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The 46° Halo and Its Arcs

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Summary. Ice crystals in the form of right hexagonal prisms have faces that form 90° prisms. Light rays were traced through these prism faces by computer calculation, and the light patterns that would be produced in the sky for a particular distribution of crystal orientations were simulated. Crystals with random orientations produce a 46° halo. Hexagonal plate crystals with nearly horizontal end faces produce circumzenithal and circumhorizontal arcs. Hexagonal column crystals with horizontal axes produce supralateral and infralateral arcs. Plate crystals spinning about a horizontal axis that is a face diagonal of the crystal produce a series of arcs touching the 46° halo. Each of these effects was simulated for several elevations of the sun.

Two large halos can sometimes be seen around the sun. This article is about the 46° halo and its associated arcs, which are less commonly observed than the smaller 22° halo and its arcs. Since the same ice crystals contribute to both sets of effects, it may be useful to review the mechanisms that produce the 22° effects.

In Wisconsin there is visible evidence of the 22° halo complex on about 160 days per year. Many of these effects are produced by light passing through alternate side faces of hexagonal ice crystals, faces that are 60° apart and hence refract light as if it were passing through a 60° prism of ice (*1-3*) (see ray a in Fig. 1). The minimum angle of deviation for light passing through such faces is 22°. Pencil crystals and plate crystals (Figs. 1 and 2), both of which are hexagonal prisms, exist in the atmosphere (*4*) and become oriented in various ways as they fall through the air (*2, 3, 5*). Light passing through the 60° faces of ice crystals whose sizes and shapes give them special orientations produces arcs tangent to the 22° halo (*6*), the circumscribed halo (*6*), sun dogs (parhelia), Parry arcs (*7*), and the arcs of Lowitz (*8, 9*). Hexagonal prism crystals that assume random orientations give rise to the 22° halo.

Hexagonal ice crystals also have faces at right angles to each other, and this produces a variety of effects in the sky. A 90° glass prism cannot transmit visible light but a 90° prism of ice, with its lower index of refraction, can indeed transmit light rays (see Fig. 1, rays b and c). The minimum angle of deviation for light passing through the 90° faces of an ice crystal is about 46°. For a random distribution of crystals there should be a concentration of light refracted at the angle of minimum deviation. This light should come to an observer's eye when he looks in a direction 46° away from the sun. The result would be a halo around the sun with an inner edge of angular radius 46°. The 46° halo and its arcs are seen considerably less often than the 22° halo—in our experience only a few days per year. The 22° halo is a large halo (not to be confused with the corona diffraction rings that encircle the sun or moon and are typically only a few sun or moon diameters across), but the 46° halo is a very large one—more than 90° in diameter. The very large scale of this halo may make it less noticeable to the casual observer.

The object of this article is to consider the effects arising from sunlight refracted through the 90° faces of hexagonal-prism ice crystals. We simulated the effects of light passing through various plausible orientation distributions of ice crystals and compared these simulations with actual observations. In some cases our simulations match observed effects; in other cases they predict arcs that apparently have not been previously reported or predicted. Although observation should not be dominated by theory, it should be encouraged by theory. We hope that the computer predictions will stimulate the search for effects that have so far escaped notice.

Method

The basic method of doing the computer simulation has been described before (*6, 7*). Instead of considering many crystals as light passes through them at different locations in the sky, we have one crystal successively assume each of the different orientations of the distribution under study. First we specify the orientation of the crystal by fixing three angles. We then apply Snell's law to determine the direction of the ray after it has entered the ice crystal through the entrance face and again as it leaves the crystal by the exit face. Once the direction of the deviated ray has been determined, the crystal can be positioned on a reference plane (for example, oriented perpendicular to the observer's line of sight to the sun) so that the refracted light is directed to the observer's eye. A dot placed at this location on the reference plane indicates where light coming from a crystal of that particular orientation would be seen. A collection of such

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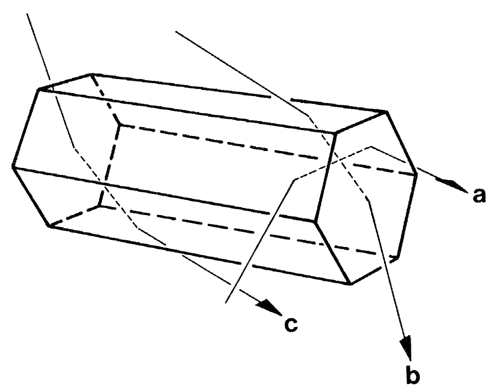
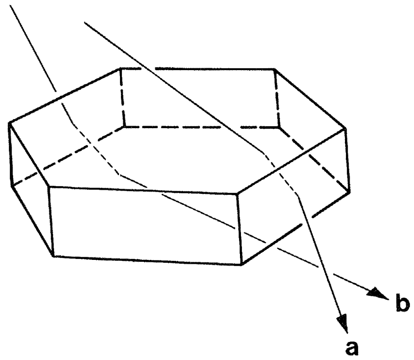
dots from all of the crystal orientations gives a pattern indicating the light distribution in the sky.

Our calculations also include intensity information. The amount of light that passes through a crystal depends on its orientation. Two factors determine how much light is transmitted: (i) the cross section of the incident light beam that enters the entrance face and impinges on the exit face inside the crystal and (ii) the light lost by reflection at both the entrance and exit faces of the crystal. The first factor is a matter of geometry; to calculate it exactly for any crystal orientation, we need to specify the ratio of length to width for the hexagonal prism. For that we choose two limiting cases. When dealing with pencil crystals we choose the limiting case of a crystal that is very long compared to its width, whereas plate crystals are represented by the other limit (a crystal that is very wide compared to its length). For the effects of 90° crystal faces we use one other approximation in the intensity calculation. We approximate the area of the hexagonal end face by using the circle that circumscribes it. With these approximations, all of the other intensity factors are calculated exactly for this geometrical-optics treatment. The second intensity factor (reflection) is not difficult to calculate. The loss of light by reflection at each face depends only on the angle of incidence of the light and the index of refraction of ice, and is given by the Fresnel expression for unpolarized light. These two factors, then, combine to give a relative intensity for each ray. We represent relative intensity on the simulations by a selective discarding of dots. The relative intensity of each dot is compared with a random number between 0 and 1 and the dot is plotted only if the intensity is larger than the random number. Thus a dot with a relative intensity of 0.2 has only a 0.2 probability of being plotted.

The only other approximations we make in this treatment are inherent in geometrical optics: diffraction effects are ignored and crystals are assumed to be of perfect geometrical form.





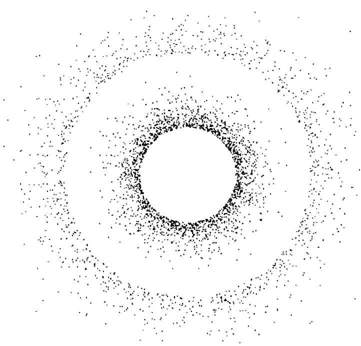
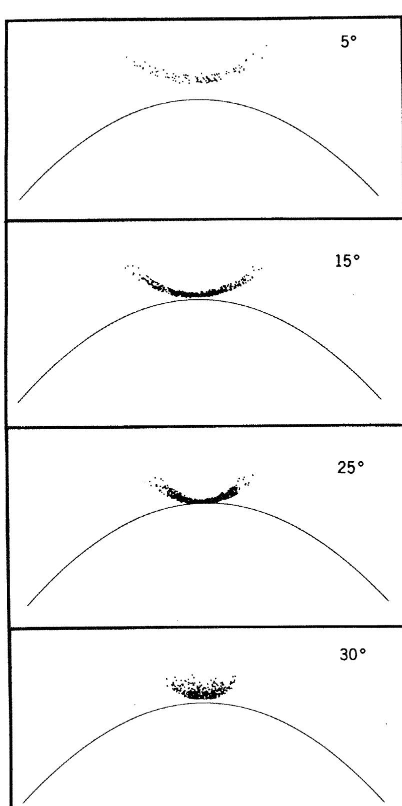


Fig. 1 (top left). Light rays passing through a pencil ice crystal. Ray a results in the 22° halo and its associated arcs; rays b and c result in the 46° halo and its associated arcs. Fig. 2 (bottom left). Light rays passing through a plate crystal. Ray a results in the circumzenithal arc; ray b, the circumhorizontal arc. Fig. 3 (top right). Simulation of the 22° and 46° halos.



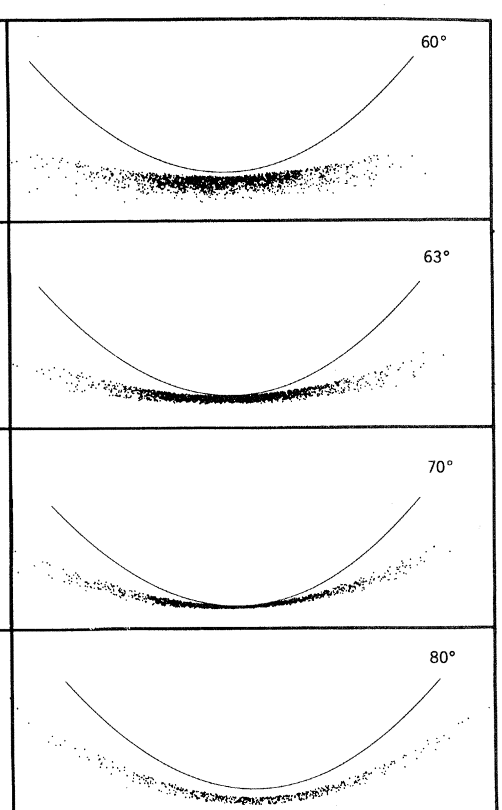


Fig. 4 (left). Circumzenithal arc simulations for four different sun elevations. They result when rays such as ray a in Fig. 2 pass through crystals whose end faces have tilts up to 3° from the horizontal. Fig. 5 (right). Circumhorizontal arc simulations for four different sun elevations. They result when rays such as ray b in Fig. 2 pass through crystals whose end faces have tilts up to 3° from the horizontal.

46° Halo

To simulate the 46° halo, we consider light rays that enter a side face and exit from an end face of the crystal, as well as rays that take the reverse path. Figure 3 shows the simulations of both the 22° and the 46° halo that result when a pencil crystal is given random orientations. (The relative intensities of the two halos are not indicated.) We believe that the primary reason for the comparative rarity of the 46° halo is its considerably lower intensity, which is due to several factors.

1) Only rays in a fairly narrow angular range will get through the 90° prism.

2) The transmitted rays for the 90°

prism have high angles of incidence on both the entrance and exit faces, resulting in a transmitted beam of small cross section.

3) The high angles of incidence result in larger reflection losses at both the entrance and exit faces.

4) Light producing the 46° halo is spread over a larger part of the sky than light producing the 22° halo, giving a fainter halo.

Circumzenithal and Circumhorizontal Arcs

Plate crystals in the appropriate size range tend to become oriented while falling so that their hexagonal end faces become nearly horizontal; refraction through the side faces then produces the well-known parhelia (sun dogs) on either side of the sun. Consider the rays shown in Fig. 2, which pass through the 90° faces of crystals whose orientations are distributed in this way. It can be shown (*3*) that a ray that enters the horizontal top face and exits from a side face will make the same angle from the vertical for any rotation of the crystal about a vertical axis (ray a in Fig. 2). The resulting pattern for such rays should be an arc above the sun, extending around the sky at constant elevation. In other words, the arc should lie on a circle centered on

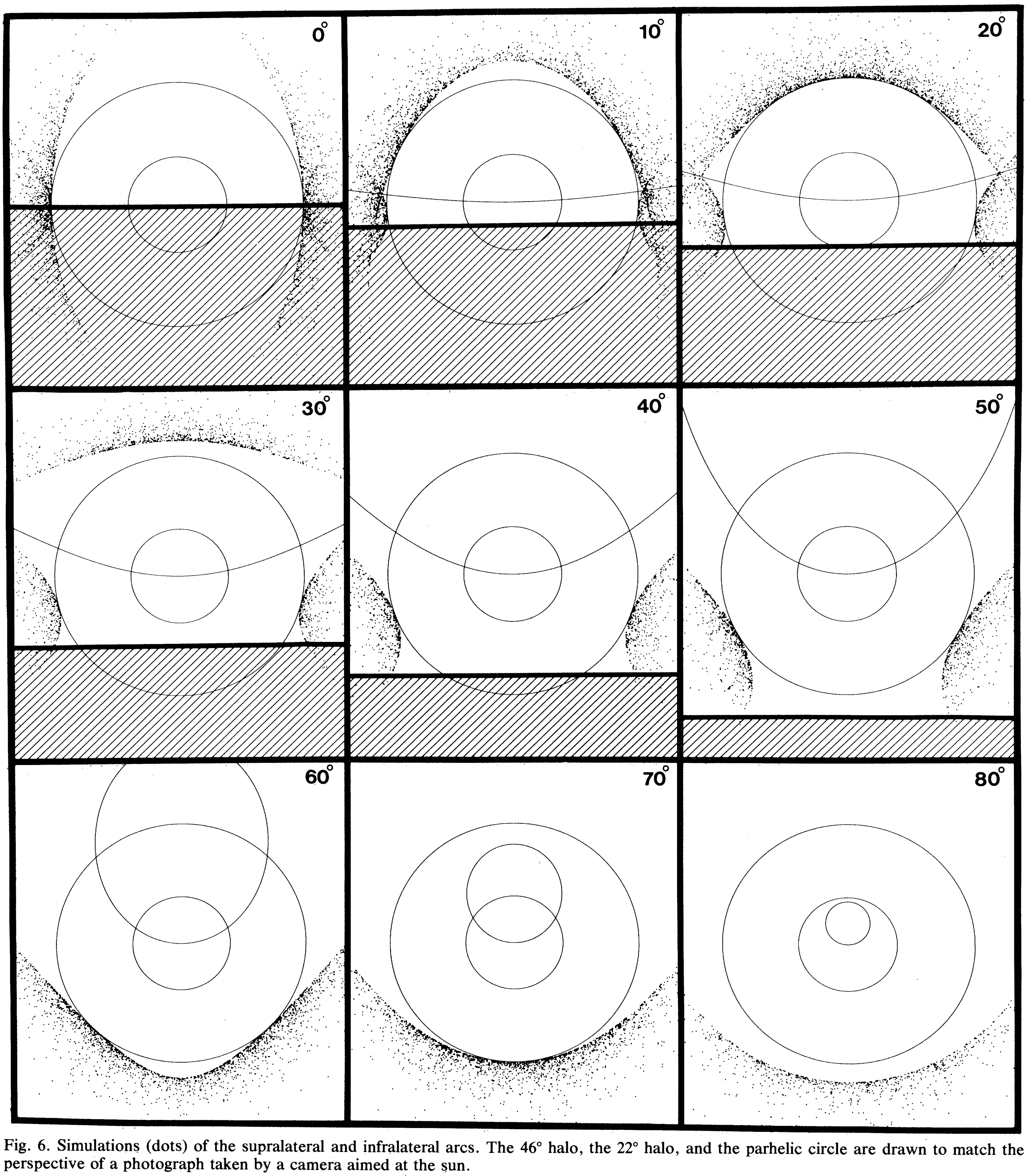


Fig. 6. Simulations (dots) of the supralateral and infralateral arcs. The 46° halo, the 22° halo, and the parhelic circle are drawn to match the perspective of a photograph taken by a camera aimed at the sun.

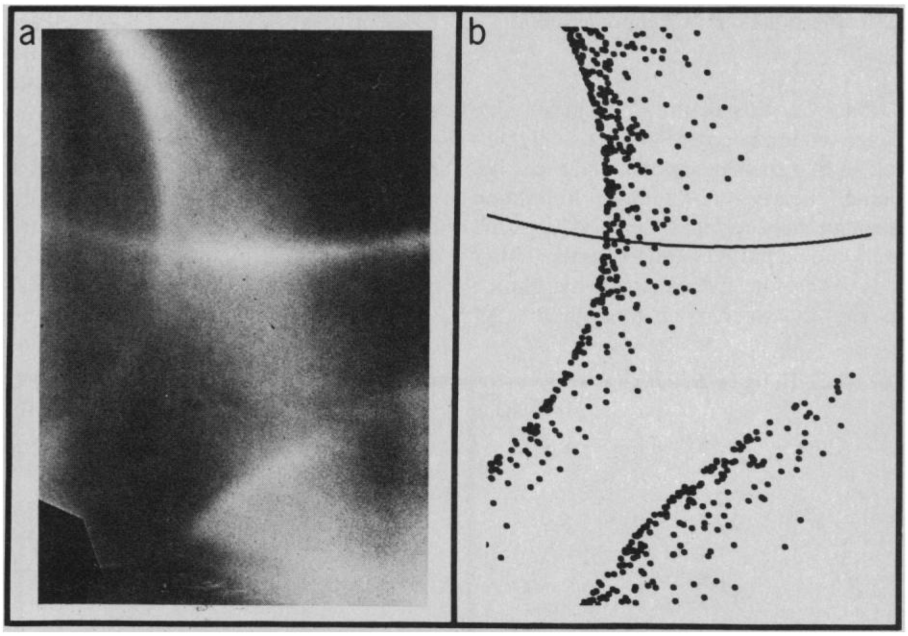


Fig. 7. Comparison between a photograph (a) and a simulation (b) of the circumscribed halo and the infralateral arc with a line representing the position of the parahelic circle. All three of these effects can result from the same distribution of ice crystals.

the zenith, hence the name circumzenithal arc. Figure 4 shows the simulations of the arc for different elevations of the sun. The curved line represents the edge of the 46° halo. The simulations are done to match the perspective of a photograph taken with a camera pointed almost directly at the arc; that is, the reference plane is oriented perpendicular to a line running from the observer's eye to a point 46° above the sun. The arc appears not as a straight line but as a curve, in the same way that a hoop, floating horizontally, would appear curved if an observer were to look up and photograph a section of it. For a sun elevation of 15° the arc extends slightly less than a third of the way around the zenith (108° of azimuth). It is limited by total internal reflection of the internal ray at the vertical side face.

There are no particular surprises in our plots of this arc. We can demonstrate characteristics that are easily calculated by other means and that have been observed (*1-3*). The arc can only be seen when the sun is at an elevation less than 32°. At a greater elevation all of the light is internally reflected at the vertical exit face. When the sun is at an elevation of about 22°, rays pass through the crystal at minimum deviation and this exit direction is little affected by slight tilts of the crystals. In this position the band for each color is narrow and the inner edge of the circumzenithal arc matches the inner edge of the 46° halo.

In all of these simulations we use 1.309 as the index of refraction of ice, which is appropriate for red light. The differing indices for other colors spread the arc into the complete spectrum, giving an arc with a red inner edge (close to the sun) and a blue outer edge. A striking feature of the circumzenithal arc is the brilliance and saturation of the spectral colors, which can be much more saturated than in either of the circular halos. The halos are minimum-deviation effects in which each color has a concentration at its minimum angle of deviation but a significant amount of light refracted at greater angles, overlapping other colors in the spectrum. The circumzenithal arc, on the other hand, is not a minimum-deviation effect, but results from rays that all have the same angle of incidence on the horizontal face of the plate crystal.

The circumhorizontal arc is a result of the same distribution of crystal orientations as the circumzenithal arc, and has many similar properties. It is caused by light entering a vertical side face of a plate crystal and exiting from the lower horizontal face (see ray b in Fig. 2). Like the circumzenithal arc it is an arc of constant elevation, but it appears below the sun. Figure 5 shows simulations that were done for the sun at different elevations to match the perspective of a photograph taken by a camera pointed 46° below the sun. The circumhorizontal arc can be seen only when the sun is at an altitude greater than 58°. It is, hence, more commonly seen in the lower latitudes, but it can be observed in the northern United States and southern Canada during summer. It has many of the same properties as the circumzenithal arc, and the high saturation of its colors makes it a spectacular display extending almost a third of the way around the horizon.

Supralateral and Infralateral Arcs

We often see parts of the circumscribed halo around the 22° halo. At sun elevations lower than about 40°, the circumscribed halo separates into upper and lower arcs tangent to the 22° halo. These effects result from light passing through alternate side faces of pencil crystals with a particular distribution of orientations (*6*). The appropriate distribution consists of pencil crystals with horizontal axes, random orientations of those axes in the horizontal plane, and random rotational orientation of the crystals about their axes. For the associated 90° phenomena, consider light that goes through the crystal as shown in Fig. 1, rays b and c. This situation differs from the one illustrated in Fig. 2 because the crystal shown in Fig. 1 can rotate about its horizontal axis; the crystal in Fig. 2 always has the upper face nearly horizontal.

Rays such as b in Fig. 1 produce the supralateral arc, which for sun elevations greater than about 15° lies mostly above the sun. Rays such as ray c in Fig. 1 produce infralateral arcs, which generally lie to the side of the sun and below it. Tricker (*9*) obtained quantitative results for the shape of the inner edge of the supralateral arc by constructing the envelope of a series of minimum-deviation curves, each for a particular tilt of the top entrance face.

Because the supralateral and infralateral arcs are assumed to be produced by the same crystal, both should occur simultaneously. Simulations of both are shown in Fig. 6. In each simulation we have drawn reference circles 22° and 46° in radius. The 46° halo need not accompany these arcs. For low sun elevations, both arcs are concentrated at the sides of the 46° halo. To the best of our knowledge, such effects have never been predicted or observed. We know of no good photographs of the supralateral arcs, although sightings have been discussed (*10, 11*). According to this model, they cannot occur if the sun is higher than 32°. Infralateral arcs have been described (*11-14*) and photographed (*15, 16*), giving us some pictures with which to compare our simulations. However, we have seen neither predictions nor descriptions that look like the low-sun-elevation simulations. At very low elevations, infralateral arcs might give the appearance of parhelia to the 46° halo, but as the elevation of the sun increases the concentration of light should fall below the parhelic circle (the circle of constant elevation passing through the sun) (*1-3*).

Figure 7a shows a photograph by Everhart (*15*). In addition to the infralateral arc, it shows a portion of a halo and the parhelic circle. Everhart had trouble reconciling the halo with any known effects. It was not circular, so it was not the 22° halo. Nor did it appear to match the dimensions of the circumscribed halo for the given sun elevation, 34°. However, when we recalculated the sun's elevation from the time (2:15 p.m., eastern standard time) and the location given by Everhart, we got an elevation of 38°. From the photograph (which was taken with a 35-millimeter lens), we estimated that the center of the picture is about 6° below the parhelic circle and at an azimuth 48° to the right of the sun. Note that 48° is the azimuthal angle (measured along the horizon), not the angular separation, which is measured along a great circle.

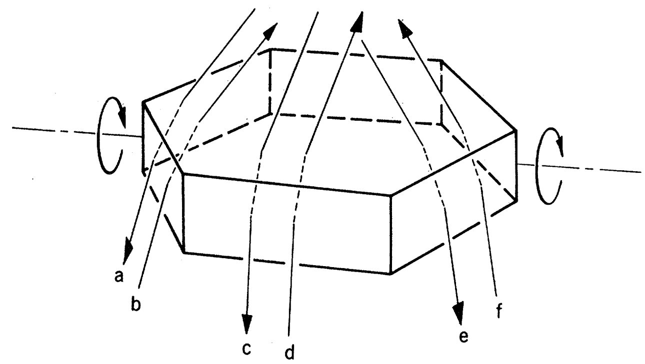
Figure 7b shows the simulation of the circumscribed halo and the infralateral arc, and we have added the computer-generated line of the parhelic circle. The agreement between the simulation and the photograph is quite satisfactory. The halo clearly matches the circumscribed halo for this sun elevation, and the simulation of the infralateral arc seems to reproduce both the form and the intensity distribution of the photograph. This gives us some confidence that our predictions of infralateral arcs that appear for other sun elevations may be confirmed by observation in the future.

Contact Arcs to the 46° Halo

There may be yet another set of arcs that touch the 46° halo. We need to consider transmission through the 90° faces of plate crystals in another set of orientations. One of the constant motions of a thin plate ice crystal falling in air is a rotation about a long axis that remains horizontal. This rotation can be illustrated by holding a playing card horizontally between fingers and thumb at the middle of a long edge and letting it roll off the supporting finger as it is released. It has been suggested (*3, 9*) that as a plate ice crystal falls, it rotates about a diagonal of the hexagonal cross section. Light passing through alternate side faces (60° faces) are thought to give rise to the arcs of Lowitz (*8, 9*). Light rays passing through the 90° faces of such rotating crystals may produce contact arcs to the 46° halo (*9*).

To predict the form of light arcs resulting from this mechanism, we must solve the six separate problems represented by the six rays shown in Fig. 8. For a given set of ice crystals, the arcs that we see in the sky should include the contributions from refraction of all of these rays. The only reason for examining them sepa-

Fig. 8. The rays considered to produce contact arcs to the 46° halo. The plate crystal has all rotational orientations about a horizontal axis that passes through a long face diagonal.



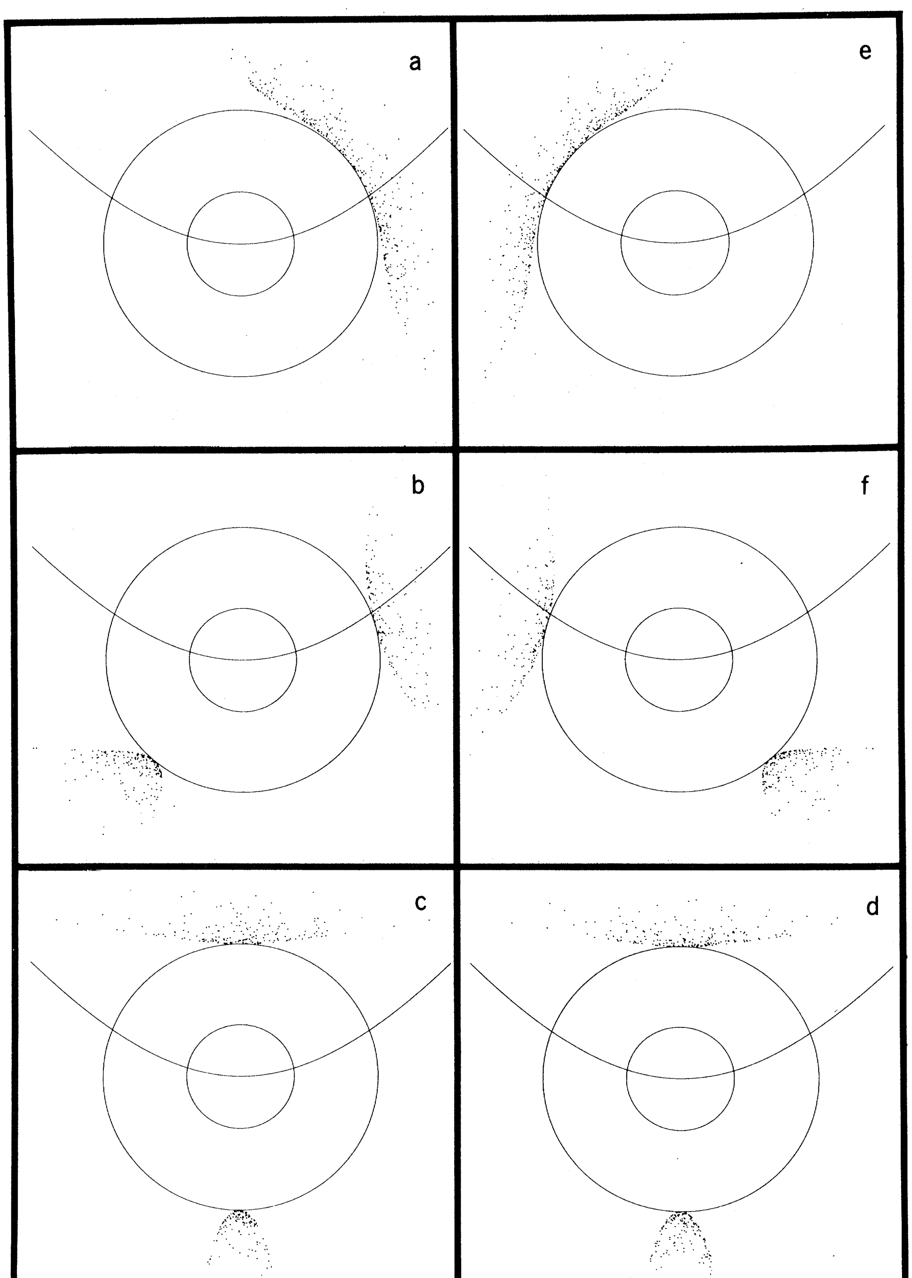


Fig. 9. Simulations of the effects resulting from the rays shown in Fig. 8.

rately is to understand which rays contribute intensity to a given part of the pattern. Figure 9 shows the results for a sun elevation of 40°. Each simulation is done with the same initial number of crystal orientations so that intensities can be compared. Figure 9 shows that each of the transmission paths contributes a significant amount of intensity. Rays c and d give intensity at the top and bottom of the 46° halo and yield the parts of the display that are suggested by the names, upper and lower contact arcs. The other rays contribute intensity at the sides of the halo.

Figure 10 shows the combined intensities for each of several sun elevations. By examining the series of simulations, we can see the evolution of each of the separate contributions as the sun's elevation changes. This strange set of arcs looks unlike anything we have observed. We consider this set of arcs to be hypothetical; however, the implications of the spinning-crystal hypothesis are laid out and are ready for comparison with observed arcs.

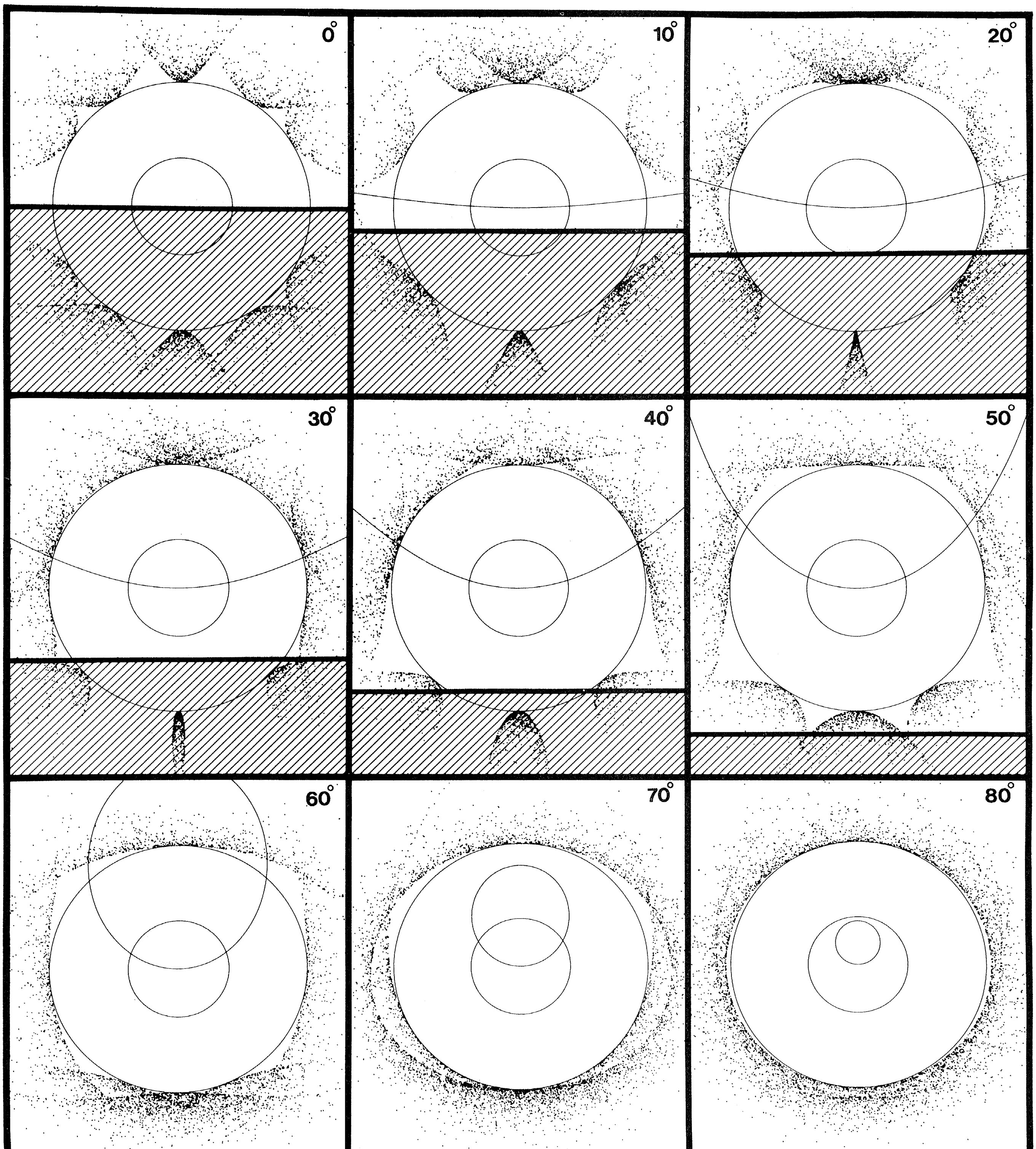


Fig. 10. Simulations of contact arcs to the 46° halo. The 22° and 46° circles and parhelic circle are added for scale, matching the perspective of a photograph taken by a camera aimed at the sun.

Combinations of Effects

The growth rate of an ice crystal in the atmosphere is influenced by temperature and the degree of saturation of water vapor. The growth of the side face or end faces of a hexagonal crystal is affected in different ways by these factors (*4*), so under one set of conditions the end face grows most rapidly (producing a pencil crystal), while under different conditions the side faces grow most rapidly (producing a plate crystal). Atmospheric conditions at different altitudes may favor the formation of one or the other of these crystal types, so there are times when the sunlight shines through a layer of pencil crystals floating at one altitude and a layer of plate crystals at a different altitude, giving rise to a combined display. It seems plausible that at some intermediate altitude, the growth rates of side and end faces are about equal, forming crystals with an aspect ratio of about one. These would assume no particular orientation as they fell.

Fraser (*5*) argues that for plate or pencil crystals to orient themselves randomly as they fall, they must have diameters less than about 7 μm. For crystals this small, diffraction effects would dominate, preventing the production of a bright halo. If the crystals were not oriented randomly, no halo would appear, only arcs. In support of this argument, Fraser claims that few people have seen the entire 46° halo—only portions of it produced by pencil crystals that are oriented fairly well. The simulations in Fig. 8 show that for sun elevations between 10° and 20°, the supralateral arc does come rather close to matching the upper part of the 46° halo. Small tilts of the horizontal axes would improve the fit so that while viewing only the portion of the display above the horizon, one might have difficulty distinguishing between the arc and the halo. Similarly, for sun elevations of 60° to 70° the infralateral arcs form a reasonable match to the part of the 46° halo below the sun. The obvious test for these theories is to examine the ice crystals in the atmosphere when a complete 46° halo is present. That, of course, is more easily said than done.

Even without the superposition of these 46° halo effects from different ice crystal forms, it may sometimes be difficult to determine whether an arc at the top of the 46° halo is, for example, a circumzenithal arc, a supralateral arc, or part of the arcs-of-contact display (*17*). We hope that the computer simulations will help to distinguish between the possibilities.

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Endothermy and Activity in Vertebrates

Albert F. Bennett and John A. Ruben

Endothermy, the maintenance of a high and constant body temperature by metabolic means, is a striking adaptation in the animal kingdom. Endothermic animals expend great quantities of energy to regulate and maintain internal thermal conditions and functional processes over a wide range of environmental temperatures. In contrast to ectotherms, they are generally warmer than their environment and are often more active animals. Although endothermy appears during intense muscular activity in several otherwise ectothermic organisms, only among mammals and birds is endothermy maintained even under resting conditions. The energetic cost of this maintenance in these groups is great, and its evolution has required substantial restructuring of many systems of the vertebrate body. In spite of these costs, mammalian and avian endothermy developed along essentially parallel lines among different groups of reptilian ancestors. The selective factors influencing its evolution must have been substantial and highly significant to have made such a profound alteration in the energetics, physiology, and behavior of two major groups of vertebrates. However, there is no general agreement among vertebrate biologists as to what those selective factors were or what the sequence of events culminating in endothermy was.

The evolution of endothermy in mammals and birds has been the subject of considerable speculation (*1, 2*), and debate has also been raised concerning metabolic thermoregulation among the dinosaurs (*3*). These discussions are generally based on the assumption that the endothermic condition is the end product of selection for a high and stable body temperature per se. That is to say, the evolution of endothermy was occasioned by only thermoregulatory considerations. Consequently, attention has been centered upon the advantages of a relatively high and constant body temperature. Among the benefits cited are stability of enzymatic catalysis, independence of timing daily activity, and resistance to freezing (*1*). These selective factors are not necessarily significant or even advantageous in all environmental circumstances in which endothermy evolved. Thus, arguments for the evolu-

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