ods," Grabiner argues, "but it did not do so in this case. The conceptual difference between the eighteenth-century way of looking at and doing the calculus and nineteenth-century views was simply too great" (p. 2). In investigating the difference, Grabiner focuses on the threads of 18th-century mathematical thought that Cauchy wove into a new basis for the calculus. They include the body of results that made up the calculus, and concepts and techniques of inequalities and approximations, onto which Cauchy imposed the rigorous proof structure of Greek geometry. He taught his new brand of analysis to "all of Europe" through his courses at the Ecole Polytechnique and his widely used textbooks, thereby laying the groundwork for the complete "rigorization" of calculus by the Weierstrass school.

Grabiner first demonstrates the difference between Cauchy's "revolutionary" understanding of rigorous analysis and his predecessors' attitudes toward the foundations of calculus. She then explores theory and practice in 18th-century algebra, with special attention to those methods of approximating roots and computing error bounds that Cauchy would transform into the basis of his calculus. With this as background, Grabiner investigates the origins of the basic concepts of limit, continuity, and convergence (as they appeared in Cauchy's 1820 *Cours d'analyse*) and his theory of the derivative and the definite integral (from the 1823 *Calcul infinitésimal*). The exposition draws equally from Cauchy's work and that of such notable predecessors as Euler, d'Alembert, Ampère, Poisson, and especially Lagrange. Indeed, Grabiner takes particular pains to demonstrate that Cauchy needed a large body of mathematical results for his innovation and that "the mathematics he needed came from the work of the major mathematicians of the eighteenth century" (p. 165).

The book is directed toward an audience with enough mathematical background to follow the proofs and to appreciate the argument regarding creative deployment of algebraic techniques in the rigorization of analysis. But Grabiner asks her readers to think historically—to distinguish between an implicit mathematical concept and a conscious statement of the same result, and to recognize the historical connection between two fields usually treated separately. Her sensitivity to changing definitions, for example, permits a reasoned reevaluation of 18th-century attitudes toward "convergence."

The reader might wish that Grabiner had not limited the book so strictly. There is little indication of where Cauchy's rigorous calculus fits into the whole of his mathematical work. Moreover, footnote material on the transmission of mathematical ideas might well be integrated into the text. By contrast, Grabiner's objections to Ivor Grattan-Guinness's contention that Bolzano influenced Cauchy interfere with the flow of her argument. The organizational pattern chosen for this book—with the story weaving back and forth between Cauchy and Lagrange—also leaves an impression of repetition. The thematic approach has its virtues, however, especially compared to chronological catalogues of mathematical results. The history of mathematics could use more such thoroughly researched and insightful studies of key themes and developments.

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Conceptions of Space

Much Ado About Nothing. Theories of Space and Vacuum from the Middle Ages to the Scientific Revolution. EDWARD GRANT. Cambridge University Press, New York, 1981. xiv, 456 pp. $59.50.

In Aristotle's physics, the whole cosmos was supposed to be finite. Its outer limit was a spherical shell containing the fixed stars. Inside this shell there were no vacuums and outside there was nothing, not even empty space. In Antiquity and the Middle Ages this view triumphed over the Stoic view, which placed the finite spherical cosmos within an infinite empty space, and also over the atomists' view, which postulated the existence of infinite numbers of atoms moving throughout an infinite vacuum. In the scientific revolution of the 17th century a revived version of the atomists' view defeated the long-held Aristotelian system.

In his classic book, *From the Closed World to the Infinite Universe*, Alexandre Koyré attempted to trace the change from Aristotle's closed spherical world to Newton's infinite universe. He pictured this change as the geometrization of space or, in other words, as "the replacement of the Aristotelian conception of space—a differentiated set of innerworldly places—by that of Euclidean geometry—an essentially infinite and homogeneous extension—from now on considered as identical with the real space of the world." Koyré pictured this geometrization of space as part of a general mathematization of physics whereby qualitative Aristotelian physics was replaced by the exact quantitative work of Descartes, Newton, and modern physics in general.

In *Much Ado About Nothing* Edward Grant resurveys territory already covered by Koyré and adds much new besides, most importantly by examining the theories of space of late medieval and early modern scholastics, a subject almost entirely neglected by Koyré. Grant argues that Newton's concept of infinite homogeneous space could not have derived from the geometrization of space, as Koyré believed, because in fact Euclid's geometry itself did not presuppose the existence of a separate, three-dimensional, infinite void space in which to locate its geometrical figures. By contrast, in the period just preceding the 17th century the discussions that one does find of the possibility of space outside the finite cosmos are frequently connected with the issue of God's omnipresence. Grant concludes (p. 263), "From the fourteenth century on, numerous medieval scholastics would associate extracosmic space with God's assumed extracosmic existence. Without the assumption of God beyond the world, the Stoic arguments about the necessity for extracosmic spatial existence would have proved of no avail against Aristotle. . . . The exclusion of scholastics from previous histories of space has limited our perspective and prevented genuine comprehension of the developments that eventually produced the fundamental frame of the Newtonian universe."

The history of late medieval and early modern concepts of space is tremendously complicated and intriguing. This book contains more than 150 pages of small-print notes, and even so often refers elsewhere for texts of the arguments under consideration. Grant does not try to show through examination of medieval and early modern geometrical works that these did not contain the concept of geometrical space needed for Koyré's argument, but his assertion that the mathematical tradition did not influence concepts of physical space is not prima facie implausible, given that medieval mathematics operated within a context shaped by Aristotelian physics. From an Aristotelian viewpoint geometry derives its concepts by abstraction from the attributes of physical bodies. Extensions are supposed to be the extensions of physical substances. According to this scheme the concept of empty three-dimensional space is anomalous because it supposes attributes such as three-dimensionality where there is nothing that has these attributes.

In the late 13th and 14th centuries, it became common to suppose that God might have created another cosmos outside of this one. Had he done so, he would be present there just as he is omnipresent in this cosmos. But he would not *move* there, since he is immutable, so he must already be in every place or space in which he might have created something. It was concluded, therefore, that God is now present outside the cosmos.

Does this imply that there is now infinite space outside the cosmos as the Stoics asserted? It is at this point that scholastic discussions of extracosmic void space become most fascinating by virtue of the intermingling of scientific, logical, and theological reasoning. According to the Aristotelian viewpoint, there will be no extension outside the cosmos unless there is substance there. This follows simply because everything that exists must be a substance or an attribute of a substance and more particularly because extension must be the extension of something. Sometimes the argument appears that for empty space to be extended there must at the very least be a sort of material scaffolding alongside it to provide it with a measure (see p. 124).

Does God's presence outside the cosmos provide a basis for extension there? Might extension be an attribute of God? This seemingly easy solution was generally blocked by the theological doctrines that had been developed to explain God's omnipresence. Although God is present everywhere, it was argued, he is not extended in space in the sense that one part of him is in one place and another part in another place. This would conflict with the Christian understanding of the nature of God. Rather God is totally present in every part of space. If this is so, although God is everywhere, he is not extended, so extension cannot be his attribute. If extracosmic extension is not God's attribute, could it be ascribed to some other entity? No: even if the requirements of strict Aristotelianism are relaxed to suppose that extracosmic space is an independent nonmaterial extended entity, theology bars the door to this solution by declaring that there can be no infinite eternal entity other than God.

Medieval scholastics, then, modified the Aristotelian view by postulating the existence of God and space outside the cosmos, but this did not lead to a concept of extended space outside the cosmos because of the combined requirements of Aristotelianism and scholastic theology. As God's attribute, extracosmic space was infinite but nonextended. The story of how this theoretical bind was eluded or sidestepped to postulate the existence of infinite extended space forms the core of the second and most important section of Grant's book. (The other major section deals with the possibility of empty space within the cosmos.) Almost every possible way out was tried. Grant believes that Henry More and Isaac Newton made three-dimensional space God's attribute and simply accepted the conclusion that God is an extended being. J. E. McGuire, however, has argued that even Newton still accepted the medieval whole-in-every-part view of God's omnipresence (see p. 253 and note 420). For the light it might cast on this problem more attention should be paid to the medieval doctrine that spiritual beings are present in space through their activity, the more powerful being having a larger sphere of activity.

Clearly there is ample material here for still further studies both of the background of Newton's concept of absolute space and with regard to understanding the dynamics of scientific change. Without doubt, in the 16th and 17th centuries metaphysical and theological considerations played an essential role even within nonscholastic physics.

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Gene Duplication

Evolution and Variation of Multigene Families. TOMOKO OHTA. Springer-Verlag, New York, 1980. viii, 132 pp., illus. Paper, $9.80. Lecture Notes in Biomathematics, vol. 37.

Much of the excitement in genetics lately comes from the discovery that a large and important part of eukaryotic genomes is made up of families of homologous genes. This finding opens up an array of new questions in population and quantitative genetics as well as evolutionary theory. In a series of papers Ohta has analyzed several specific problems related to multigene families. In this book she provides a compilation of these papers, with the exception of some of her most recent ones. Some previously unpublished results are also included. The compilation will be useful because it presents the results in a logical order and with a consistent algebraic notation.

Despite the overly general title this is not meant to present a general theory of multigene families, but only a treatment of certain problems. The word "evolution" in the title could have been replaced by "neutral evolution" for most of the work. Such topics as the coexistence of functional and nonfunctional genes in the same family are not considered. The number of genes per family is assumed to be fixed, so that the many important questions dealing with the evolution of multiplicity itself are bypassed. The term "multigene families" is also very narrowly defined to mean genes of high multiplicity arranged in direct, tandem repeats. The prevailing view now is that multigene families of low multiplicity (2 to 10 copies) are much more typical, and many of these are widely dispersed throughout the genome rather than tandemly repeated. Moreover, movable genetic elements have been discovered in several organisms and can occupy as much as 10 to 20 percent of the euchromatic genome. The existence of such elements poses the intriguing possibility that action at the molecular level can be separate from, and even opposed to, selection at the organismal level. No precise theory exists for handling these and many other questions related to multigene families.

The scope of the book can be summarized as follows: Consider a very long sequence of tandemly repeated genes undergoing selectively neutral mutations at a constant rate. Each new mutation is assumed to be unique, thus increasing variability in the sequence. Meanwhile, unequal crossovers between homologous regions shifted by one or more positions tend to decrease variability by creating duplications and deletions. Ohta examines the properties of the equilibrium at which these opposing forces are balanced and presents a description in terms of identity coefficients, correlations between loci, and so on. She then compares this description to existing data on amino acid sequences of immunoglobulins to argue that, with appropriate choice of model parameters, they agree. She also uses this approach to compare opposing mutational hypotheses (somatic versus germ line) for the origin of hypervariable regions. By examining within- and between-species variability, she shows that the data fit her model better under the germ-line hypothesis.

A consistent mathematical strategy is employed throughout the book. We start by defining some variable or set of varia-