Measurement of Fluctuating Magnetic Gradients Originating from Oceanic Internal Waves

Abstract. *Fluctuating magnetic gradients over oceans come from electric currents flowing in seawater arising from its motions across the earth's magnetic field. Gradients of 0.3 to 0.6 picoteslas per meter for each meter of internal wave displacement have been measured at frequencies of 2 to 5 millihertz with a superconductive magnetic gradiometer supported 7 meters above the surface of water 18 meters deep about 1.5 kilometers offshore from San Diego, California.*

Fluctuating magnetic gradients over oceans come from electric currents driven along crests of waves by seawater oscillating across the earth's steady magnetic field. We report here the first measurements of fluctuating magnetic gradients originating from oceanic internal waves. The gradients are minute, being of the order of 1 pT/m (*1*), but the unprecedented sensitivity of superconductive quantum interference devices (*2*) presents heretofore unavailable means of measuring them. We use a superconductive magnetic gradiometer formed by two coplanar pickup loops (*3*) to measure a transverse gradient from internal waves passing an oceanographic research tower (*4*) located about 1.5 km off Mission Beach near San Diego, California, in 18 m of water.

Figure 1 shows the fluctuating gradient 7 m above the surface coming from an internal oscillation that displaces a sharp thermocline 7 to 10 m through two cycles in 16 minutes. Levels of isothermal water, marked for 13° and 15°C isotherms, trace the oscillation of the thermocline within which the water temperature falls about 1°C per meter of increasing depth through a layer 6 to 8 m thick in water 18 m deep. Dashed curves delineate the thermocline profiles at the start and second minimum, $3\tau/2$, of the almost sinusoidal oscillation having a period, τ, of 480 seconds. Horizontal seawater currents flowing onshore at 4, 8, 12, and 16 m above the bottom show a fluctuating shear flow, which is profiled at the same instants as the thermocline.

The magnetic gradient fluctuates with the period of an isotherm oscillation but lags behind it by about 60 seconds. Its variation from crest to trough ranges from 2.6 to 3.6 pT/m, corresponding to isotherm displacements ranging from 7.5 to 10 m and velocity variations of 10 to 12 cm/sec and 16 to 22 cm/sec at 16 m and 4 m above the bottom, respectively

The gradiometer sits in a gimbal mount fixed to a nonmagnetic cantilever truss that juts it 20 m from the south face of the tower about 7 m above the surface. Two axes of rotation and one axis of tilt (± 45°) of the gimbal allow every orientation of the pickup loops within a 90° solid angle about the vertical. We orient the pickup loops in order to suppress noise both from fluctuating magnetization currents in the steel structure of the tower and from their slight motion in its steady gradient field (*5*). The cantilever truss (*6*) keeps the orientation of the gradiometer stable to 5 seconds of arc.

With pickup loops oriented to suppress noise (*7*), we simultaneously record the voltage of the gradiometer once a second together with voltages from four biaxial flowmeters of an electromagnetic type (*8*) and from an array of three moored chains, each supporting 17 thermistors spaced 1 m apart. The four flowmeters measure seawater velocities directed onshore (reckoned positive eastward) and alongshore (reckoned positive northward) at 4, 8, 12, and 16 m above the bottom in the center of the tower. The array of three moored chains gives temperature at 1-m intervals throughout the water column at three locations: (i) beneath the gradiometer, (ii) 14.3 m due north of the gradiometer near the south face of the tower, and (iii) 36.6 m west and 16.5 m north of the gradiometer.

Typically, we record data at night (for 9 hours or so) to mitigate noise from thermal and wind loads on the truss. Our data base comprises two periods: (i) 19 days from 8 May 1978 to 5 June 1978 preceding the establishment of the seasonal thermocline (*9*), during which we determined the gradiometer orientation giving least noise, and (ii) 35 days from 17 July 1978 to 24 August 1978 spanning the peak of the internal wave season, during which we consistently measured gradients originating from internal oscillations (*10*). Strong regular internal oscillations of the kind shown in Fig. 1 are rare. The oscillations are usually irregular, comprising periods from 200 to 500 seconds, with root-mean-square amplitudes of 1 to 2 m (*11*). We correlate gradients from irregular oscillations with fluctuations of isotherms and seawater currents to obtain statistical estimates of the frequency dependence of the amplitude and the

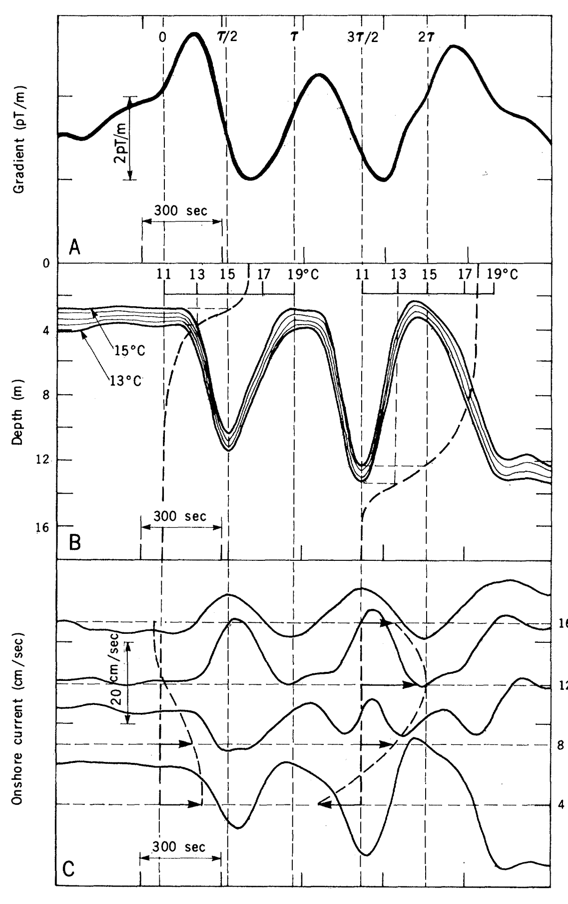

Fig. 1. Magnetic gradient fluctuation (A) coming from an internal oscillation depicted by simultaneous fluctuations of isotherms (B) and seawater currents (C). Oscillations above 5 mHz are filtered, and time extends from 06:35 to 07:05 on 22 July 1978. Light, vertical dashed lines mark each one-half cycle of a sinusoidal oscillation continuing through two cycles with a period, τ, of 480 seconds. Heavy dashed lines in (B) profile temperatures, corresponding to scales marked from 11° to 19°C, throughout the water column at two instants, 0 and $3\tau/2$. Heavy dashed lines in (C) delineate profiles of the shear flow at the same instants. Light, horizontal dashed curves mark the origins for velocity fluctuations measured at 4, 8, 12, and 16 m above the bottom.

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phase of gradients from internal waves. Here, we compare estimates of the empirical frequency response (*12*) with gradients expected from Ampere's law.

By using Ampere's law together with a linear description of internal oscillations (*13*), we expect the real part of the expression

$$\Gamma(\mathbf{r}, h, t; \mathbf{k}, \omega) = (\mu_0\sigma/1)\mathbf{B} \cdot (\hat{z} - i\hat{k})e^{i(\omega t - k \cdot r)} \times \int_0^D V_k(z)\, e^{-k(h+z)}kdz$$

to describe the vertical gradient, Re[$\Gamma(\mathbf{r}, h, t; \mathbf{k}, \omega)$, of the horizontal component of the magnetic field at horizontal position $\mathbf{r}$, height h above the surface, and time t that comes from an internal wave progressing horizontally in a direction $\hat{k}$ with wave number k at frequency ω in an ocean of depth D (*14*). Here, $V_k(z)$ is a profile of horizontal velocity; $\mathbf{B}$ is the earth's steady magnetic field (*15*); $\hat{z}$ is a unit vector directed vertically downward; σ is the electrical conductivity of seawater ($\sim$ 4 mho/m); and $\mu_0 = 4\pi \times 10^{-7}$ H/m. Profiles of horizontal velocity depend on the thermocline shape and the modal composition of the internal oscillations (*16*). The expression tells us that the magnitude of the gradients is proportional to an exponentially weighted integral over a profile of horizontal velocity and that their phase depends on wave heading alone.

At the tower, the fundamental mode of the thermocline dominates the internal oscillations, and waves head predominantly eastward. For a first comparison, we represent thermocline shape by a linear decrease in temperature between a warm isothermal layer 2 m thick at the surface and a cold isothermal layer 5 m thick at the bottom. We choose its slope to obtain an effective buoyancy frequency of 7.75 mHz (0.049 rad/sec), which typifies stratification at the tower and gives phase speeds consistent with the value 25 cm/sec estimated from phase differences between the 15°C isotherm of each chain.

Figure 2 compares gradients expected from Ampere's law with statistical estimates of the frequency dependence of the magnitude and phase of gradients per unit speed relative to onshore seawater currents at 4 and 16 m above the bottom. Estimates come from a data sample 6.8 hours long extending from 22:15 on 31 July 1978 to 05:05 on 1 August 1978 (*17*). Although the frequency resolution is limited by the narrow bandwidth of the internal oscillations, estimates of the magnitude per unit speed from each flowmeter increase with frequency as expected. Estimates of phase are consistent with

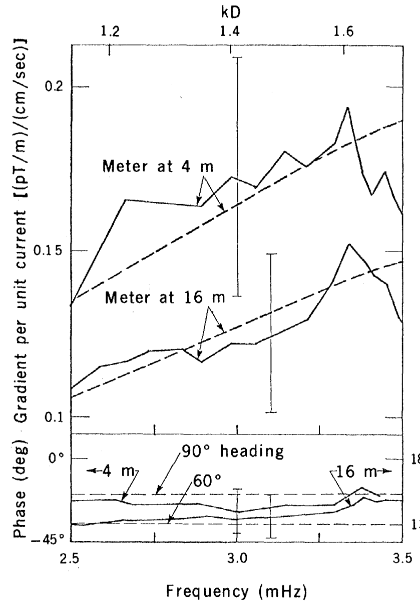

Fig. 2. Comparison of the theoretical frequency response to statistical estimates. Heavy dashed curves delineate the frequency dependence of the magnitude of gradients expected per unit speed from onshore seawater currents at 4 and 16 m above the bottom. Jagged lines trace corresponding statistical estimates. Estimates of phase from the flowmeter at 4 m correspond to the left scale, marked from −45° to 0°, and estimates from the flowmeter at 16 m correspond to the right scale, marked from 135° to 180°. In each case, horizontal dashed lines mark the limits of phase lags expected for waves heading between 60° and 90° east of north. Error bars mark 50 percent confidence limits. The square of coherence between gradient and velocity fluctuations is 0.6 over the frequency band of internal oscillations and falls to 0.12 immediately outside the band. The 95 percent level of significance is 0.25 for 32 degrees of freedom.

the phase expected for waves heading predominantly eastward.

As Fig. 2 shows, a linear representation gives gradients that closely fit statistical estimates made from weak irregular oscillations. It fails, however, to describe gradients from strong oscillations of the kind shown in Fig. 1, which distort the thermocline profile. In particular, the triangular array shows that the strong oscillation depicted in Fig. 1 heads 22° north of due east, and so we expect the gradient fluctuation to lag behind the isotherm displacement by 13 seconds rather than 60 seconds as observed.

Measurement of fluctuating magnetic gradients over oceans provides a direct integral measure of oceanic internal waves. The measure is insensitive to details of ocean stratification and depends separately on both wave amplitude and heading.

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References and Notes

1. We use meter-kilogram-second units throughout and the international convention defining the tesla (T) as a weber per square meter, so that 1 pT/m = 10^{-12} T/m = 10^{-10} G/cm = 10^{-3} gamma/m.
2. J. Clarke, *Science* **184**, 1235 (1974).
3. G. Gillespie, W. Podney, J. Buxton, *J. Appl. Phys.* **48**, 354 (1977).
4. The tower is an oceanographic research facility of the Naval Ocean Systems Center at San Diego, Calif.
5. We repeatedly turn the gradiometer about each of the three gimbal axes in succession to find a set of three angles at which gradient noise is least by displaying its noise spectrum from 0.1 to 25 Hz on a Spectral Dynamics Model 330A analyzer.
6. Report MRI-2882-TR2 (Mechanics Research, Inc., McLean, Va., 1976) describes the structural design of the cantilever. It is a four-sided space frame comprising a rectangular base section 8 m long made of aluminum tubing, which attaches to the tower, and a 12-m tapered section made of a fiber glass-reinforced plastic that supports the instrument platform at its tip. Measurements made with a tiltmeter sensitive to 10^{-9} radian (Autonetics Division of North American Rockwell, Inc., part SE541ALRA-P-120-XX), show that a root-mean-square deviation of 5 seconds of arc in the frequency band 2 to 5 mHz typifies platform stability under calm winds and light swell at night.
7. The loops lie approximately in a vertical plane facing due west, and so we effectively measure a vertical gradient of the horizontal component of the magnetic field directed westward.
8. J. R. Olson, *Mar. Technol. Soc. J.* **6**, 19 (1972).
9. J. L. Cairns and K. W. Nelson, *J. Geophys. Res.* **75**, 1127 (1970).
10. Earlier measurements over 27 days from 1 September 1977 to 1 October 1977 during the ebb of internal waves gave inconclusive results, largely because the first techniques used to suppress noise from the tower were not precise enough.
11. We observe internal waves at the tower that are remarkably similar to those reported 20 years ago by E. O. LaFond [in *The Sea*, M. N. Hill, Ed. (Wiley, New York, 1962), vol. 1, part 2, p. 731].
12. We use a standard fast Fourier-transform algorithm to compute spectral and cross spectral estimators and then normalize cross spectra to estimate frequency response as prescribed by J. S. Bendat and A. G. Piersol [*Random Data: Analysis and Measurement Procedures* (Wiley-Interscience, New York, 1971), pp. 196–208].
13. W. Podney, *J. Geophys. Res.* **80**, 2977 (1975); _____ and R. Sager, in *Future Trends in Superconductive Electronics*, B. S. Deaver, Jr., C. M. Falco, J. H. Harris, S. A. Wolf, Eds. (American Institute of Physics, New York, 1978). p. 95.
14. Above the surface, the magnetic vector of a wave is circularly polarized in a vertical plane normal to the wave crests. It originates from electric currents flowing horizontally along wave crests and so is effectively independent of the electrical conductivity of suboceanic strata. Magnetic fields of electric currents flowing in vertical planes vanish above the surface. Because magnetic fields from seawater oscillations diffuse over a distance of a wavelength in a small fraction of a wave period, they keep pace everywhere with wave motions, and induction is negligibly small.
15. Our measurements of the steady magnetic field near the tower give a magnitude of 46,847 nT, a dip angle of 60°, and a declination of 18°.
16. O. M. Phillips, *The Dynamics of the Upper Ocean* (Cambridge Univ. Press, London, 1977), pp. 207–252.
17. We filter voltage fluctuations above 300 mHz to preclude aliasing and record data once a second on magnetic tape. We then digitally filter the tape-recorded data above 50 mHz, average 12 seconds of data to obtain each point of a 2048-point time series that after Fourier transformation gives spectra having 1024 frequencies spaced uniformly over the bandwidth 0.041 to 41.7 mHz, and average over 32 frequencies to reduce the variance of the spectral estimates.

18. We gratefully acknowledge support provided by the Ocean Measurements Group of the Naval Ocean Systems Center, under the direction of D. Good. We appreciate stimulating criticisms and encouragements offered by P. Selwyn during the course of the work. Supported by the Advanced Research Projects Agency of the Department of Defense under order 3370 and was monitored by the Office of Naval Research under contract N00014-77-C-0254.

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