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15. Sonicated or hand-shaken gum arabic emulsions provided the most stable preparations (10 percent gum arabic and olive oil, 2:1 by volume). Similar results were obtained with sodium oleate-stabilized emulsions (1 percent oleate), but these emulsions were less stable at *p*H 6.5. Solutions of gum arabic (Sigma) were first dialyzed to remove divalent salts [W. J. Brown, A. A. Belmonte, P. Melius, *Biochim. Biophys. Acta* **485**, 313 (1977)].
16. To avoid committing ourselves to structure type, this second nonbirefringent phase is here called simply "viscous isotropic" to distinguish it from the first birefringent phase which exhibited optical textures of a neat soap or "acid-soap" (*22*) of smectic (possibly lamellar) type. Two liquid crystalline phases have been identified in pure aqueous-monoolein systems at ambient temperatures by polarizing microscopy and x-ray diffraction [Lutton (*17*); K. Larsson, *Z. Phys. Chem. N. F.* **56** (1967)]. Between 4 and 22 percent H_2O a lamellar phase, and between 25 and 41 percent H_2O a "viscous isotropic" phase occurs. At higher H_2O contents the latter phase coexists with excess water. Both freeze-etch electron microscopy [Lundberg (*18*)] and x-ray diffraction [K. Larsson, *Chem. Phys. Lipids* **9**, 181 (1972)] suggest that the viscous isotropic phase of the monoolein-water system consists of spherical water droplets in a cubic face centered lattice surrounded by a hydrocarbon continuum. The retention of the optical viscous isotropic appearance of this phase with incorporated fatty acids in a bile salt solution suggests a similar molecular structure for the viscous isotropic phase during fat digestion.
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A Two-Fluid Approach to Town Traffic

Robert Herman and Ilya Prigogine

A kinetic theory of vehicular traffic has been developed over the past 20 years to attempt to describe the characteristics of traffic on multilane highways (*1–3*). In this theory we have examined the evolution of the speed distribution function in terms of a number of important processes: the relaxation or speeding up process, which expresses the attempts of drivers to achieve their own desired speeds; the interaction or slowing down process, which arises in the conflict between a faster driver and a slower driver; and the adjustment process, which reduces the variance around the local mean speed. We have tried to avoid examining many specific details—for example, the details of all the vehicles in queues and the details of passing maneuvers. In addition, since the beginning of this work, we have been searching for an extension of the theory that would describe in a similar overall fashion the traffic in towns—that is, traffic in an extended network.

Traffic Observations

In order to obtain some feeling for such a complex problem we have made a number of observations. Recently, we analyzed a large amount of data in the form of speed-time histories of vehicles reported by Johnson *et al.* (*4*) for many cities in the United States. The data were generated by following vehicles in each area studied, and they consist of speed-time histories and usage patterns of randomly selected vehicles operated under different traffic conditions as well as on various roadways. Using these data, we have investigated the possibility of evaluating relative traffic quality and finding ways to characterize traffic on different roadway types with various flow levels in nine metropolitan areas (*5*). We have examined traffic attributes such as average speed, stopped time, speed distribution function, and acceleration and speed noise (defined as the standard deviation of acceleration or speed), as well as the ratio of speed noise to average speed (the coefficient of variation of speed), in order to measure quantitatively the effects of road type, traffic conditions, and driver behavior on traffic phenomena.

We have found that although the traffic in various areas is different, similar relations appear to exist between a num-

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ber of traffic variables and the average speed. For example, the acceleration noise and the ratio of speed noise to average speed are correlated with average speed, and the stop time is linearly related to trip time, the reciprocal of average speed. These relations are surprisingly simple and global in character. For this reason we have pursued an examination of the relation between trip time and stop time to see whether a theoretical basis could be developed for such seemingly general relations between some of the pertinent traffic variables. We note that Smeed (*6*) has also examined town traffic in a similar spirit of trying to develop simplifying overall relations.

Summary. A two-fluid model of town traffic has been developed by extending ideas formulated in an earlier kinetic theory of multilane traffic. The two fluids are taken to consist of moving cars and cars stopped as a result of traffic conditions. The average speed of the moving vehicles is assumed to be proportional to the fraction of the vehicles that are moving raised to a power that reflects the "goodness" of the traffic. It is then found that the trip time per unit distance is essentially linearly related to the stop time per unit distance, in general accord with data obtained in many cities. Relations are developed on this basis for flow, among other variables, versus average speed. These relations contain a new parameter that is identified with the quality of the traffic network system.

The trip time on the links of a transportation network is a significant and useful variable for many facets of transportation engineering and planning. Travel time studies are often carried out to measure the effectiveness of a transportation system in terms of traffic engineering improvements and cost-benefit analyses. Trip time is perhaps the major factor in determining a driver's route choice and appears to be the most reliable single variable in the traffic assignment process.

Herman and Lam (*7*) found that when the stop time was compared to the running time of the same trip in the Detroit suburban area, the stop time, for all practical purposes, varied linearly with the running time as well as with the total trip time. Similar results were obtained with two sets of previously reported data, one collected on a specific route in Berkeley, California (*8*), and the other collected on all the major arterial streets in Fresno, California (*9*). The slopes and intercepts obtained from linear fits of the data for stop time versus running time were reasonably consistent for the different data sets from different city areas.

More recently, we obtained additional trip time versus stop time data. In particular, in connection with studies of urban traffic and fuel consumption, we accumulated data for a subcompact car circulating in the Detroit metropolitan area (*10*). In addition, we analyzed data for London reported by Roth in 1959 (*11*), as well as more recent data for Melbourne reported by Johnston *et al.* (*12*). Results of these analyses as well as the analysis by Chang and Herman (*5*) of speed-time histories from nine U.S. cities are summarized in Table 1. The information presented in Table 1 includes the regression coefficients for the linear fit as well as the coefficients of correlation. The general consistency of the regression coefficients combined with the high values of the coefficients of correlation provides evidence for the general nature of the trip time versus stop time relation. It should be noted that for the two routes to work studied in the Detroit suburban area (*7*), the average values of the regression coefficients *A* and *B* (see Table 1) are 1.73 and 2.14 minutes per mile, respectively, and for the Berkeley route (*8*) they are 3.21 and 1.73 minutes per mile. These values compare favorably with those given in Table 1.

Figure 1 shows trip time as a function of stop time. Linear trends are plotted for most of the data mentioned above. Although there is a considerable amount of scatter in the data and the lines vary in slope and intercept, they do fall in a fairly consistent overall trend. The dashed curve in Fig. 1 represents data derived from results reported by Wardrop (*13*) for traffic in London. These data also lie well inside the general trend.

The existence of a general trend in the data for trip time versus stop time suggested that there might be some underlying theoretical basis for such a result. This led us to consider a two-fluid model for traffic in towns.

Two-Fluid Model

The concept of a two-fluid model appeared in our kinetic theory of multilane highway traffic when the transition to the so-called collective flow regime was achieved at sufficiently high vehicular concentrations. In this case we found that the average speed depends on the fraction of the cars that are immobilized. Since the two-fluid model discussed here stems from this earlier result, we will briefly sketch the background concepts. According to equation 4.39 in (*1*), we can write for the average speed, v, in terms of the average concentration, k

$$v = \frac{1}{kT(1 - P)} \quad (1)$$

We emphasize that throughout this article all the quantities discussed are meant to be averages taken over the entire system. This result is strongly mechanism-dependent, since the probability of passing, P, depends on concentration as well as on quantities such as lane configuration and driver behavior. The relaxation time to traffic perturbations, T, also depends on these quantities. Similarly, the fraction of stopped cars, f_s, is a complicated function of concentration. However, there is a remarkably simple relation between $1 - f_s$ and the average velocity that follows from equations 4.39 and 4.40 in (*1*), namely

$$v = u_0(1 - f_s) = u_0 f_r \quad (2)$$

where $f_r = 1 - f_s$ is the fraction of moving cars and u_0 is a characteristic quantity depending on the "desired speed" distribution function (the speeds which the drivers desire to achieve). As noted above, f_r and f_s are the fractions of vehicles that are moving or stopped ($f_r + f_s = 1$). The two-fluid model is the basic concept we wish to take over in considering traffic in town networks. We will show that this concept leads in a straightforward manner to a quantitative interpretation of the trip time versus stop time relation.

Application to Town Traffic

We now proceed to develop a relation between the trip time and the stop time according to the ideas previously indicated. At the collective transition in the case of highway traffic, the velocity distribution for cars splits into two parts—one corresponding to moving vehicles and the other to vehicles that are stopped because of local conditions such as traffic jams (*1*). Likewise, the traffic in a network in a city may be considered to consist of two traffic fluids—one composed of moving cars and the other of cars that are stopped as a consequence of congestion, traffic signals, stop signs, and so on, but not in the parked condition. Parked cars are ignored as not being a component of the traffic; they

Table 1. Summary of regression results for $T = A + BT_s$ for various urban areas, where T is the trip time (minutes per mile) and T_s is the stop time (minutes per mile).

Data base	Number of data points	A (min/mile)		B (min/mile)		Standard error of residual	Correlation coefficient
		Estimate	Standard error	Estimate	Standard error		
Various U.S. metropolitan areas*	30	2.168	0.085	2.157	0.0956	0.333	0.974
Detroit metropolitan area†	92	1.932	0.068	2.576	0.162	0.523	0.859
London metropolitan area‡	39	2.741	0.176	1.988	0.108	0.775	0.950
Melbourne§	22	1.953	0.082	1.547	0.053	0.287	0.988

*Chicago, Detroit, Los Angeles, New York City-Newark, Phoenix, Saint Louis, San Francisco, Salt Lake City, and Washington, D.C. (5). †Data from (10). ‡Data from (11). §Data from (12).

form part of the street configuration. Extending the ideas in our kinetic theory of traffic, we assume that the average speed of the moving cars, v_r, depends on the fraction of cars that are moving. We might expect a relation of the form

$$v_r = v_m(1 - f_s)^n \quad (3)$$

where v_m is the average maximum running speed. Obviously, the boundary conditions are satisfied for $f_s = 0$ and 1.

The designation two-fluid model is used in analogy with the Bose-Einstein condensation, which at sufficiently low temperatures leads to splitting of the molecular distribution function into two parts—one representing molecules in the ground state and the other molecules in excited states (14). In the case of the Bose-Einstein condensation

$$\frac{\theta}{\theta_c} = \left(1 - \frac{\rho_0}{\rho}\right)^{2/3} \quad (4)$$

where θ and θ_c are the temperature and the critical temperature and ρ_0/ρ represents the fraction of condensed molecules. Thus, the thermal energy of the excited molecules is proportional to a power of the fraction of excited molecules. Likewise, the speed of the moving vehicles depends on the fraction of the vehicles that are moving. It is in this sense that the word analogy has been used.

Returning to the traffic problem, note that $v_m = 1/T_m$, where T_m is the average minimum trip time per unit distance. The trip time can be written as

$$T = T_r + T_s \quad (5)$$

where T_r and T_s are the running time per unit distance and the stop time per unit distance, respectively. In addition, we may expect that

$$f_s = \frac{T_s}{T} \quad (6)$$

This statement can be considered as a type of ergodic condition relating ensemble averages to time averages. We have then the implication that the fraction of the time stopped for a vehicle circulating in the network is equal to the average fraction of stopped cars in the system over the same time period. If the overall concentration varies widely—that is, fluctuates rapidly during the time of the trip—this condition may not be satisfied. The concentration must vary slowly over the time scale during which T_s/T and f_s are measured.

From Eq. 3 we can write

$$v_r = \frac{1}{T_m}(1 - f_s)^n \quad (7)$$

Since by definition

$$v = v_r f_r \quad (8)$$

then

$$v = \frac{1}{T} = \frac{1}{T_m}(1 - f_s)^n f_r = \frac{1}{T_m}(1 - f_s)^{n+1} \quad (9)$$

or

$$\frac{T_m}{T} = (1 - f_s)^{n+1} \quad (10)$$

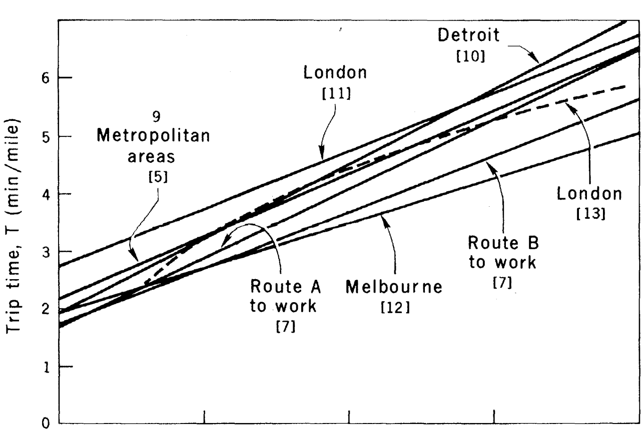
Since $f_s = T_s/T$, we have

$$1 - \frac{T_s}{T} = \left(\frac{T_m}{T}\right)^{\frac{1}{n+1}} \quad (11)$$

and finally

$$T_s = T - T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \quad (12)$$

An urban network might be characterized by the (T_m, n) pair of values. In Fig. 2 we show a plot of T versus T_s in units of minutes per mile. The quantity T_m is taken as 2 min/mile, corresponding to an average maximum speed of 30 miles per hour. Three curves are given for $n = 1$, 2, and 3. The curvature is small in all cases, so that we essentially have a linear relation between the trip time and the stop time. Furthermore, the curve for $n = 2$ corresponds fairly well to the gen-



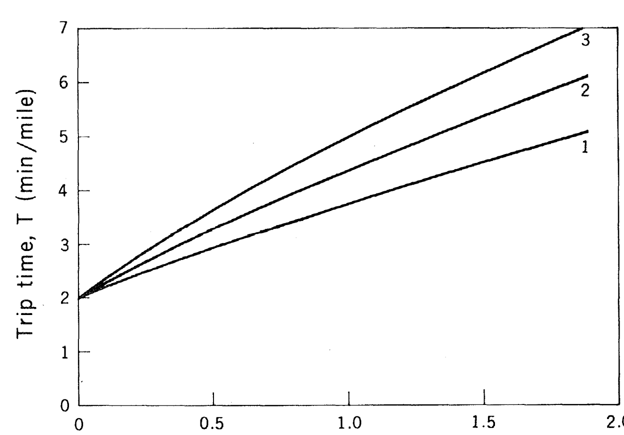


Fig. 1 (left). Trip time versus stop time. The straight lines are linear regressions fitted to observational data obtained in various studies (reference numbers are given in brackets). The dashed curve represents trip time–stop time values calculated from data in (13). Fig. 2 (right). Trip time versus stop time from Eq. 12 with $T_m = 2$ min/mile and $n = 1$, 2, and 3.

eral overall trend of the data given in Fig. 1. We have not attempted to fine-tune the functional representation by adjusting T_m and n because we are seeking mainly the character of the overall trend.

We have shown the reasonableness of the assumption that the average speed of the moving vehicles depends on the fraction of the vehicles that are moving. The question now arises whether we can go further in our attempt to describe traffic in towns. One important aspect of city traffic compared to highway traffic is that city traffic always involves stopped time arising from one cause or another.

The fraction of the vehicles stopped in a network will depend on the number N of vehicles on lanes of length L, with average concentration k given by

$$k = \frac{N}{L} \tag{13}$$

Note that the concentrations of moving and stopped vehicles, k_r and k_s, are related; that is

$$k_r + k_s = k \tag{14}$$

and that by definition

$$f_s = \frac{k_s}{k} \tag{15}$$

We also note that the fraction of vehicles stopped must depend on the "goodness" of the road facilities and traffic control system. In our search for a single independent variable to which traffic variables such as T, T_s, k, v, and q (flow) might be related, we suggest that the fraction of the stopped vehicles, f_s, be given in terms of concentration, k, as

$$f_s = \left(\frac{k}{k_m}\right)^p \tag{16}$$

where k_m is an average maximum concentration at which the traffic "jams." The parameter p must in some sense be a measure of the quality of the traffic network as it determines the fraction of stopped vehicles for a given fractional concentration k/k_m.

Remembering from Eqs. 6 and 11 that

$$f_s = 1 - \left(\frac{v}{v_m}\right)^{\frac{1}{n+1}} \tag{17}$$

and combining Eq. 17 with the postulated relation for f_s in terms of k from Eq. 16, we obtain the result

$$k = k_m \left| 1 - \left(\frac{v}{v_m}\right)^{\frac{1}{n+1}} \right|^{\frac{1}{p}} \tag{18}$$

The average flow q is then given by

$$q = kv = k_m v \left| 1 - \left(\frac{v}{v_m}\right)^{\frac{1}{n+1}} \right|^{\frac{1}{p}} \tag{19}$$

All of the relations we have been discussing are stated in terms of the averages of the traffic variables taken over the entire system.

The curves in the family represented by the q, v relation in Eq. 19 have a maximum q value at various values of v that depend on the values of the parameters n, p, and k_m. There is not only the regime in which v decreases as q increases up to q maximum, but also a regime in which v continues to decrease as q turns around and decreases. This latter regime is not normally seen in highway data because flow data are ordinarily taken over relatively long time periods.

Discussion

It would be most interesting to obtain comprehensive data over a sizable traffic network to see if the fraction of vehicles stopped might be an independent variable through which the character of the traffic could be described. It may be hoped that the two simple relations of the type

$$v_r \propto (1 - f_s)^n \tag{20}$$

and

$$f_s \propto \left(\frac{k}{k_m}\right)^p \tag{21}$$

may prove to be useful in organizing various aspects of town traffic. We are examining these theoretical ideas and are also exploring the possibility of obtaining, say from aerial photography, global information about the two traffic fluids postulated in this discussion.

Simple relations of the type developed can be very useful in discussing further questions such as the consumption of fuel in a town. Earlier work on low-speed urban traffic and fuel consumption (*10, 15*) led to a relation for ϕ, the fuel consumed per unit distance, in terms of the trip time per unit distance, T, namely

$$\phi = a + bT$$

$$\text{for } v < \sim 40 \text{ miles per hour} \tag{22}$$

where $T = 1/v$, and a and b are parameters related to the characteristics of the vehicle—the mass of the vehicle and the idle fuel flow rate, respectively. This result can be merged with the traffic description to give some idea of the fuel consumption in a town when the concentration of vehicles, traffic intensity, and so on in the town are given functions of the geometry.

In addition, it might be possible in this manner to explore the influence of town planning, which, of course, is intimately related to the dependence of the concentration of vehicles on the spatial geometry. This would involve examining the effects of inhomogeneities in the distribution of housing, industry, and services as well as in the street and highway systems on the character of the vehicular traffic. Models of this type are being studied by colleagues of one of us in Brussels (*16*).

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