# Contributions of a Mathematical Physicist

**E. B. Christoffel.** The Influence of His Work on Mathematics and the Physical Sciences. P. L. BUTZER and F. FEHÉR, Eds. Birkhäuser, Boston, 1981. xxvi, 762 pp. $44.95.

The mathematical and scientific work of E. B. Christoffel covers very wide ground. A student principally of Dirichlet, Christoffel viewed himself as a mathematical physicist, particularly at the beginning of his career. He was interested in elasticity of solid bodies, dispersion of light, hydrodynamics, and acoustics. He became one of the pioneers in the theory of shock waves. Probably his greatest contribution to physics, however, was the indirect one that he made in helping to lay the mathematical foundation for relativity theory. Indeed, Christoffel was a mathematician par excellence. He had an instinct for problems of decisive importance, and he made fundamental contributions in the areas of quadrature formulas, orthogonal polynomials, continued fractions, conformal mapping, potential theory, differential geometry, and invariant theory.

Christoffel was born in 1829 in a small textile center called Montjoie, which today is the idyllic town of Monschau in western Germany, near Aachen. To mark the 150th anniversary of his birth, an ambitious project was undertaken to survey the impact of his work on mathematics and the physical sciences. An interdisciplinary effort was required because of the broad range of topics that required coverage. The project included a conference, the International Christoffel Symposium, which was held in Aachen and Monschau in November 1979. It has culminated in the publication of the present volume, which includes the 12 invited papers read at the conference, 43 additional papers solicited to shed light on the issues raised, and 22 short papers and commentaries. Though most of the papers are in English, a significant number are in German.

The book is organized into 13 sections, one devoted to historical matters, 11 to the main subjects, and one to the short papers and commentaries. For a good overview of the contents, I recommend P. L. Butzer's informative outline of the life and work of Christoffel. Here it is only possible to sample a few items.

Sections 2 and 3 deal with the interrelated subjects of quadrature formulas, orthogonal polynomials, continued fractions, and Padé approximations. These are all old subjects in which there is much modern interest, in part because of new possibilities with computers. A Gauss-Christoffel quadrature formula has the form

$$\int_a^b f(t)w(t)\,dt = \sum_1^n \lambda_j f(t_j) + R_n(f)$$

where $t_1, \ldots, t_n$ and $\lambda_1, \ldots, \lambda_n$ depend only on the weight function $w(t)$ and $R_n(f) = 0$ for every polynomial $f(t)$ of degree less than $2n$. The formula has many variants. A superb 76-page survey by W. Gautschi traces the history of quadrature formulas from Newton and Cotes through Gauss, Jacobi, Mehler, and Christoffel, to the present. The paper gives copious references to the modern literature on each topic, for example quadrature formulas for Cauchy principal value integrals, complex weight functions, remainder estimates, efficient computer algorithms, and numerical tables.

Continued fractions and Padé approximations can be used to replace slowly converging series by more rapidly converging processes or to extend the region of convergence to values of a parameter for which the series itself diverges. Interesting papers by P. Wynn and W. J. Thron discuss new and old problems and applications for continued fractions. Papers by J. S. R. Chisholm and A. K. Common and by G. A. Baker and J. E. Gubernatis explore some of the possibilities with Padé approximations for Fourier, Chebyshev, and Legendre series, including applications to potential scattering theory.

Sections 8 and 9 contain ten papers on differential geometry and related topics. It was here that Christoffel made his great contributions that helped prepare the way for relativity theory. In his work on the equivalence problem for quadratic differential forms, Christoffel introduced the symbols $\Gamma_{ij}^k$ and $\Gamma_{ij,k}$, which bear his name, and the operation now known as covariant differentiation. These ideas and modern generalizations, as well as applications to relativity theory and field physics, are discussed in an authoritative survey by J. Ehlers.

In connection with the problem of determining the shape of the earth from local measurements, Christoffel proved that a smooth convex surface is determined up to translation by its mean curvature as a function of the outward normal vector. This result is one of the topics treated in a survey paper by W. Klingenberg. It is also discussed from the point of view of geodesy in an extensive commentary by E. W. Grafarend.

Other chapters cover, for example, complex function theory, shock waves, and potential theory. The inclusion of the last subject is a surprise, since Christoffel's work on potential theory escaped notice until now. M. Brelot has contributed an assessment of Christoffel's papers on the subject. The assessment is very favorable, and even though Christoffel's results have since been largely superseded, Brelot writes that one paper "remains actually partly new and suggestive."

Written by experts from diverse fields, the book aims to communicate both historical perspectives and modern ideas across disciplinary boundaries. Careful editing has preserved coherence in the exposition. The book is of a rare genre, and it is highly successful.

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## Papers from Erice

**Rigorous Atomic and Molecular Physics.** Proceedings of a school, Erice, Italy, June 1980. G. VELO and A. S. WIGHTMAN, Eds. Plenum, New York, 1981. viii, 496 pp. $59.50. NATO Advanced Study Institutes Series B, vol. 74.

The conceptual framework of quantum mechanics was studied by Von Neumann and Weyl immediately following the inception of the theory. The rigorous study of the explicit equations (for, say, atomic physics) is more recent. Although one should mention earlier work by K. Friedrichs and F. Rellich, it is fair to say that the subject, often called "the theory of Schrödinger operators," was born only in 1951 with a basic paper by T. Kato. There has been vigorous growth since, especially in the past 15 years.

The book under review, the proceedings of the fourth International School of Mathematical Physics, is in two rather disjoint parts. The first 326 pages contain a comprehensive overview of the subject of Schrödinger operators by a set of authors who are almost a who's who of the subject. All seven papers on this subject are valuable and all but one by

Combes (which I found too diffuse in presentation and too terse in its technical parts) are very well written. Special mention should be made of an encyclopedic review by E. Lieb of the work (most of it by Lieb and his collaborators) on Thomas-Fermi and related equations and of a paper by J. M. Combes, P. Duclos, and R. Seiler on the Born-Oppenheimer approximation, which is the most complete presentation to date of a set of ideas that the authors announced over five years ago and that have previously been presented only in bits and pieces. It should be mentioned that the title is somewhat misleading concerning this part of the book. Roughly half the material deals with atomic and molecular problems, but the other half discusses general nonrelativistic quantum mechanics.

The second 163 pages discuss Coulomb systems. All four papers on this subject are excellent, but with the partial exception of J. L. Lebowitz's paper on free energy and correlation functions of Coulomb systems their connection with the first seven papers is tenuous at best. The editors (who were the organizers of the school) write in the preface that the recent work of D. Brydges and P. Federbush shows for the first time from first principles that Debye screening is a consequence of Schrödinger's equation. This work (which Brydges and Federbush discuss in the book) and all other rigorous proofs of screening have been for classical Coulomb systems, however. Papers by M. Aizenman and by J. Frohlich and T. Spencer are even further removed from atomic problems; they deal with Coulomb systems in one and two space dimensions respectively, where Coulomb potential diverges at infinity. So only about a third of the book deals with atomic and molecular physics, but virtually all of the book is rigorous.

The editors might have included the papers on Coulomb systems in order to make the point that there is too little known about the special features of quantum mechanics with Coulomb potentials. For example, we don't even know how to prove that for atoms with more than three electrons it takes more energy to remove the second electron from the atom than the first. Since there isn't enough rigorous quantum Coulomb theory, why not present rigorous quantum theory side by side with rigorous classical Coulomb theory?

The book would have had greater coherence and would probably have been more useful had the three papers on classical Coulomb systems been replaced by papers on some of the items missing or only briefly discussed in the overview of Schrödinger operators: properties of wave functions, the Mourre theory, eigenfunction expansions, the complex scaling theory of resonances, and path integrals. It would have been exciting if the book had included something from the more mathematically inclined chemical physicists.

In spite of these reservations, the high quality of the papers makes this a most welcome addition to the literature. The caliber is so high that many mathematical physicists will want to have a copy on their shelves.

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## Far Eastern Archeology

**Prehistory of Japan.** C. MELVIN AIKENS and TAKAYASU HIGUCHI. Academic Press, New York, 1981. xviii, 358 pp., illus. $37.50. Studies in Archaeology.

A rapid economic development in a small archipelago that has been occupied for thousands of years churns up archeological remains. In order to salvage part of what would otherwise be completely lost, 11,000 archeological excavations are planned for the fiscal year ending in March 1982. There is an avalanche of excavation reports, which the law requires, reports of scientific analyses, which are increasing with the number of interdisciplinary investigations, and a large number of interpretative works on specialized topics. An up-to-date synthesis of the whole of Japanese prehistory has become an impossible task that no prudent person would undertake.

In *Prehistory of Japan*, Aikens and Higuchi have wisely chosen to present a collection of summary translations of site reports, with introductory and concluding chapters where they set forth the central theme of cultural continuity for both the Ainu and the Japanese. In its matter-of-fact descriptive approach the book is somewhat unusual for the Studies in Archaeology series, which contains a number of highly theoretical works. Detailed summaries of 17 out of some 1000 Paleolithic sites, 18 of about 10,000 sites of the Jomon period (around 10,000 to 300 B.C.), and nine Yayoi (300 B.C. to A.D. 300) and seven Kofun (A.D. 300 to 700) sites, out of several thousands each, are carefully interwoven with connecting passages where the theme is developed.

The work and skill involved in producing such coherent and readable summaries will be appreciated by those who have tackled Japanese site reports, which are often voluminous and not always well coordinated. The use of original reports as the main source, however, has obvious drawbacks. The authors arrive at the surprisingly early dates of 400,000 years ago for Sozudai and over 65,000 years ago for Cultural Stratum 3 (which, by the way, consists of chert, not quartzite, specimens) of Hoshino by basing their account on site reports published in the 1960's. In later publications, C. Serizawa, the investigator of the two sites, states that Sozudai may be about 100,000 years old and Stratum 3 of Hoshino is younger than 21,000 years.

In any event, these "Early Paleolithic" assemblages, according to the authors, do not demonstrate human workmanship. What the authors accept as the earliest indications of human presence are the stone tools that date to between 25,000 and 30,000 years ago. Some of these assemblages, such as those described here, actually contain nothing that could not be understood as the continuation of the Early Paleolithic tradition, whose existence I happen to accept. Within the specified time range are other assemblages, not mentioned in the book, that include edge-ground axes and scrapers, as well as parallel-sided blades. If the blades signify the "Advanced Palaeolithic," then, the authors' "Aurignacian-like" industries began about the same time in Japan as their "Mousterian-like" industries. The only benefit of using European terminology for Japanese Paleolithic, it seems to me, is to imply a long-range diffusion. According to Aikens and Higuchi, the Japanese Paleolithic is to be seen as part of "the evolutionary process . . . that involved both autochthonous development, and diffusion, going on more or less simultaneously throughout Eurasia" (p. 326).

The rest of Japanese prehistory is also to be understood in terms of the combination of indigenous development and diffusion, without invoking major population replacements. The transition from the Paleolithic to the Jomon was effected when ceramics were introduced from a yet unknown locality on "the continent via Korea" (pp. 114, 328) about 12,500 years ago. The continuity in stone tools from the Late Paleolithic to the early phase of the Initial Jomon is stressed, and the emergence of "the mature Jomon tradition" about 9500 years ago is described as the results of readaptation of Japan's Ice Age occupants to the new environmental conditions (p. 328). Simi-