuring process. First given by von Neumann (*19*), the description of the measuring process has received much attention, clarification, and specification. Examples have been worked out explicitly (*20*) and the best of the QM textbooks now even discuss it in some detail (*21*).

In the first part of the measuring process the system and the apparatus enter into mutual interaction according to the dynamical laws of QM (a unitary evolution, that is, one that conserves the probabilities of system and apparatus together). This results in general in a coherent superposition of states each containing an eigenstate of A together with that state of the apparatus that indicates the "pointer position" appropriate to the eigenvalue of that state. But the classical (macroscopic) nature of the apparatus does not allow such a coherence. Therefore one obtains an incoherent superposition of these states; the phase relations are destroyed, the states are disjoint. Finally, one of these states leaves a permanent record, the final pointer position. What this position is cannot be predicted for one single experiment. This is what Bohr meant by "uncontrollable interaction." But the theory does predict the exact relative probabilities for all possible pointer positions to occur; the tally of a large ensemble of experiments is therefore predicted and can be confirmed.

The "collapse of the wave packet" is thus rather similar to the toss of a coin where the probability of heads is 1/2 until the coin comes to rest, at which point the 1/2 "collapses" to either probability 0 or probability 1. The analogy fails in that the dynamics of the classical coin can, in principle, be analyzed so that the final position of the coin is determined by the initial conditions of the tossing; such detailed dynamics does not exist in the quantum world (*22*).

The change brought about in the physical system due to the interaction with the measuring apparatus should not be interpreted to mean that "reality is created by the measurement." Just as two dice are real before they hit the table, and not only after they show snake eyes, so does a quantum mechanical system exist in a real state (specified by a wave function) before the interaction with the apparatus sets in. Reality is not created by observation. The system is present all the time.

There are certain questions about quantum mechanical systems that have no answers. These are questions motivated by the common language used to describe the system. As indicated earlier, that language can be badly misleading. For example, a photon is emitted when an atom makes a transition from an excited state to the ground state. One might ask how this photon was created since it was not present before the transition took place. This is a meaningless question in QM and it has no answer. The mathematical language does not lend itself to asking such a question.

We can summarize what QM tells us by saying that it answers epistemic questions very differently from classical physics. If we were to hold this against QM because we do not like the answers (even though they are confirmed by experiment), we would be committing the error of taking the epistemology of classical physics as normative. We would enter a world far removed from our common knowledge with preconceived notions.

Conclusions

Einstein contributed more to the development of QM than is generally realized (*23*). Nevertheless, he did not accept it on philosophical grounds. His famous dictum "God does not play dice" characterizes his view objecting to the probabilistic element in the theory. As a believer in classical determinism he regarded probabilistic QM as necessarily incomplete and expected some type of hidden variables theory to provide for complete knowledge. He thus became the leading figure among the opponents to standard QM.

This probabilistic nature of QM has now received new and strong experimental support. The new results show that no hidden variables theory based on Einstein locality can be in agreement with experiments.

Complete knowledge of the state of a classical system at a given time is synonymous with exact knowledge of all observables at that time. Complete knowledge of the state of a quantum mechanical system is ensured by the exact knowledge of only a subset of all observables at the same time. That set is a complete set even though it excludes many and often the majority of other observables. This is a consequence of Planck's constant h not being zero. In the limit $h \to 0$ one recovers the classical theory.

Only when one asks for the value of an observable not in the complete set of observables that are knowable simultaneously does one encounter the probabilistic nature of QM. In this sense the probabilistic element enters QM through the measurement process, which is bound to change the observed system if that system is not in an eigenstate of the quantity that is being measured. The various possible outcomes of such a measurement occur with probabilities that can be predicted exactly. The outcome of any one such measurement (like the throw of dice) cannot be predicted.

This feature does not make QM incomplete. Even though only a subset of all observables can be sharp simultaneously, the theory accounts correctly for all observations. The treatment of the measuring apparatus as "just another system" provides for the universality of the theory. The classical idealization of ignoring the effect of the apparatus on the measured system has been dropped.

Just as the epistemology of QM changes our classical views drastically, so does its ontology. The existence of interference between probabilities in QM has no classical analog. Nevertheless, it has been confirmed in innumerable experiments. The two recent experiments in Paris are examples of these in a very specific context.

Some draw the conclusion from all this that "the universe does not 'exist out there' independent of all acts of observation," and that reality is created by the observer. But that view is not shared by the overwhelming majority of physicists.

The standard ontology of QM is a realist position. It accepts a qualitative difference between the quantum world and the classical world. The world of electrons, protons, and all the rest does exist out there even if we do not observe it, and it behaves exactly as QM tells us it does (*24*). The point is that physical reality on the quantum level cannot be defined in classical terms as was attempted by Einstein, Podolsky, and Rosen (*3*). The superposition principle, which is responsible for adding probability amplitudes instead of probabilities, is one of the laws of nature which we must accept as we accept the universal constancy of the speed of light. That principle is responsible for the peculiar interference phenomena that lead to quantum correlations quite different from classical ones. It makes the quantum world no less real than the classical world. And it teaches us that the reality of the common experience in the classical world is only a small part of what there is.

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11. Here I follow A. Peres and W. H. Zurek [*Am. J. Phys.* **50**, 807 (1982)].
12. D. Bohm, *Phys. Rev.* **85**, 166 (1952); *ibid.*, p. 180.
13. Both local and nonlocal HVT's are reviewed in detail in F. J. Belinfante, *A Survey of Hidden Variables Theories* (Pergamon, New York, 1973).
14. N. Bohr and L. Rosenfeld, *Phys. Rev.* **78**, 794 (1950).
15. It must be remembered that some quantum phenomena such as superconductivity and superfluidity can be observed over macroscopic distances. The Paris experiments discussed above show "quantum behavior" over a distance of 12 meters. Therefore, the quantum world is not restricted to phenomena that take place on atomic scales only.
16. Here I discuss only systems in "pure" states, that is, in states for which we have maximal information.
17. For observables which do not occur in smallest units (quanta) but which can take on a continuum of values (such as position), "sharp" here means "very narrow width."
18. N. Bohr, in *Albert Einstein Philosopher-Scientist*, P. A. Schilpp, Ed. (Tudor, New York, 1951), pp. 199–241; in *Atomic Physics and Human Knowledge* (Wiley, New York, 1958).
19. J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton Univ. Press, Princeton, N.J., 1955); this is a translation of the original German edition published in 1932.
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22. The simplistic Descartian determinism has long been recognized as often insufficient to account even for classical dynamical behavior. Over long periods of time the dynamics of many systems can become nondeterministic (technically, the equations of motion are nonintegrable). A well-known example is a pinball machine; the motion of the ball comes to "branching points": only probabilistic information is available on whether it takes a path to the right or to the left of the pin. Quantum mechanics is much closer to such actual classical mechanics than is commonly realized. Indeterminacy also exists in the nonquantum domain.
23. A. Pais, *'Subtle is the Lord . . .': The Science and Life of Albert Einstein* (Clarenden, Oxford, 1982).
24. Throughout this article, considerations were restricted to "nonrelativistic quantum mechanics." This theory is now well established because its domain of validity is known. Relativistic quantum mechanics (quantum field theory), developed during the last 50 years, has greatly extended our knowledge beyond QM. That theory reduces to QM in the small velocity limit, which gives additional strong support to its correctness.

25. I am indebted to A. Shimony for his valuable comments on this article.