dustrial application of composting for anything other than paper products or sewage sludge. If their demonstration is successful, Epstein says, other companies are very likely to follow suit.

Recycling, chemical treatment, and biodegradation are the least controversial and most acceptable methods for disposal of hazardous wastes. Their primary advantage is that the waste material is either used or destroyed, so there is no further need for containment or monitoring. Of all disposal techniques, they alone have met little citizen opposition to siting of facilities—the recycling and treatment processes because they are more or less conventional industrial processes, and land farming because the sites have generally not been near communities. That situation could change if compost facilities sited near cities should produce noxious odors, but such a development should be preventable with care. The chief disadvantage of the techniques is that they are applicable to only a limited percentage of hazardous wastes. Despite the desirability of these approaches, other techniques must be found for the majority of wastes.—THOMAS H. MAUGH II

Isadore Singer and Differential Geometry

More an art form than a science, it is one of the "purest" fields of mathematics

Isadore M. Singer of the University of California at Berkeley is a quiet, collegial man, famous among mathematicians for his innovative work in global differential geometry, the study of properties of surfaces, often in higher-dimensional spaces. Geometers are concerned with how surfaces twist, what their shapes are, what sorts of holes are in them, what their curvatures are, and what kinds of metrics, or distance formulas, they have.

This is the third of a series of occasional articles about mathematics as seen through the eyes of its most prominent scholars.

There are relatively few differential geometers among mathematicians, partly because the field is highly abstract and partly because it requires so much background. However, differential geometry is attracting wider interest of late because it now seems to have close ties to elementary particle physics. Singer discussed these connections to physics in an interview with *Science* and explained the concepts and character of differential geometry research.

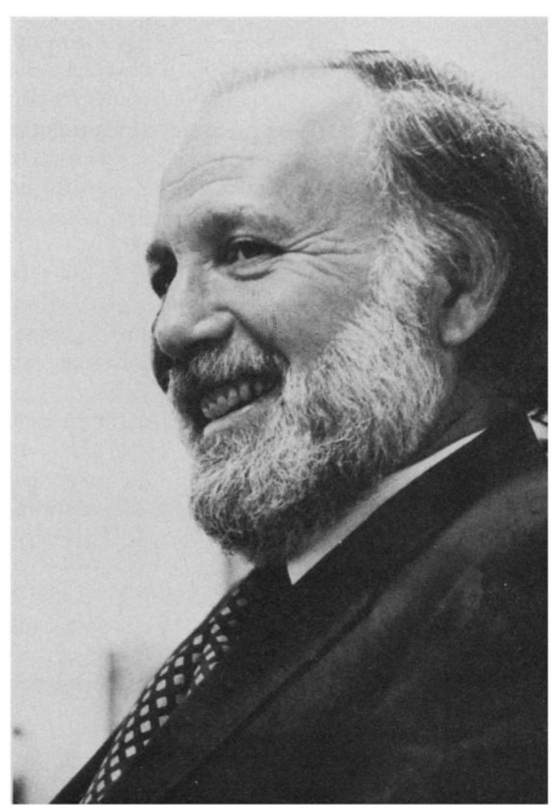
Singer is well known for his ability to explain his work to those outside his field. George Pimentel of the National Science Foundation tells a story that illustrates Singer's appeal. When Singer first came to Berkeley, he agreed to give a series of seminars on differential geometry. The first seminar was scheduled for the usual, fairly small room, but Singer drew such a crowd that the seminar had to be moved to larger quarters. Thinking the large crowd was a one-time phenomenon, the mathematics department scheduled the second lecture for the smaller room, but the lecture had to be moved again. By that time, the math department had learned its lesson and scheduled the rest of Singer's seminars for the large room.

Although Singer's appeal is genuine, the large crowd at his lectures included physicists as well as mathematicians and was probably partly due to the new interest in differential geometry among physicists. Coincidentally, or, as Singer thinks, perhaps not so coincidentally, physicists, independently of mathematicians, were led to a central concept of differential geometry, that of objects called fiber bundles. The physicists implicitly use the theory of fiber bundles in what they call gauge theories—the presently favored means of explaining the three types of forces between elementary particles: the electrodynamic, the strong nuclear, and the weak forces. It is Singer's conjecture that this convergence of mathematics and physics suggests the nature of the way the human brain works, the way people view the world. "I stand with Einstein in thinking that the explanation of the natural world is geometrical," he says.

Singer is currently exploring these connections between differential geometry and elementary particle physics as part of his long-standing interest in physics. An undergraduate physics major, he began studying mathematics because he felt he did not have the mathematical background to understand quantum mechanics and relativity. He intended to return to physics, he says, "but now physics has come back to me."

Singer is careful to emphasize that differential geometry's ties to physics are not the be-all and end-all of his field. Although exciting, they are only a small part of differential geometry. These applications, Mary Gray of The American University explains, "lend a certain cachet these days."

Fiber bundles, which are central to differential geometry, are a way of constructing abstract spaces. Each fiber bundle consists of a base space together with other spaces known as fibers. For example, the base space could be a circle and the fibers could be spikes projecting from the circle so that a spike goes through every point of the circle. The fiber bundle is the prescription, or set of rules, telling how the fibers fit together.



Isadore M. Singer

0036-8075/79/0601-0933$00.50/0

Singer explains that if the base space in the example is a Moebius strip (a circular band with one twist in it) rather than a cylinder, the fibers will twist as they go around the circle. When the fibers meet at the starting point of the circle, the fibers at the end of the circle will be upside down relative to those at the start. Both the base space and the fibers can be quite complicated, and the prescription for fitting the fibers together can be even more so.

The fiber bundles applicable to elementary particle physics an special ones. Physicists consider the base space to be the space in which we live, whereas differential geometers often study far more intricate base spaces. Singer says that a motivating principle in both physics and geometry is symmetry. "Until gauge theories came along, the symmetries were rigid in physics," he says. So far, some gauge theories fit the experimental data on elementary particles and it may prove to be the answer to the physicists' problem of how to unify weak and strong forces.

Physicists, independently of mathematicians, were led to a central concept of differential geometry.

According to Singer, fiber bundles have some practical import in areas other than elementary particle physics. For example, they may aid mathematicians in solving nonlinear equations that commonly arise in mathematical models. He explains that these equations often have singularities, which are points where the solutions blow up. These problems can be recast in terms of fiber bundles. "This approach has not yet reached the point at which we can solve practical problems," Singer says. "But it can be used to put the problems in a conceptual framework where we can deal with them."

Within the past year, a number of mathematicians and physicists used these concepts from differential geometry to solve nonlinear equations that, in a certain four-dimensional space, are called instantons and are of interest in elementary particle physics (*Science*, 14 October 1977, p. 180). Physicists had previously found some but not all solutions to these equations. But, working together, mathematicians and physicists recently found all solutions to these equations in a form as explicit as the original solutions. "Furthermore, it was shown that if the solutions are put on fiber bundles and the bundles are twisted, there are no singularities," Singer explains.

Although he has made many important contributions to his field, Singer is best known for a result he proved with Michael Atiyah of Oxford University about 15 years ago. Known as the Atiyah-Singer Index Theorem, the result is extremely useful to physicists and mathematicians. It combines geometry, topology, and partial differential equations in a startling way. "We were able to count the number of global solutions to differential equations. Our theorem unified many special cases that looked different," Singer says. He explains that new applications of this theorem are still being discovered.

The story of the Atiyah-Singer Index theorem illustrates the mood of research in differential geometry. Atiyah and Singer began collaborating when Singer visited Oxford on a sabbatical. "Everything clicked," Singer says. They proved the theorem in 9 months and announced their result. Shortly afterward, a seminar on their results was held at the Institute for Advanced Study in Princeton. These seminar notes were later written up (although not by Singer or Atiyah) and published, but it was several years before Atiyah and Singer got around to publishing the details of their theorem.

As Singer and Atiyah's leisurely publication schedule indicates, competition is much less prevalent in differential geometry and in pure mathematics than in other sciences. For example, Singer explains that, in distinct contrast to researchers in the more competitive sciences, differential geometers feel no rush to publish their results since the reason for publishing in his field is not to establish priorities but to enable others to study the results. Differential geometry resembles an art form more than a science because most of its results are dictated by the internal structure and the esthetics of the field rather than by practical problems. In this sense, differential geometry is one of the "purest" fields of mathematics.

Differential geometry is also one of the most difficult fields for students to enter. It is highly abstract, yet relies on visual imagery. Students must acquire a substantial mathematical background before they can reach the frontiers of knowledge. Since the field draws heavily on algebra, analysis, and topology, students must be quite well prepared before they tackle research problems.

One reason for the depth of preparation required to enter differential geometry is that it is constantly taking on new tools and new problems. It is an old field originally using only the calculus. Now it uses and illuminates differential equations and algebraic topology. It is finding and solving new problems in theoretical physics. "Geometry is not constrained," says Singer. "At any given moment there are explicit problems to solve. But much of the subject's development takes place by crystallizing diffuse ideas and combining them in novel ways. When there are a lot of people focusing on one problem, there is a lot of competition. When the problems are open-ended, there is room to breathe and not much competition."

Like many other mathematicians, Singer feels he needs uninterrupted time for research. When he was younger, he used to begin work at 11:00 at night. "I got very little sleep in those days," he says. Now he sets aside three afternoons a week. His family, even his 2-year-old daughter, knows not to bother him then, even if it does not look like he is working. What he needs is the time, and then the ideas eventually come. But the pace can be slow. "If I get one new idea in a year, that's terrific," he says.

With his academic responsibilities, his graduate students, his chairmanship of the National Academy of Sciences Committee on Science and Public Policy, and his family, Singer feels constantly pressured by a lack of time. He has practically given up reading, although he reads the *New York Times* about three times a week. Occasionally, he will take his family and dash off to a bookstore, where he will buy as many as 25 books. But then Singer's family, and not Singer, reads them. For relaxation, he listens to jazz and plays tennis.

When asked their opinion of Singer, scientists who know him say how unabrasive and generally likable he is. "Iz is one of the *nicest* people I know," Gray says. "Of all the stars in mathematics, he is one person whose judgment I trust. He knows so much and is so willing to share." Research aside, this willingness to share has gained him a considerable reputation as a liaison between differential geometry and other areas of mathematics and science, explaining this abstract field to interested colleagues.

—Gina Bari Kolata