absent (*11*). The onset of generally cooler summers after 4000 to 5000 years ago, indicated by the overall decline in timberline elevations (*12*), is compatible with the Holocene glacial record of the Cordillera, which shows an increasing incidence of glacial activity culminating in the "Little Ice Age" advances of the last few centuries (*13*).

The Hypsithermal remains one of the most striking features of late Quaternary climate, and evidence is mounting in support of rapid and complex changes in temperatures during this period in western North America. Archaic settlement data from the Colorado Front Range (*14*) indicate major discontinuities in the settlement record of this area between ~ 7000 and 6500 years ago and ~ 6000 to 5500 years ago, perhaps as a result of prolonged severe droughts. Analyses of the annual rings of fossil bristlecone pine snags from New Mexico (*15*) reveal major fluctuations in atmospheric ^{14}C between ~ 8000 and 5000 years ago, with a substantial departure ~ 7500 years ago. These variations in atmospheric ^{14}C are thought to be proxy data for fluctuations in solar activity (*16*) and therefore possibly for climatic changes. We believe that the complex and apparently episodic oscillations in summer temperatures exhibited by the Jasper timberline record support the picture of a multiphase Hypsithermal.

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Ages Estimated from a Diffusion Equation Model for Scarp Degradation

Abstract. *The diffusion equation derived from the continuity equation for hillslopes is applied to scarp erosion in unconsolidated materials. Solutions to this equation allow direct calculation of the product of the rate coefficient and the age of the scarp from measurements of scarp morphology. Where the rate coefficient can be estimated or can be derived from scarps of known age, this method allows direct calculation of unknown ages of scarps.*

In the past 25 years many efforts have been made to mathematically model the evolution of hillslopes (*1, 2*). These models are constrained by the diversity and complexity of natural hillslopes and by difficulties in determining the ages of many landforms. Scarps in unconsolidated materials are common landforms that are simpler than other kinds of hillslopes. They often have specifiable initial conditions, and they change fast enough to test the temporal predictions of mathematical models. Recent studies of scarps (*3–6*), especially fault scarps, have produced many measurements of scarp morphology (Fig. 1A). The purpose of this report is to explicitly solve the diffusion equation for scarp boundary conditions and to demonstrate how the solution can be used to estimate scarp age.

Changes in the morphology of scarps in unconsolidated deposits are controlled by transport-limited processes, primarily soil creep, raindrop impact, and slope wash. For these processes the supply rate of loose debris is not a controlling factor, and process rates are generally proportional to powers of the slope distance and the sine or tangent of the slope angle (*2*):

$$S = cx^m \frac{\partial^n y}{\partial x^n} \quad (1)$$

where S is the rate of downslope transport, c is a rate constant, x is horizontal distance, and y is elevation. For raindrop impact and creep processes, which are the dominant processes on many scarps, $m = 0$ and $n = 1$ are generally accepted (*1, 2*). For slope wash m and n may vary considerably, but in some cases $m = 0$ and $n = 1$ (*7*). Slope wash on the relatively short, steep slopes of scarps seems likely to conform to these values. Where $m = 0$ and $n = 1$ the rate of downslope transport is simply proportional to the surface gradient.

Most models of hillslope evolution are based on continuity considerations and assume no change in density of the surficial material (*1, 2*). This requires that the change in elevation of a point be equal to the difference between the amount of material transported to the point and the amount of material transported away from it, or

$$\frac{\partial y}{\partial t} = \frac{\partial S}{\partial x} \quad (2)$$

Combining the continuity equation (Eq. 2) with the dependence of downslope transport on surface gradient (Eq. 1 with $m = 0$ and $n = 1$) gives the diffusion equation

$$\frac{\partial y}{\partial t} = c \frac{\partial^2 y}{\partial x^2} \quad (3)$$

This equation is well known as a description of many processes in chemical diffusion, conductive heat flow, and flow through porous media. Equation 3 has also been derived for a variety of slope models (*1, 2*), including several for scarp evolution (*4, 8*). For scarp boundary conditions it has been solved by finite difference methods (*4*) and by analytical methods (*8*) similar to those presented here.

The diffusion equation applied to scarps implies that the change in elevation of a point is proportional to the profile curvature at that point. Thus with time the crest and the toe become round-

ed and the midslope angle decreases (Fig. 1B).

The solution of the diffusion equation for the profile of a scarp in unconsolidated deposits is analogous to that for one-dimensional heat flow in a semi-infinite solid having a specific initial temperature distribution and a surface kept at zero temperature. The analogous scarp is one whose initial configuration is specified and whose midpoint is fixed at the origin. The model is for half the scarp; the other half behaves symmetrically. Only changes in elevation in cross section are considered—effects of the length or curvature along the length of the scarp are assumed to be negligible. The model also assumes (i) no overall lowering of the landscape and a constant base level for the scarp (these are reasonable approximations for most scarps, which change much more rapidly than the general landscape) and (ii) that the scarps are formed by a single event and then left to degrade under constant conditions.

The general solution to Eq. 3 subject to these boundary conditions (*9*) is given by

$$y(x,t) = \frac{1}{2(\pi ct)^{1/2}} \int_0^\infty f(u) \left(\exp\left[\frac{-(x-u)^2}{4ct}\right] - \exp\left[\frac{-(x+u)^2}{4ct}\right]\right) du \quad (4)$$

where $f(u)$ is the initial temperature distribution (or elevation profile). This solution has been used (*10*) to obtain hypothetical sequences of hillslope profiles. From Eq. 4 the slope ($g = \tan\theta$) at $x = 0$ is

$$g \equiv \frac{\alpha x}{\alpha y_{(x=0)}} = \frac{1}{2(\pi)^{1/2}(ct)^{3/2}} \int_0^\infty uf(u) \exp\left(\frac{-u^2}{4ct}\right) du \quad (5)$$

Different initial configurations are conceivable for different kinds of scarps (*11*). For a vertical initial scarp, $f(u) = d/2 + g_1 u$ ($g_1 = \tan\alpha$) and Eq. 5 reduces to

$$g = \frac{d}{2(\pi ct)^{1/2}} + g_1 \quad (6)$$

As t approaches zero, g goes to infinity (vertical slope); as t approaches infinity, g approaches g_1.

Solving for the product ct yields

$$ct = \frac{d^2}{4\pi}\left(\frac{1}{g - g_1}\right)^2 \quad (7)$$

Thus for each scarp profile the product of the rate constant and the scarp age can be calculated from the initial and final configurations of the scarp.

Most scarps, especially fault scarps, are formed with free faces, steep segments having angles commonly more than 50° (*12*). These free faces are modified by slumping, spalling, and other processes until the scarp slope reaches approximately the angle of repose of the material in which the scarp is developed. The angle of repose of unconsolidated deposits depends on climate and materials, but angles of 25° to 35° are common (*1, 2*).

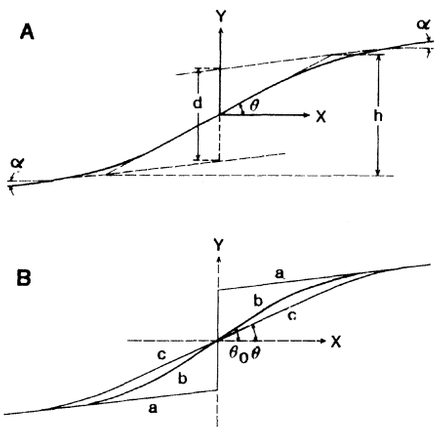

Fig. 1. Scarp morphology measurements and progression of forms. (A) Definition of common morphological measurements: h, scarp height; d, surface offset, θ, maximum scarp angle; and α, surface slope angle. Modified from (*5*). (B) Progression of scarp forms: a, vertical initial scarp, $f(u) = d/2 + g_1 u$ ($g_1 = \tan\alpha$); b, scarp with angle θ_0 at time t_0 when the diffusion equation model begins to apply; and c, observed scarp with angle θ at time t.

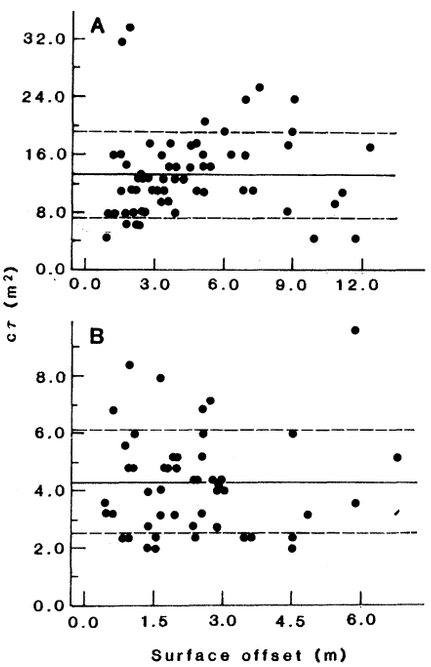

Fig. 2. Calculated $c\tau$ values for two scarps in Utah. Solid lines indicate means; dashed lines represent ±1 standard deviation. (A) Bonneville shoreline: initial angle 30.0°, mean $c\tau$ 13.5, standard deviation 5.9. (B) Drum Mountains fault scarps: initial angle 28.0°, mean $c\tau$ 4.3, and standard deviation 1.8. See text for calculation methods; original morphological data are from (*5*).

The processes that remove the free face from a scarp and lead to a slope at the angle of repose do not satisfy the assumptions used in Eq. 1. Therefore, the diffusion equation model applies only after the scarp has reached its angle of repose and is controlled by creep, raindrop impact, and slope wash. For fault scarps in the western United States the time necessary to remove the free face appears to vary from a few tens of years to a few hundred years (*12*). This time then is the starting time for the diffusion equation model. Because the diffusion equation does not apply to the processes that remove the free face from a scarp, we divide the age of the scarp (t) into t_0, the time required to remove the free face, and τ, the time for which the diffusion equation applies to scarp degradation. Thus t_0 is the time required to go from profile a to profile b in Fig. 1B and τ is the time required to go from profile b to profile c. To calculate $c\tau$ we first obtain $ct = c(t_0 + \tau)$ and then subtract ct_0, both calculated from Eq. 7 by using a vertical initial profile. These calculations are combined in Eq. 8:

$$c\tau = \frac{d^2}{4\pi}\left[\frac{1}{(g - g_1)^2} - \frac{1}{(g_0 - g_1)^2}\right] \quad (8)$$

where $g_0 = \tan\theta$, the initial angle (angle of repose). The true age of the scarp, then, is τ plus the time required to remove the free face.

The angle of repose θ_0 for the materials in most scarps is unknown. Trial calculations show that relatively high or low values of θ_0 produce calculated $c\tau$ values that are correlated (positively and negatively, respectively) with surface offset. These correlations result from the fact that the time t_0 to reach θ_0, calculated by the diffusion equation, depends on the size d of the scarp. However, $c\tau$ clearly should not correlate with or depend on the size of the scarp. We estimate θ_0 by trial-and-error calculations to determine the initial angle that results in the lowest correlation coefficient between $c\tau$ and surface offset. Estimates of θ_0 for actual data sets (Fig. 2) are a few degrees lower than what one might expect for these materials (*1, 2*). Therefore, θ_0 is more accurately described as the angle at which the diffusion equation begins to apply to the degradation of the scarp, rather than as the angle of repose.

As an example of the utility of the diffusion equation model for estimating ages, we applied Eq. 8 to two sets of scarp morphology data from Utah (*5*). The data consist of measurements of θ, α, and d for each of several profiles measured along the length of the scarps. Both time τ and diffusion or rate coefficient c in Eq. 8 are unknown. The rate coefficient must in some way be estimated or determined in order to solve for time. In rare cases of independently dated scarps, the rate coefficient can be calculated directly from Eq. 8.

The Bonneville shoreline is a widespread wave-cut scarp in Utah (*5*), whose abandonment is radiocarbon-dated at about 15,000 years ago (*13*). The Bonneville shoreline probably had a rounded initial profile with a midslope near the angle of repose. The time to reach the initial angle for the diffusion equation model was probably a small fraction of the scarp age. Using 15,000 years and the calculated $c\tau$ value of 13.5 ± 5.9 (Fig. 2A) gives an estimate for c of 9.0×10^{-4} m^2/year.

The Drum Mountains scarps are a set of single-event fault scarps near former Lake Bonneville that are younger than the Bonneville shoreline (*5*). The scarps have a calculated $c\tau$ value of 4.3 ± 1.8 (Fig. 2B). The c value of 9.0×10^{-4} m^2/year calculated for the Bonneville shoreline yields an age of about 4800 years for the Drum Mountains scarps. To this age must be added the time required to remove the free face from the scarps, probably a few hundred years.

The diffusion equation model has potential for estimating the ages of many scarps. The most serious limitation at present is difficulty in estimating rate coefficient c.

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$$g = g_1 + (g_0 - g_1)\, erf\left(\frac{L}{(4ct)^{1/2}}\right)$$
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Anthropogenic Chlorofluoromethanes in the Greenland and Norwegian Seas

Abstract. *The concentrations of two industrially produced chlorofluoromethanes, CCl_3F (F-11) and CCl_2F_2 (F-12), have been measured in the water column and in the marine atmosphere of the Greenland and Norwegian seas. Measurable concentrations of these two chlorofluoromethanes have penetrated to the deep basins of both of these regions, and the general characteristics of their vertical distributions are similar to those of the bomb-produced radioisotopes injected into the atmosphere on a similar time scale. The data have been fitted to a time-dependent box model based on deep convective mixing in the Greenland Sea and lateral exchange between the deep basins. The model calculations for the two chlorofluoromethanes in the Greenland Sea give similar results, with a time scale for deep convection of about 40 years. The time scale for lateral mixing between the deep Greenland Sea and the deep Norwegian Sea is estimated to be 20 to 30 years, although the agreement between the calculations for the two chlorofluoromethanes is limited by analytical uncertainties at the low concentrations found in the deep Norwegian Sea and by uncertainties in the model assumptions.*

The potential value of dissolved atmospheric CCl_3F and CCl_2F_2 as conservative time-dependent tracers of ocean circulation and mixing has been recognized for some years (*1*). These compounds are extremely stable in the troposphere and in natural waters, they have no natural sources, and their histories of release to the atmosphere are fairly well known (*2*). Because of their importance in stratospheric chlorine chemistry and in the modulation of the earth's O_3 layer (*3*), the global atmospheric distributions of these gases are closely monitored. Unlike the transient increases in tritium and radiocarbon resulting from atmospheric nuclear-weapons testing, which are now being used as oceanic tracers on a similar decadal time scale, the atmospheric distributions of CCl_3F and CCl_2F_2 are not strongly dependent on latitude, and their surface-water concentrations can be expected to come into relatively rapid solubility equilibrium with the atmosphere (*4*). The presence of two separate chlorofluoromethanes (CFM's) with different rates of increase in the atmosphere provides an additional time-dependent constraint on models of subsurface circulation and mixing.

Until recently, the development of oceanic CFM measurements has been limited by analytical difficulties. Although the early measurements of Hahne *et al.* (*5*) and Hammer *et al.* (*6*) established the general vertical distributions of these tracers, these measurements were significantly affected by sample contamination and limited analytical precision. Recent measurements by Gammon and his co-workers in the North Pacific (*7*) and North Atlantic (*8*) have demonstrated that these difficulties can be largely overcome. The measurements we report here were made with the use of a modified version of their technique.

Sample collections and analyses were carried out aboard C.S.S. *Hudson* expedition 82-001 to the Greenland and Norwegian seas during February through April 1982. The major objectives of this expedition were to observe possible deep convection during times of maximum surface density and to study wintertime physical and chemical properties in this important region of deep water formation. The locations of stations at which CCl_3F and CCl_2F_2 measurements were made are shown in Fig. 1. We discuss here the results of our CCl_3F and CCl_2F_2 measurements at station 60 (74°N, 1°W) and at station 88 (70°N, 11°E), located in the deep basins of the Greenland and Norwegian seas, respectively.

Concentrations of CCl_3F and CCl_2F_2 were measured in seawater and in air samples by shipboard electron-capture