

speculate that great earthquakes in southern California may be initiated by a Parkfield quake, but for different reasons. They have found that both the 1934 and 1966 Parkfield quakes resulted from the concentration of stress at a slight bend in the San Andreas. This stress was transmitted through the bend when a strong foreshock broke the fault on the northwestern end of the bend. In each case, 17 minutes later the additional stress broke the fault toward the southeast, producing the main shock. This progression is usually stopped, Bakun and McEvilly suggest, by another irregularity on the fault. But once every few hundred years, they say, these moderate quakes could break through the obstructing irregularity and trigger a great earthquake.

If Parkfield quakes can be triggers, an accurate prediction of the next great quake in southern California would require not only predicting the next Parkfield quake but also anticipating whether the locked section of the San Andreas is ready to rupture under the additional stress transmitted by the Parkfield shock. In one of Brune's worst-case scenarios, a long section of a fault such as

the San Andreas could be triggered without the appearance of any precursors except possibly in the small area of the triggering quake. "We really don't know if classic premonitory signals would show up along the entire fault or only on part of it," notes Barry Raleigh, head of the tectonophysics branch at Menlo Park.

Because precursors may not be available, researchers are giving increasing attention to measuring physical properties that should be more directly related to the ultimate failure of any fault. One is stress itself. In principle, the stress in the earth's crust can be measured by pressurizing a section of a drill hole, but the difficulties of working in the highly fractured rock of a fault zone are great. Acceptable approaches to measuring stress 5 to 10 kilometers beneath the surface, where earthquakes and the asperities that seem to control them occur, are only beginning to be tested. Measurement of the crustal deformation caused by accumulating stress is being expanded, but interpretation of such data is not always straightforward—witness the still enigmatic Palmdale bulge. The significance of this swelling of part of southern California remains unknown. Still, deforma-

tion could be the phenomenon most easily related to the cause of earthquakes, and Raleigh "finds it difficult to imagine how as much strain as is released during a large earthquake could build up without it being noted."

Whether these or any of the other approaches being pursued will contribute to reliable predictions must await the capturing of more—a lot more—moderate or larger earthquakes within dense instrument networks. Coyote Lake was the first. When the United States' prediction program's funding was tripled in 1977, the official view was that, considering average levels of seismicity, it would take 10 years to capture the ten quakes needed to determine the practicality of earthquake prediction. So far, earthquakes in California have not kept up that pace. Unfortunately, few instruments other than seismometers were even near the recent quake near El Centro in southern California. Obviously, patience is needed—patience to wait for many more earthquakes, both here and abroad, and patience to develop a basic understanding of how earthquakes work. "There's so much we don't know," Wesson observes.—RICHARD A. KERR

## Mathematicians Amazed by Russian's Discovery

*An unknown Russian discovers a new solution to linear programming problems. But his result goes unnoticed for months*

A Russian mathematician, L. G. Khachian, who apparently is unknown to U.S. scientists, has made a discovery of great theoretical, and probably practical, importance. He has found a surprising new way to solve linear programming problems, which are among the most common problems solved on computers. "There is a linear programming package at almost every computer installation," says Ronald Graham of Bell Laboratories. One consequence of Khachian's discovery is related to a major problem in computer science.

A typical linear programming problem faced by a business is to select its sources of supplies so as to minimize its shipping costs. The suppliers may place some constraints on the business by requiring that all orders be at least some minimum size. The resulting problem involves finding the best solution to a set of linear inequalities by considering a function that must be maximized or mini-

mized. In the case of the business, shipping costs must be minimized. Such problems are an everyday part of life for businesses and economists, and they also occur in such fields as engineering, biology, agriculture, and the social sciences. According to Vasek Chvatal of McGill University, linear programming problems commonly involve thousands of inequalities.

Khachian's discovery went unnoticed for months after its publication last January. Computer scientists and mathematicians explain their missing the result by saying that it appeared as an abstract in the Soviet journal *Doklady*, which they seldom read. Many, in fact, cannot read it because it is published in Russian.

Virtually no U.S. scientists were aware of Khachian's result until early summer, when Eugene Lawler of the University of California at Berkeley brought it to the attention of the computer science community. Lawler heard of it

in May while attending a conference in Germany. There Rainer Burkhard of the University of Cologne showed Lawler the abstract and asked him what he knew about it. Lawler, who was unfamiliar with the abstract, had it translated and sent it out to prominent computer scientists, asking if they knew about it. None did, but all were intrigued.

Through Lawler's efforts, Laslo Lovász of the University of Szeged in Hungary and Peter Gács of the University of Rochester heard of the result. These two Hungarian mathematicians were visiting at Stanford and decided to work together in an attempt to reconstruct the mathematical proofs from Khachian's abstract, thereby establishing the validity of the result. Not only did they fill in the proofs but they also improved on the method, making it more efficient. Lovász reports that their task was "medium hard" and that it took him and Gács "a couple of days." But Lovász, at age 29, is a rising

young star of mathematics of whom it has been said that he only works on the hard problems.

The immediate importance of Khachian's result is theoretical. It is tied to what is said to be the major unsolved problem in computer science (*Science*, 8 November 1974, p. 520). This problem arose about 10 years ago, when computer scientists found that a large class of seemingly simple problems are all alike in that if an efficient way could be found to solve one of them, all could be solved efficiently. But no one has ever been able to find such an efficient solution, despite the fact that there is an economic as well as an intellectual stimulus to do so. These equivalent problems are immensely important to corporations. For example, one is the traveling salesman

are not members of the class of equivalent problems and yet do not have polynomial time solutions. They suspected that linear programming might be one of these borderline problems and had even shown that linear programming has its own little orbit of problems that are equivalent to it and that have no known polynomial time solutions.

Thus Khachian's method, which requires only polynomial time, came as a surprise. George B. Dantzig of Stanford University, who developed the simplex method—an exponential time method in universal use for solving linear programming problems—says, "A lot of people, including myself, spent a lot of time looking for a polynomial time algorithm for linear programming. I feel stupid that I didn't see it."

ulate that Khachian's method might eventually replace the simplex method, at least for certain classes of problems. Lovász explains, for example, that Khachian's method is relatively insensitive to the number of constraints on the problem, whereas the simplex method becomes increasingly difficult as the number of constraints grows large. Thus Khachian's method might prove superior for problems with numerous constraints. All these experts point out, however, that the practicality of Khachian's method cannot be decided until people gain much more computing experience with it. They have had 47 years of experience with the simplex method, during which time they have constantly improved its efficiency. Still, Lovász has managed to program Khachian's method on his pocket calculator to solve problems with six inequalities and six unknowns. "It's not clear that you could program the simplex method [to solve similar problems] on a pocket calculator," Graham says.

Khachian's method may also be useful for a far broader class of problems than the simplex method. As Bland points out, Khachian's algorithm does not require that the systems of equations or inequalities be linear. Most equations of mathematical models are nonlinear, but they are very hard to work with so they usually are approximated by linear equations. Bland believes that Khachian's method might turn out to be quite useful for solving certain classes of nonlinear equations.

Lovász sees still another use for the method, in solving combinatorial optimization problems. This is a class of problems typically involving large but finite sets of objects. The problem is to find the best elements of such a set. Graham explains that it is a combinatorial optimization problem to find the best way to schedule nurses in a large hospital. There may be 300 or more nurses, each with particular days off and each with particular specialties. The idea is to pick the best of all possible schedules—the best being the one that uses fewest nurses.

In light of the importance of Khachian's method, it is ironic that so little is known about this mathematician. No one, not even Lovász in nearby Hungary, has any idea of what his training was or what led him to his discovery. Since he seems not to have published previously, Lovász and others speculate that Khachian's surprising result may be part of his doctoral thesis. It is clear, however, that his days of anonymity are over as the impact of his method begins to be felt.—GINA BARI KOLATA

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problem: Find the shortest tour of a group of cities so that no city is visited more than once. The telephone company must solve a traveling salesman problem when it plans collections from the coin boxes of pay telephones.

By an efficient solution, computer scientists mean a polynomial time method. This is one in which the number of computations (and thus the computer time) increases as a polynomial function of some parameter of the problem, such as the number of cities in the salesman's tour. The only known general solutions to these problems are highly inefficient exponential time ones, which means that the number of computations increases as an exponential function of some parameter of the problem.

The contrast between polynomial and exponential time methods can be astonishing. For example, a polynomial time algorithm requiring  $x^3$  steps takes only 1/5 second of computer time when  $x$  is 60. An exponential time algorithm requiring  $3^x$  steps takes billions of centuries of computer time when  $x$  is 60.

The major unsolved question is whether it is possible to solve one, and thus all, of the class of equivalent problems with polynomial time methods. So far, most investigators suspect that the answer is no, although they have not been able to prove that it is. But they have shown that if the answer is no, there must be problems that fall between the cracks—that

Yet Khachian came at the linear programming problems from quite a different direction than other investigators. Robert Bland of Cornell University describes the method as being in the spirit of numerical analysis. It involves the construction of a sequence of ellipsoids in multidimensional space that close in on the optimal solution. In contrast, the simplex method involves a polygon in multidimensional space, whose boundaries are determined by the constraints on the problem, and whose vertices are all possible solutions. The optimum solution is found by moving from vertex to vertex, seeking the highest (if one wants a maximum) or the lowest (if one wants a minimum).

Although the simplex method is an exponential time one, this only means that in the worst case it could take forever to solve a problem. On the average, however, the method works quite well. As Dantzig explains, thousands or even tens of thousands of linear programming problems are solved each day with the method and all are solved in reasonable lengths of time. Yet computer scientists can easily cook up problems that foil the method and cannot be solved, for all practical purposes. Investigators are deeply puzzled by the algorithm's success. They cannot explain what property of practical problems it is that allows the method to work for them.

A number of computer scientists spec-