Contributions of a Mathematical Physicist

E. B. Christoffel. The Influence of His Work on Mathematics and the Physical Sciences. P. L. BUTZER and F. FEHÉR, Eds. Birkhäuser, Boston, 1981. xxvi, 762 pp. $44.95.

The mathematical and scientific work of E. B. Christoffel covers very wide ground. A student principally of Dirichlet, Christoffel viewed himself as a mathematical physicist, particularly at the beginning of his career. He was interested in elasticity of solid bodies, dispersion of light, hydrodynamics, and acoustics. He became one of the pioneers in the theory of shock waves. Probably his greatest contribution to physics, however, was the indirect one that he made in helping to lay the mathematical foundation for relativity theory. Indeed, Christoffel was a mathematician par excellence. He had an instinct for problems of decisive importance, and he made fundamental contributions in the areas of quadrature formulas, orthogonal polynomials, continued fractions, conformal mapping, potential theory, differential geometry, and invariant theory.

Christoffel was born in 1829 in a small textile center called Montjoie, which today is the idyllic town of Monschau in western Germany, near Aachen. To mark the 150th anniversary of his birth, an ambitious project was undertaken to survey the impact of his work on mathematics and the physical sciences. An interdisciplinary effort was required because of the broad range of topics that required coverage. The project included a conference, the International Christoffel Symposium, which was held in Aachen and Monschau in November 1979. It has culminated in the publication of the present volume, which includes the 12 invited papers read at the conference, 43 additional papers solicited to shed light on the issues raised, and 22 short papers and commentaries. Though most of the papers are in English, a significant number are in German.

The book is organized into 13 sections, one devoted to historical matters, 11 to the main subjects, and one to the short papers and commentaries. For a good overview of the contents, I recommend P. L. Butzer's informative outline of the life and work of Christoffel. Here it is only possible to sample a few items.

Sections 2 and 3 deal with the interrelated subjects of quadrature formulas, orthogonal polynomials, continued fractions, and Padé approximations. These are all old subjects in which there is much modern interest, in part because of new possibilities with computers. A Gauss-Christoffel quadrature formula has the form

$$\int_a^b f(t)w(t)\,dt = \sum_1^n \lambda_j f(t_j) + R_n(f)$$

where $t_1, \ldots, t_n$ and $\lambda_1, \ldots, \lambda_n$ depend only on the weight function $w(t)$ and $R_n(f) = 0$ for every polynomial $f(t)$ of degree less than $2n$. The formula has many variants. A superb 76-page survey by W. Gautschi traces the history of quadrature formulas from Newton and Cotes through Gauss, Jacobi, Mehler, and Christoffel, to the present. The paper gives copious references to the modern literature on each topic, for example quadrature formulas for Cauchy principal value integrals, complex weight functions, remainder estimates, efficient computer algorithms, and numerical tables.

Continued fractions and Padé approximations can be used to replace slowly converging series by more rapidly converging processes or to extend the region of convergence to values of a parameter for which the series itself diverges. Interesting papers by P. Wynn and W. J. Thron discuss new and old problems and applications for continued fractions. Papers by J. S. R. Chisholm and A. K. Common and by G. A. Baker and J. E. Gubernatis explore some of the possibilities with Padé approximations for Fourier, Chebyshev, and Legendre series, including applications to potential scattering theory.

Sections 8 and 9 contain ten papers on differential geometry and related topics. It was here that Christoffel made his great contributions that helped prepare the way for relativity theory. In his work on the equivalence problem for quadratic differential forms, Christoffel introduced the symbols Γ_{ij}^k and $\Gamma_{ij,k}$, which bear his name, and the operation now known as covariant differentiation. These ideas and modern generalizations, as well as applications to relativity theory and field physics, are discussed in an authoritative survey by J. Ehlers.

In connection with the problem of determining the shape of the earth from local measurements, Christoffel proved that a smooth convex surface is determined up to translation by its mean curvature as a function of the outward normal vector. This result is one of the topics treated in a survey paper by W. Klingenberg. It is also discussed from the point of view of geodesy in an extensive commentary by E. W. Grafarend.

Other chapters cover, for example, complex function theory, shock waves, and potential theory. The inclusion of the last subject is a surprise, since Christoffel's work on potential theory escaped notice until now. M. Brelot has contributed an assessment of Christoffel's papers on the subject. The assessment is very favorable, and even though Christoffel's results have since been largely superseded, Brelot writes that one paper "remains actually partly new and suggestive."

Written by experts from diverse fields, the book aims to communicate both historical perspectives and modern ideas across disciplinary boundaries. Careful editing has preserved coherence in the exposition. The book is of a rare genre, and it is highly successful.

JAMES ROVNYAK
*Department of Mathematics,
University of Virginia,
Charlottesville 22903*

Papers from Erice

Rigorous Atomic and Molecular Physics. Proceedings of a school, Erice, Italy, June 1980. G. VELO and A. S. WIGHTMAN, Eds. Plenum, New York, 1981. viii, 496 pp. $59.50. NATO Advanced Study Institutes Series B, vol. 74.

The conceptual framework of quantum mechanics was studied by Von Neumann and Weyl immediately following the inception of the theory. The rigorous study of the explicit equations (for, say, atomic physics) is more recent. Although one should mention earlier work by K. Friedrichs and F. Rellich, it is fair to say that the subject, often called "the theory of Schrödinger operators," was born only in 1951 with a basic paper by T. Kato. There has been vigorous growth since, especially in the past 15 years.

The book under review, the proceedings of the fourth International School of Mathematical Physics, is in two rather disjoint parts. The first 326 pages contain a comprehensive overview of the subject of Schrödinger operators by a set of authors who are almost a who's who of the subject. All seven papers on this subject are valuable and all but one by