How Does Fluid Flow Become Turbulent?

Myriad instabilities that precede turbulence in a simple model (Taylor-Couette flow) defy understanding 60 years after the first one was found

How do smoothly flowing fluids become turbulent? The answer affects us all—from the seaside poet pondering the mysteries of the ocean to the engineer planning a pipeline to a passenger traveling on a jetliner, who wonders why the air is always choppiest at mealtimes. A workshop held last month in Eugene, Oregon, dealing with the flow of liquids between concentric rotating cylinders (Couette cell) did not come up with an answer to this question.* An important reason for the lack of an answer is that the mathematical equations that describe fluid flow, though well established, defy solution outside the smooth laminar flow regime, even for the highly symmetric Couette cell.

All is not bleak, however. Theoretical and experimental investigations of fluid flow in Couette cells have been on a rising curve in the last few years, pointed out J. Trevor Stuart of Imperial College (London) at the close of the workshop. The use of lasers to make accurate measurements of fluid velocity (laser-Doppler velocimetry) and of computers to collect and rapidly massage vast amounts of data have made more stringent tests of theory possible. Experimentalists are also finding ways to isolate particular aspects of the fluid flow from a field of highly complex behavior. Supercomputers make feasible numerical simulation of the nonlinear, partial differential Navier-Stokes equations that describe fluid flow and provide even more grist for the theorists' mills. And most exciting of all, the first hard evidence was presented that at least some transitions to turbulence may be described by relatively simple nonlinear equations (with only a few variables) whose solutions lie on mathematical structures called strange attractors.

The history of Couette flow dates back to the tail end of the 19th century and the work of Arnulf Mallock in England and Maurice M. Couette in France. These investigators measured the viscosity of water and other liquids in a cell comprising two concentric cylinders. Liquid between the cylinders flows azimuthally (that is, in a circle around the axis of the cylinders) when one or both of them is rotated. Both scientists noticed that the fluid flow becomes unstable as its velocity increases, but Couette got his name on the apparatus.

A major event in understanding the Couette cell was Sir Geoffrey Taylor's 1923 discovery of the nature of this instability. Taylor quantitatively showed theoretically and verified experimentally that the instability set in at particular values of the rotational speeds of the inner and outer cylinders that depended on their dimensions and the viscosity of the fluid. The instability consists of the formation of a stack of toroidal vortices like doughnuts around a pole. The net fluid motion in these Taylor vortices, as they are now known, is a helical superposition of the azimuthal Couette flow and the circular flow around the axis of the vortex. The direction of the latter flow alternates from vortex to vortex, so that at one boundary between vortices (the inflow boundary) fluid rushes toward the inner cylinder, whereas at the next (the outflow boundary) it rushes toward the outer cylinder.

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An interesting feature of Taylor vortices is that they never form if only the outer cylinder rotates. Moreover, they form most easily if only the inner cylinder rotates. In the latter case, as the inner cylinder rotates, it drags nearby fluid with it and spins it away. Fluid initially near the outer cylinder rushes in to fill the "vacuum," thereby initiating the vortex flow. However, if only the outer cylinder rotates, fluid near the outer wall is accelerated outward. Since there is no place for this fluid to go, nothing happens, just as people in the spinning barrel at a carnival are pressed against the wall before the bottom of the barrel drops away.

Although the general nature of Taylor vortices is known, most details are not, because there is no exact solution to the Navier-Stokes equations. Taylor's calculations involved making perturbations of laminar Couette flow that were small and linearizing the equations. He determined at what cylinder speed the vortices came into existence by observing when the perturbations grew in size (linear stability analysis).

One outstanding issue is the diameter of the vortices. The naïve expectation is that the diameter is the same as the distance between the two cylinders, and indeed this is what linear stability analysis predicts at the critical inner cylinder speed for the onset of vortices. However, experiments and calculations since Taylor's day agree that for cylinder speeds above the critical value, the diameter of the vortices in the direction of the axis of the cylinders can be larger or smaller. If there is a range of allowable diameters when the cylinder speed exceeds the critical value, how does the flow choose which one to assume? Last year, Lorenz Kramer, Eshel Ben-Jacob, and Helmut Brand at the Institute for Theoretical Physics in Santa Barbara, and Michael Cross of Bell Laboratories addressed part of this problem. They calculated that in systems, such as Couette cells, under conditions where part of the system was below the critical value for the formation of periodic spatial patterns, such as Taylor vortices, and part was above, the resulting pattern would have only one repeat distance or wavelength. An additional stipulation was that the spatial transition from below to above critical conditions must be very gradual. If it is too abrupt, a band of wavelengths appears.

This work brings up the important point that the Couette problem is of far wider interest than it might first appear. Its well defined and highly symmetric geometry provides a test-bed for the solution of nonlinear equations in general. Kramer and his colleagues, for example, were interested in the problem of pattern selection that is described by nonlinear equations in several contexts. An especially important one is the formation of dendritic, snowflake-like patterns under certain conditions when molten metals freeze and crystallize.

In the present case, the theoreticians found that if the wavelength is determined uniquely anywhere in the system, then it is determined everywhere. Since it is fixed at the critical condition, this criterion is satisfied. The precise value of the wavelength could be calculated when the form of the transition from sub- to

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supercritical conditions was specified and not too complex.

Their calculation was partially verified by the experiments of David Cannell, Marco Dominguez-Lerma, and Günter Ahlers of the University of California at Santa Barbara. These physicists devised a Couette cell such that the outer cylinder was tapered over part of its length. The result of the tapering was that the distance between the two cylinders was too small to support Taylor vortices at the bottom of the Couette cell but was well above the size needed to allow their formation at the top.

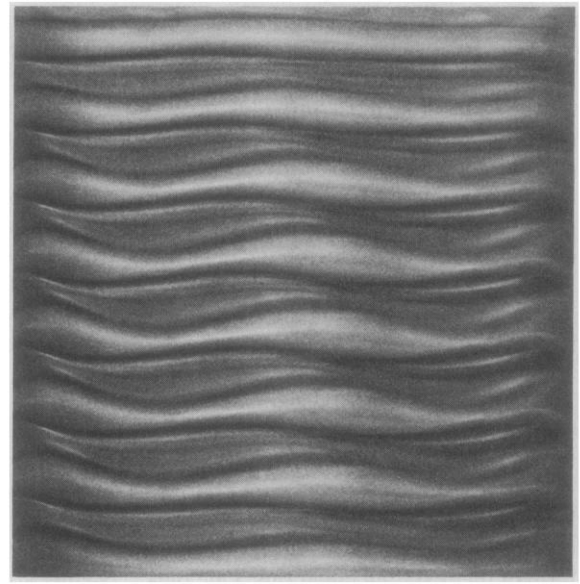
With a very gentle taper, the experimenters found that a single vortex diameter occurred in the straight section of the cell and that it was always equal to the distance between the cylinder walls. With a more severe taper, they found that a small range of vortex diameters was possible. However, the diameters actually chosen depended on the length of the cylinders. At a given length, only one diameter ever appeared. As the length increased, the wavelength would rise and fall repetitively with a period equal to twice the separation between the cylinder walls. In contrast to the theory, the investigators found that the range of allowable diameters decreased very rapidly as the speed of the cylinder increased.

A second issue relates to the effects of the ends of the Couette cell on the overall pattern of Taylor vortices. Vortices at the ends of a Couette cell form at lower cylinder speeds than those in the center, for example. The issue arises because most theoretical work has treated mathematically convenient cells of infinite length, whereas experiments necessarily involve shorter lengths. One obvious solution (subject to technical limitations) is to make cells very long, a few meters, in relation to the cylinder separation of a centimeter or less.

In 1981, Kwangjai Park, Gerald Crawford, and Russell Donnelly of the University of Oregon suggested that the problem was more complicated than this. The Oregon investigators measured the difference in the critical cylinder speed for the onset of Taylor vortices as the cylinder speed was increased from below and decreased from above the critical value (hysteresis). From the measured hysteresis at different acceleration (ramping) rates, with different cell lengths and cylinder spacings, and with fluids of varying viscosity, they concluded that the characteristic time for a fluid to settle was proportional to the length of the cell, thereby limiting the maximum useful cell length to that dictated by the patience of the observer. A cell only 20 centimeters long with a cylinder spacing of 0.8 centimeter filled with water takes about 1 day to reach the steady state. Any flow patterns observed before that are transients, argue the Oregon group.

Wavy mode

The photograph shows a side-view of a stack of Taylor vortices with traveling waves on them. Dark lines are outflow boundaries and bright lines are inflow boundaries. Because of the lighting, the inflow boundary shows up only in alternating pairs. [Source: Harry Swinney, University of Texas]



This interpretation remains controversial, but it seems to leave experimenters in a pretty pickle because long cells with presumably negligible end effects are not practical to study and short cells have large enough end effects that comparison with theory should be difficult. The spirit of the workshop was, to paraphrase several participants: "For 60 years experimentalists have been trying to devise experiments that fit the theory. Now it is time for theorists to model realistic, short cells." Some theorists, such as T. Brooke Benjamin of Oxford University have been complying with this request for some time.

However, there are more complex flow patterns than Taylor vortices awaiting attention. From Taylor's time onward, researchers observed that increasing the cylinder speed much beyond that for the onset of Taylor vortices introduced a transition to the wavy mode. One can imagine that either the inflow boundary, the outflow boundary, or both oscillate as one proceeds in the azimuthal direction around the cylinders. For a stationary wave, this pattern would remain fixed in place, but most wavy modes are traveling waves, so that the oscillation travels azimuthally in time (see photo). The number of waves or periods in a mode can vary, but there is always an integer number, starting with 1 at the lowest cylinder speed. At faster speeds, the number of waves first increases to as many as ten then decreases to as many as three.

From the discussion so far, one would not expect to be able to specify only the geometry of the Couette cell and the speeds of the cylinders and thereby to dictate which wavy mode would appear. Indeed, in 1965, Donald Coles of the California Institute of Technology demonstrated that as many as 26 distinct spatial states (combinations of the number of Taylor vortices, which ranged from 18 to 32, and the number of traveling waves on each of them, which varied from 3 to 7) could occur for a single cylinder speed. At the workshop, Coles renewed an old challenge. He would give to anyone who could explain it a three-dimensional model depicting the allowed combinations of vortices and waves as a function of cylinder speed that he made in 1968.

Donnelly listed several other features of wavy mode flow that the theorists have been unable to model accurately. The onset of wavy modes occurs at higher cylinder speeds in short cells than in long ones of the same cylinder spacing. And, as the cylinder speed rises and more waves are generated, the number of Taylor vortices decreases at certain speeds. Finally, there are defects in the wavy modes that can arise, persist indefinitely, and move about. For example, two Taylor vortex pairs carrying waves can merge into one pair while a third vortex pair divides into two. Computer simulation of the Navier-Stokes equations may be able to provide additional help, but realistic simulations seem to require access to a supercomputer.

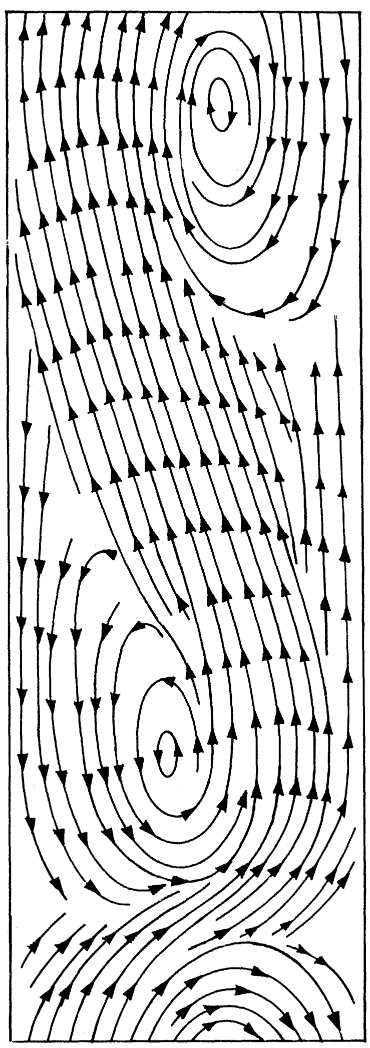
Philip Marcus of the Massachusetts Institute of Technology has access to the National Center for Atmospheric Research's Cray supercomputer, which is used there for the modeling of the earth's atmosphere. Marcus explained at the workshop that numerical simulation lies somewhere between theory and experiment. Simulation itself provides no explanation for what is calculated but can give theorists leads as to where interesting and feasible analyses might be.

Marcus uses what those in the trade call a pseudospectral technique. The solution to the Navier-Stokes equations is a velocity field that gives the three components of the fluid velocity at each point in space and at successive times. The velocity field is approximated by a truncated polynomial series with 33 terms for the radial dependence of the velocity, 32 terms for the azimuthal velocity, and 32 terms for the axial velocity. Spectral refers to the Fourier transform of the resulting equations. One outcome of this procedure is that the partial differential equations are converted into a set of ordinary difference equations that can be integrated on a computer. Pseudospectral means that some steps in the integration are carried out in the real coordinate system rather than in the Fourier transform coordinate system. In the end, Marcus obtains the velocity field on a grid of 33 by 32 by 32 points.

One result already turned in indicates that the resulting simulation of wavy modes is right on track. Marcus calculated the speeds of the waves and compared them with those recently measured by Gregory King, Ying Li, Wai Lee, and Harry Swinney of the University of Texas at Austin. In three different combinations of cylinder speed and vortex diameter, the agreement between experiment and simulation was within 0.1 percent. The simulations also suggest a physical mechanism for the formation of wavy modes. The first mode obtains its energy by breaking up the strong fluid jet that builds up at the outflow boundary due to the faster Taylor vortex flow at higher driving cylinder speeds. Conversely, the flow in normally isolated vortices is strongly mixed at the inflow boundary where the jet is weaker (see figure).

The simulation was, however, for an infinitely long Couette cell and a certain adjustment had to be made to relate the simulation to the experiment with a finite cell length. Asked from the floor at the workshop when the method could be applied to simulate end effects in long cells, Marcus estimated 5 to 10 years. Part of the problem is the computational resources needed to simulate a complex system. Improved simulation techniques and cheaper computing may make the end-effect simulation affordable in the stated time period. Marcus says it now takes about 1 hour to do a simulation, but the time scales with the square of the speed of the rotating cylinder.

Wavy modes are not the end of the story. As the cylinder speed in a Couette cell rises further, modulated wavy modes appear. The 1975 discovery of these modes at the City College of the City University of New York by Jerry Gollub of Haverford College and the University of Pennsylvania and Swinney popularized the use of laser-Doppler velocimetry in time-dependent fluid-flow research. By measuring the frequency shift of laser light that is scattered after being focused onto a small volume of moving fluid, one can determine the velocity of the fluid very accurately. In experiments of this type, Gollub and Swinney observed four distinct flow regimes when they made Fourier transforms of the velocity over a certain time interval. In the Taylor vortex (time-independent flow), there was no structure in the spectrum. At the onset of wavy modes, there was a single sharp peak at a particular frequency (and multiples or harmonics of the fundamental frequency) that corresponds to the speed of the traveling wave. As the cylinder speed increased still further, they observed a second fundamental frequency in the Fourier spectrum. Wavy modes with two frequencies are now termed modulated wavy modes.



Computer simulation

The figure shows a two-dimensional projection of the three-dimensional velocity field of a wavy mode. The horizontal axis is radial and runs from the inner to the outer cylinder. The length of the vertical axis is that of one vortex pair. [Source: Philip Marcus, Massachusetts Institute of Technology]

Subsequent experiments at Texas culminated in the report last year by Rob Shaw of the University of California at Santa Cruz, David Andereck of Texas, Leslie Reith of Bell Laboratories, and Swinney that definitively established that the second frequency belonged to a second traveling wave with the same or a different number of waves as the first but a different speed. A key part of the investigation was taking movies of the flow with a camera mounted on a rotating table. By adjusting the table speed to match one or the other of the waves, one can unravel their behavior.

Many physicists have found the most interesting feature of Gollub and Swinney's report to be the fourth regime, the turbulent state at the highest cylinder speeds. In the turbulent state, the Fourier spectrum is dominated by noise. Up to that time, the prevailing line of thinking started from the conjecture of the late Soviet physicist Lev Landau that there would be an infinite succession of transitions, with each transition accompanied by the addition of a new frequency to the Fourier spectrum. A noisy, turbulent state would never occur, whereas a successively more complicated but still periodic one would.

The direct transition from two frequencies to turbulence tied in nicely with ideas circulating among physicists, mathematicians, and others that turbulence in fluids might be one of many behaviors that could be determined by mathematical objects with the name strange attractors. Turbulence might be one example of a more general phenomenon called chaos (*Science*, 5 November 1982, p. 554). The notion is an enticing one partly because, rather than having to contend with a velocity field with a very large number of components, it would be possible to study the transition to turbulence with nonlinear equations having only a small number of variables. Several experiments have since produced evidence that some geometrically constrained fluids behave in accordance with models of chaos, but until recently, no strange attractors were deduced from experimental data. Anke Brandstäter and Swinney at Texas have now done this for Couette flow.

Attractors are the general name for the steady-state trajectory of a dynamical system. In the case of a fluid, the trajectory at each point in time is described by the values of the components of the velocity field at each point in space, a very large number of variables. For Taylor vortex flow, the attractor is a single point in this multidimensional space because the flow is time-independent. For wavy modes, the attractor is a closed loop formed as the velocity oscillates between the maximum and minimum values during the passage of a wave. For a type of flow with two frequencies similar to modulated wavy modes (quasi-periodic flow), the attractor is a torus. One frequency corresponds to motion around the major axis of the torus, and the other corresponds to motion around the minor axis. Motion on a strange attractor, which has a complex shape, is unpredictable, giving a noisy Fourier spectrum in the turbulent regime.

It is clearly impractical to construct a trajectory for a real fluid. But theorists have shown that an equivalent can be made much more simply by measuring the velocity, at a single location in the fluid. A point in the trajectory at time t is defined by the velocity at this location at times t, $t + \tau$, $t + 2\tau$, $t + 3\tau$, and so on. A practical limit to the number of multiples of τ needed is reached when the form of the trajectory fails to change with the addition of more terms.

With the aid of extremely accurate laser-Doppler velocimetry measurements, the Texas researchers were able to construct trajectories at different values of the cylinder speed. The trajectories had an important feature that branded them as belonging to a strange attractor. Trajectories starting from almost identical but slightly different initial points diverge exponentially on a strange attractor. This behavior is measured by a parameter called a Lyapunov exponent, whose value must be positive. From the experimental data, Alan Wolf and Jack Swift of Texas, calculated a positive Lyapunov exponent whose value increased as the cylinder speed rose.

Another important characteristic of strange attractors is that their dimension need not have an integer value. Swinney told the workshop that Doyne Farmer and Erica Jen of the Los Alamos National Laboratory have used the Texas data to calculate the dimension of the strange attractor. They found it to be small in accordance with chaos models and to vary continuously from 2 to 4 over the range of cylinder speeds studied.

There is as yet no specific chaos model that applies to Couette flow, but theorists are said to be working on one. Moreover, at a NATO advanced research workshop at Haverford College in early June, Pierre Bergé and Monique DuBois of the French Nuclear Studies Center in Saclay and George Buzyna and Richard Pfeffer of Florida State University presented evidence for strange attractors in other fluid systems.†

All in all, if the transition to turbulence remains a murky topic for the moment, fluids researchers seem to have more promising leads than they have had in quite a while.—**Arthur L. Robinson**

†NATO Advanced Research Workshop on Testing Nonlinear Dynamics, Haverford College, Haverford, Pennsylvania, 6 to 9 June 1983.

An Early Glacial Two-Step?

The most recent ice age seems to have exited in two steps, the earlier one perhaps coming sooner than had been thought possible

When the end of the last ice age came more than 10,000 years ago, it came quickly. After 100,000 years of gradual accumulation from winter snows, ice piled as high as 3000 meters over Canada, Greenland, and northern Europe wasted away to nearly nothing in only a few thousand years. This suddenness has been an awkward problem for geologists, a problem complicated by the recent detection of an apparent pause in the disintegration of the great ice sheets. But new evidence suggests that the first step of deglaciation, the one before the pause, may have been driven largely by forces other than the gradual warming of the climate.

Scientists generally agree that the ultimate cause of the end of the last ice age was changes in the orbit and the axial tilt of Earth, the Milankovitch or orbital variations that seem to pace the comings and goings of the ice ages (*Science*, 21 January, p. 272). Orbital variations produced a smooth increase in the solar energy or insolation falling during the summer on the Northern Hemisphere, where most of the ice to be melted was located, so that by 17,000 years ago solar insolation conditions resembled those of today. By 11,000 years ago, summer insolation on Northern Hemisphere ice sheets peaked at a level no more than 9 percent higher than today's. The timing of that peak nicely fit geological evidence from the continents and isotopic evidence from the deep sea for the occurrence of the maximum rate of ice sheet disintegration at about 11,000 years ago. The strongest sunshine melted the ice the fastest, or so it seemed.

Nothing is that tidy anymore. Jean-Claude Duplessy of the National Center for Scientific Research, Gif-sur-Yvette, France, and colleagues there and at the University of Bordeaux have concluded from their oxygen-isotope study of deep-sea sediments from the Bay of Biscay, south of the English Channel, that at least one-third of the ice melted between 16,000 and 13,000 years ago. That was long before increasing insolation could have melted much ice. Further complicating matters, melting of the ice sheets appeared to pause between 13,000 and 10,000 years ago, just when the fastest melting had been deduced from earlier studies. A second episode of melting apparently led to the present volume of ice by about 6000 years ago. A similar study of equatorial Atlantic sediments presented at a recent meeting* in Airlie, Virginia, by Alan Mix and William Ruddiman of the Lamont-Doherty Geological Observatory also found two steps in the deglaciation. The second step (10,000 to 8,000 years ago) coincided with Duplessy's, but the first came between 13,000 and 11,000 years ago. Despite their disagreement on timing, these two studies convinced most of those at the Airlie meeting that something slowed or stopped the disintegration of the ice sheets at the least likely time.

*Conference on the Timing and Mechanism of the Last Deglaciation, held at Airlie House, Virginia, 2 to 6 May; William Ruddiman, Lamont-Doherty Geological Observation, and Jean-Claude Duplessy, CNRS, Gif-sur-Yvette, France, cochairmen.