The Mutual Embrace of Electricity and Magnetism

In the development of his electromagnetic field theory Maxwell relied heavily on this suggestive image.

M. Norton Wise

Scientific imagination has often been guided by visual images. Notable in the physical sciences among those who claimed to think in pictures, images, or embodied mathematics are Bohr, Einstein, and Maxwell. As historically important as visual images have been, however, the process of concept formation through imagery has not often been easy to unravel. Therein lies the intended significance of the present article for, once uncovered, the creative role of Maxwell's imagery is relatively transparent. Moreover, it explains much that has been obscure in the emergence of his electromagnetic theory—for example, how his ideas were related to Faraday's and how the well-known reciprocal symmetry of electric and magnetic fields first appeared. Before considering electromagnetism, however, a few remarks on the general function of images may serve at the outset to focus attention on those aspects which seem most clearly exemplified in Maxwell's work.

Without attempting to distinguish sharply between visual images, physical analogies, and models, I will use the phrase visual image to refer to those pictorial representations of natural phenomena which function primarily as symbols and which often have metaphorical connotations. Kepler's representation of the cosmos by a sun-centered sphere, in metaphorical analogy to the Christian trinity as a sphere with God the Father at the center, was such a symbol. So was Bohr's "formal representation" of electron stationary states by definite orbits, particularly when coupled with his metaphor of "free will" of the electron in transitions between states. Visual images as symbols, or hieroglyphs, depict more or less abstractly those characteristics of a phenomenon seen to be core characteristics, while merely evoking or standing for much that is not depicted, sometimes a physical model or analogy, sometimes a mathematical structure. The metaphorical connotations of visual images, on the other hand, often reveal strong commitments—psychological, religious, and philosophical—which help to explain their power in concept formation and their continual reappearance in the work of a single individual.

Partly because of these symbolic and metaphorical aspects, powerful images suggest relations and concepts that extend far beyond the empirical subject matter they are taken to represent. Much as models and analogies, but with less concreteness, images serve as heuristic devices, as guides to what one hopes to discover, as well as symbols for what one supposedly knows. All such heuristic devices share the property of generating problems whose answers force reformulation of the representations from which they arose. But images conceived specifically as symbols have a special characteristic, not appropriate to models and analogies. A symbol for physical reality may remain the same while the content symbolized changes radically. In this sense, the function of visual images is like that of words and of mathematical symbols, which also shape perceptions and are reshaped in meaning by the perceptions they produce. But visual images are easier to grasp as symbols than are words and they possess more immediate perceptual significance than abstract mathematical symbols. For that reason an historical examination of the role of imagery in the formation of a powerful concept may display quite sharply the process of shaping and reshaping of content.

With the latter goal in mind I present here an analysis of a striking example, an image of mutually embracing curves that was present throughout the formative stages of Maxwell's electromagnetic theory, roughly 1855 to 1870. Through the metaphor of mutual embrace, symbolized as two interlocked rings, Maxwell first conceived the reciprocal dynamics of electric currents and magnetic forces. His original conception was physically and mathematically incomplete, yet it acted as both motivation and guide for completion. The immediate result was a symmetrical set of relations between steady-state currents and magnetic forces. This initial success, however, more nearly opened the problem than solved it and led to a thoroughgoing reformulation that we now recognize as Maxwell's equations. In order to see this process clearly, it will be helpful first to survey the 19th-century prelude to Maxwell's transformation of electricity and magnetism.

Replacing Laplacian Imagery

Following the examples of Newton's universal gravitation between atoms of matter and his corpuscular theory of light, a coherent and fruitful program for all of physics emerged in France in the early 19th century. This program—which has been called Laplacian, after its most illustrious exponent—attempted to describe each of the areas of physical phenomena, such as gravity, electricity, magnetism, and heat, in terms of a separate "matter" whose action was the sum of the independent actions of its supposed constituent particles. Two such actions were possible: either the particles exerted forces, such as the gravitational force, on points at a distance, or the particles themselves were transmitted to points at a distance, as the corpuscles of light. Thus there were electric and magnetic "fluids" between whose respective particles inverse square forces acted; heat was also identified with a fluid, the self-repulsive caloric fluid, but conduction of heat through normal matter took place as radiation of particles of caloric between molecules of matter. In each area, then, the controlling image was one of direct action at a distance between independent particles, whether through forces or secondary particle transmission. This single image for many phenomena represented a coherent explanatory program for physics. It did not, however, unify the various branches on a shared physical basis,

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a demand that would soon be widely felt and that Maxwell's theory would partially satisfy.

The Laplacian type of mechanical reduction was not conceived by its supporters as merely a heuristic device to guide research; Poisson, Biot, Laplace, and others insisted that it was the only acceptable mode of explanation (*1, 2*). As a mathematically sophisticated and very successful program, therefore, it demanded an equally powerful opposition before its associated imagery could be replaced. The most serious challenges came after 1815 in the form of highly articulated mathematical alternatives, presented by Fresnel in his wave theory of light and by Fourier in his macroscopic description of heat conduction as a flow process. Beneath the mathematics of their descriptions lay new and very simple imagery. Both Fresnel and Fourier substituted indirect transmission of effects by propagation through a largely unspecified medium for direct transmission of particles or forces to a distance. Fourier in particular highlighted the positive, descriptive character of his heat conduction analysis, which would elevate it beyond any hypothesis concerning the true nature of heat. He provided neither a new model nor (directly) a new analogy, but rather a new technique of analysis and a new image. In a picture consisting only of flow across constant-temperature surfaces, he stressed the simple linear relation between the amount of heat crossing any unit surface per unit time (flux) and the rate of change of temperature across the surface (temperature gradient). In vector notation, this is $\mathbf{Q} = k\mathbf{I}$, where **Q** is flux of heat ("quantity," in the language of Faraday and Maxwell), **I** is temperature gradient ("intensity"), and k is relative conductivity of the medium.

Fourier's description effectively made temperature gradient the cause of heat flow through a medium. That would later be seen to have raised the concept of the gradient to independent and fundamental status, analogous to the steepness of a hill, whereas in the Laplacian picture the gradient had been only a secondary, purely mathematical, expression. Similarly, the wave theory of light substituted two concepts, displacement and tension in a medium, for the former flux of corpuscles. Both new propagation schemes, in retrospect, characterized propagation by two parameters, one a quantity and the other an intensity.

Fourier's treatment of heat transformed the Laplacian picture of direct transmission to a distance into an indirect propagation view. This generalized statement, however, contains an implication that Fourier never intended. Because the older picture applied not only to the transfer of material corpuscles, such as caloric, but also to nonmaterial electric or magnetic action at a distance, the transformed picture could potentially be applied to the transmission of forces as well as to heat conduction. That potential was recognized to varying degrees by several mathematically oriented physical scientists, notably Green, Gauss, Whewell, and especially William Thomson (later Lord Kelvin). Thomson began to develop the analogy explicitly and extensively in the 1840's, but primarily as a mathematical analogy. He stopped short of a physical analogy, which would have involved a quantity-intensity distinction. When Maxwell learned the mathematics from Thomson in the early 1850's, however, he learned it as what may be called physical geometry. He gave the mathematical form itself a certain reality. The full flow picture, therefore, with a quantity-intensity distinction displayed, would come to play a major role in his new synthesis of electricity and magnetism. The quantity-intensity distinction will be seen below to be one of the basic sources for his dual-field description of electric and magnetic forces, with two fields for each force (**E**, **D** and **H**, **B** in modern notation).

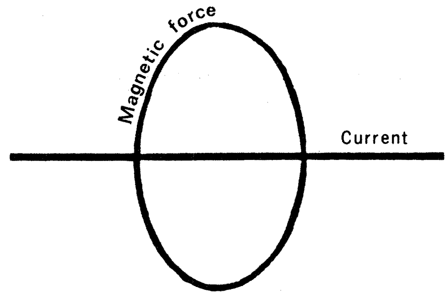


Fig. 1. Oersted's picture of the magnetic force in the vicinity of a current-carrying wire. For a complete representation Oersted imagined magnetic circles drawn at all radii from the wire.

Interconvertibility and Faraday's Imagery

Although Fourier and Fresnel upset the coherence of Laplacian physics, they did not mount the fatal challenge. That challenge derived from several discoveries of the interconversion of powers or forces in nature: chemical to electric in the voltaic cell (Volta, 1800); electric to magnetic in electromagnetism (Oersted, 1820); and magnetic to electric in electromagnetic induction (Faraday, 1831). Such interconversions could not occur between independently conserved fluids. At the very least all aspects of electricity and magnetism had to be reduced to a common basis. Today we recognize the demand for unity through conservation of energy, which was just emerging as a principle in the 1840's (*3*). During that emergence unity was sought in at least three different directions. One could attempt to salvage the action at a distance view by reducing many phenomena to complex forces between only electric particles. Among continental analysts this approach was widely adopted, and with remarkable success. Wilhelm Weber's velocity- and acceleration-dependent law of force was the classic attainment of the 1840's. A second alternative, which attempted more but initially achieved less, was to reduce electricity and magnetism, along with heat and light, to mechanical processes in the luminiferous ether that underlay the wave theory of light. The all-subsuming ether was originally most popular among British natural philosophers, such as Whewell, Herschel, and Challis. It led directly to the mathematical work of Thomson and Maxwell, although with a strong influence from a third alternative.

The third alternative was pursued by Michael Faraday. Instead of subtle fluids and ethers, Faraday spoke of natural powers and forces and attempted to investigate them experimentally. Focusing on the relations evident between powers in interconversion, he hoped to uncover the unity of all natural powers. But describing relations between powers, while avoiding concrete models, is not a straightforward task, and therein lies the importance of imagery to Faraday's discussion. All of Faraday's descriptions and analyses of electric and magnetic phenomena were couched in graphic images of lines of force, which represented, ostensibly without prejudice, whatever action might actually have been occurring. If we look at these pictures closely, they will take us a long way toward understanding the theoretical structure that Maxwell extracted from them.

The 19th-century image of electromagnetism derived from Oersted's discovery that the magnetic power of an electric current is directed in circles around, and is perpendicular to, a wire carrying the current, as shown in Fig. 1. These two "axes of power," the current line and the magnetic line, became Faraday's fundamental lines of force, although each line had other important representations. More complex pictures, even models, emerged from Ampère's suggestion that all magnetism derives from currents. The lodestone, for example, would consist of molecules of iron surrounded by tiny currents. Any small magnet, similarly, could be replaced by a current loop, and the well-known attractive and repulsive effects of magnets would apply equally to currents. Figure 2 shows typical resulting configurations. Working in the Laplacian tradition, Ampère reduced these interactions to an action at a distance law of force between infinitesimal sections, or elements, of current. In simplified form, adjacent parallel elements of current always attract, while adjacent antiparallel elements repel.

Faraday accepted neither the reality of action at a distance nor the propriety of arbitrarily defined current elements, but he did agree that two adjacent lines of electric force, representing parallel currents and taken as a whole, would attract laterally. Any single closed line, furthermore, would tend to elongate, as though antiparallel elements on opposite sides of the loop repelled each other. These lateral and longitudinal relations of lines of force, represented in Fig. 3, Faraday took to be the proper descriptive basis for understanding the nature of the electric power as it appeared in currents: electric current lines attract laterally and extend longitudinally.

Considering the closely associated magnetic lines of force, Faraday thought of them as a series of little magnets placed end to end, with the net effect that magnetic lines contract longitudinally and repel laterally, as represented for the lines of a bar magnet in Fig. 4a. Thus magnetic lines and electric current lines each have their own lateral and longitudinal relations, but if the bar magnet is replaced by a bundle of current loops, as shown in Fig. 4b, those two sets of relations are seen to be reciprocal. Lateral attraction between current lines has the same effect as longitudinal contraction in magnetic lines, and longitudinal extension in current lines has the same effect as lateral repulsion between magnetic lines. With a simplicity that would soon rivet Maxwell's attention, Faraday symbolized this mutual relation of the two axes of power as two linked rings, perpendicular to each other, but unified in their reciprocity. That image is reproduced in Fig. 5.

The perpendicular rings of electricity and magnetism, Faraday observed, exhibit a certain unity, a "oneness of condition of that which is apparently two powers or forms of power, electric and magnetic" (*4*). This statement is from a

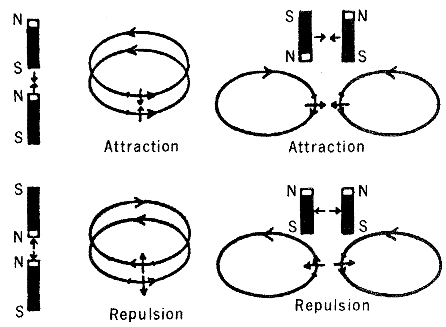


Fig. 2. The forces between current loops conceived as magnets led Ampère to reduce their actions to forces between elements of current (short sections along the currents). In simplified form, adjacent parallel elements attract and adjacent antiparallel elements repel.

paper written in 1851, after Faraday had struggled for at least 20 years with the nature of lines of force. He had by then developed a large body of associated concepts, of which the most immediately important for Maxwell were the notion of conduction of lines of force, and an associated distinction, similar to Fourier's, between quantity and intensity of the power represented by the lines.

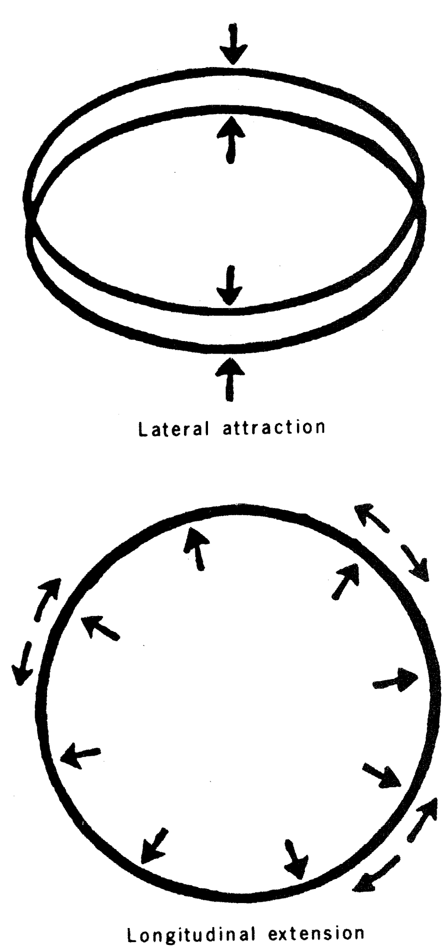
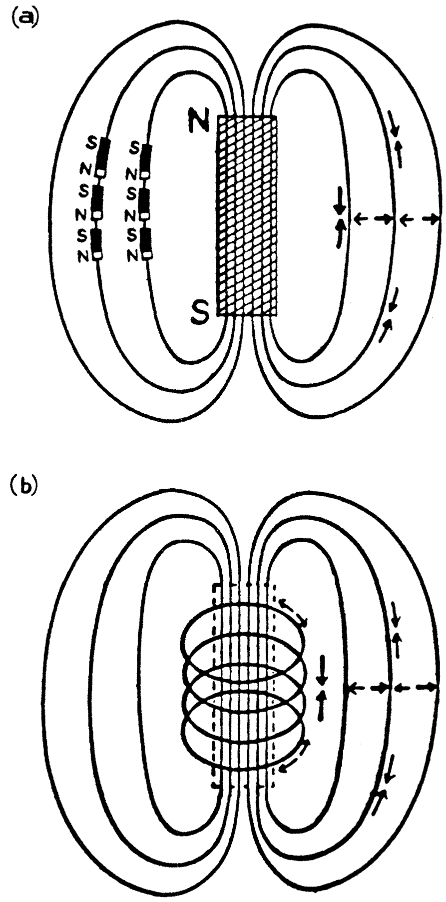


Fig. 3. Faraday replaced Ampère's forces between current elements by a dynamics of lines of force: adjacent electric current lines attract laterally, but any particular line tends to extend longitudinally.

All of these ideas developed originally out of Faraday's analysis of electrochemical processes, such as those in voltaic cells. He imagined the electric tension between the two plates of a voltaic cell to inhere in chains of polarized water molecules produced by differential affinities of the two plates for the oxygen and hydrogen of water. Each such chain represented a line of electric force. If the plates were connected by a conductor—a material that could not support electric tension—the entire chain would momentarily break up, relieving its tension through the conductor, only to immediately recombine and break up again. Continuation of the process would constitute a current of electricity through the circuit.

In this picture it is apparent that the quantity of the current would depend on how many lines of tension were undergoing breakup and recombination; that is, on the size of the plates or on the number of lines through any lateral section cutting all of them. Similarly, the power of the current to overcome any resistance to conduction in the connecting circuit, its intensity, would depend on the power of the cell to develop longitudinal tension in the lines of force, or on the relative affinities among its components. Every current, then, had two characteristics, quantity and intensity, the first a lateral measure of power and the second a longitudinal one. In any resisting circuit, the actual quantity of the current would be proportional to its intensity and to the conducting power of the circuit, yielding a description very similar to Fourier's earlier conduction equation [although to Faraday the meaning was quite different (*2*, pp. 142-148)]. It is understandable, then, that the mathematically inclined Maxwell would read the two versions as one, $\mathbf{Q} = k\mathbf{I}$, particularly when he had learned the conduction equations of Ohm and Kirchhoff.

Faraday applied his schema for conduction of lines of force between contiguous particles successively to electrostatics and magnetism, developing by 1850 a highly articulated version for magnetic lines conducted through magnetic materials and through space. To the continental notion of force at a point arising from the action of point particles at a distance, he specifically opposed the notion of conduction of powers through an intervening medium, or through space. Ponderomotive forces, he reasoned, were exerted on objects only as a result of their participation in the conduction process, or in the field of force. When Maxwell came to Faraday's work in 1854 the lateral and longitudinal properties of quantities and intensities in conduction were an integral corollary to the reciprocal lateral and longitudinal dynamics of electric and magnetic lines, although Faraday admitted to an only imperfect understanding of what the exact relation might be. This, I will argue, was the primary problem Maxwell sought to resolve in his first assault on electromagnetism.



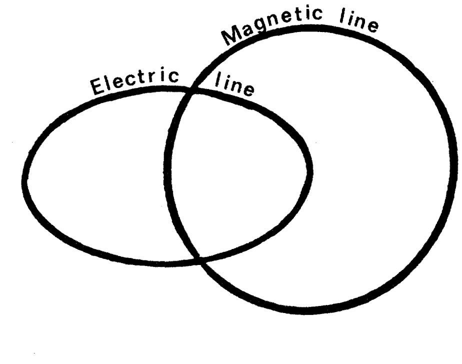


Fig. 4 (left). (a) Dynamics of Faraday's magnetic lines of force. Each line tends to contract along its length, but adjacent lines repel laterally. The result is a dynamic balance. (b) Bundle of current loops and associated lines of magnetic force. When the two sets of lines are taken as a system it is apparent that electric and magnetic effects are reciprocal. Longitudinal contraction in the magnetic lines has the same effect as lateral attraction between current lines, and lateral repulsion between magnetic lines has the same effect as longitudinal extension of current lines. Fig. 5 (right). Faraday's symbol of oneness between electric and magnetic axes of power. This would soon become the central image of Maxwell's first paper on electromagnetism, where he would label it mutually embracing curves.

Maxwell's Original Conception

Maxwell set out in 1854 to develop further the relation of electric currents to magnetic forces in the manner of Faraday's conception of a field of force. He reported his earliest ideas to Thomson (*5*):

> I have heard you speak of "magnetic lines of force" and Faraday seems to make great use of them, but others seem to prefer the notion of attractions of elements of currents directly [Ampère]. Now I thought that as every current generated magnetic lines and was acted on in a manner determined by the lines through which it passed that something might be done by considering "magnetic polarization" as a property of a "magnetic field" or space and developing the geometrical ideas according to this view.

Maxwell went on to sketch out a geometrical description of the magnetic field in relation to currents. As would soon be typical of him, he attempted to distill the entire conception into a simple set of basic theorems. The two theorems presented below were a direct attempt to express Faraday's reciprocal dynamics of the linked rings in the language of quantities and intensities.

Here and throughout the remainder of this article, I have added in brackets modernized equations for the benefit of mathematical readers. Table 1 shows the stages of development of these equations and compares them with present-day Maxwell's equations. In translating the mathematics I have consistently used the terms quantity and intensity with the notations **Q** and **I** in order to maintain a focus on symmetry (and missing symmetry) between electric and magnetic descriptions. This corresponds to Maxwell's first image and to the continuing geometrical significance of his variously denoted field variables, which he ultimately grouped under fluxes (quantities) and forces (intensities) (*6, 7*). Although the evolution of Maxwell's equations can be conveniently expressed mathematically, nothing in the present discussion depends on the bracketed equations, and they should not be allowed to obscure the primary power of Maxwell's imagery. In his initial letter to Thomson, Maxwell himself provided no equations for his theorems, which is an essential part of the story. That it is essential provides a general clue, I believe, to the way in which Maxwell developed his later arguments. With this in mind, we may return to the original dynamics of electric and magnetic lines.

A natural measure of lateral attraction of contiguous lines of magnetic force was the quantity or number of the lines crossing a unit area, $\mathbf{Q}_m$, and a natural measure of the total tendency of a line to contract longitudinally was the sum of intensity $\mathbf{I}_m$ along the line. Through a translation of this kind, I suggest, Faraday's identity between the longitudinal contraction of a magnetic line and the lateral attraction $\mathbf{Q}_e$ of any current lines linked through it became Maxwell's first theorem, which I express in the slightly simplified and retrospective form:

1) The magnetic intensity summed around a closed loop is measured by the total quantity of current through the loop (*8*).

$$\left[\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} \propto \iint \mathbf{Q}_e \cdot d\boldsymbol{\sigma}\right]$$

where $d\boldsymbol{\lambda}$ is a line element and $d\boldsymbol{\sigma}$ is a surface element.

That much might be seen as merely a result of Ampère's work on the relation of magnetic forces to currents. Now called Ampère's law, the relation had been utilized and extended by both Gauss and Thomson well before Maxwell. However, they employed only the single theorem, for force to them had only a single aspect. Maxwell's use of a second theorem has no precedent in any work but Faraday's. If I am correct, the second theorem was a loose attempt, following the associations of theorem 1, to express an identity between lateral attraction of magnetic lines passing through any area and longitudinal extension of any current line surrounding the area, thereby completing the symmetry of Faraday's linked rings (again simplified):

2) The total magnetic quantity through any surface is measured by the current around its edge.

$$\left[\oint (\text{current?}) \cdot d\boldsymbol{\lambda} \propto \iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}\right]$$

To the degree that these two theorems were the foundation of Maxwell's electromagnetic theory, and it was a very large degree, his deepest insight was to have made two laws out of what had been one. That move was more than merely reminiscent of Fourier's transformation of heat conduction; it was much the same transformation, here arrived at by reinserting Faraday's quantity-in-

Table 1. Forms of Maxwell's equations (abbreviated set) as they evolved from his initial letter to Thomson in 1854 to the "Electromagnetic theory of light" in 1868. The problem of energy and its relation to moving forces is omitted. The last column translates quantities and intensities into present-day symbols.

1854 (reconstruction of verbal description)		1855 (here limited to the differential forms)	1868 (for source-free space)	Present (including sources)
		$\mathbf{Q}'_e = k'_e \mathbf{I}_e$	$\mathbf{D} = \epsilon \mathbf{E}$	
$\mathbf{Q}_m$, $\mathbf{I}_m$ (distinguished geometrically)	Law II	$\mathbf{Q}_m = k_m \mathbf{I}_m$	$\mathbf{Q}_m = k_m \mathbf{I}_m$	$\mathbf{B} = \mu \mathbf{H}$
$\mathbf{Q}_e = k_e \mathbf{I}_e$ (simplified)	Law IV	$\mathbf{Q}_e = k_e \mathbf{I}_e$		$\mathbf{j} = \sigma \mathbf{E}$
$\nabla \cdot \mathbf{Q}_m = 0$ (implicit)		$\nabla \cdot \mathbf{Q}_m = 0$	$\nabla \cdot \mathbf{Q}_m = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \cdot \mathbf{Q}_e = 0$ (closed currents)		$\nabla \cdot \mathbf{Q}_e = 0$	$\nabla \cdot \mathbf{Q}'_e = 0$	$\nabla \cdot \mathbf{D} = 4\pi e$
$\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} = \iint \mathbf{Q}_e \cdot d\boldsymbol{\sigma}$	Law III	$\nabla \times \mathbf{I}_m = \mathbf{Q}_e$	$\nabla \times \mathbf{I}_m = 4\pi \frac{\partial \mathbf{Q}'_e}{\partial t}$	$\nabla \times \mathbf{H} = 4\pi \mathbf{j} + 4\pi \frac{\partial \mathbf{D}}{\partial t}$
$\oint$ (current?) $\cdot\, d\boldsymbol{\lambda} = \iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}$	Law I	$\nabla \times \mathbf{I}_o = \mathbf{Q}_m$		$\nabla \times \mathbf{A} = \mathbf{B}$
$\oint \mathbf{I}_e \cdot d\boldsymbol{\lambda} = -\frac{\partial}{\partial t} \iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}$	Law VI	$\mathbf{I}_e = -\frac{\partial \mathbf{I}_o}{\partial t}$	$\nabla \times \mathbf{I}_e = -\frac{\partial \mathbf{Q}_m}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

tensity physics into Thomson's mathematical heat analogy for electric and magnetic forces.

Now two observations can quickly be made regarding the second theorem: first, it is wrong as stated, since the magnetic quantity would depend on the size and shape of the loop and not only on the current; second, the usage of "current around the edge" cannot be quite "intensity of the current, $\mathbf{I}_e$, summed around the edge" (as it would have to be for symmetry with theorem 1), because magnetic quantity has no direct relation to current intensity, or tension. Maxwell was therefore either not following Faraday's dynamic reciprocity, or he was in something of a predicament concerning its exact mathematical expression. All the evidence points to the latter explanation. I suggest that we are seeing here, in the loose verbal statements of the theorems, the initial metaphorical usage of an image which, if stated precisely, Maxwell would immediately have seen to be false. Once used loosely, however, as a means for visualizing a complex physical situation, it suggested creative new ways of treating the old problem.

A Formalized Mutual Embrace

Only a little more than 1 year after his theoretical sketch, Maxwell had generated, in his first paper on electricity and magnetism, a new structure. He did so by inventing a new mathematical description to fulfill Faraday's unifying image of linked rings. In fact, he now raised Faraday's captivating but rather dry description of the linked axes of power to a completely new level of metaphorical appeal with the term "*mutually embracing* curves" (*9, 10*). The process of Maxwell's creation is of considerable interest for the history of science because it illustrates simply and clearly the importance of pure *imag*ination. Maxwell was at this time neither a sophisticated mathematician (although his talent was great) nor an experimentally or theoretically learned electrician. The mathematics he employed came entirely from Thomson's recent work in electricity and magnetism—to such a degree that Maxwell hesitated to continue his own efforts, thinking Thomson must have "the whole draught of the thing lying about in loose papers" (*11*).

We recall that the mutually embracing curves symbolized two sorts of relations between the curves: a conduction description for each, involving lateral quantities and longitudinal intensities, and dynamic reciprocity, interrelating lateral and longitudinal forces. Maxwell's seminal paper "On Faraday's lines of force," was in two parts, which mirror these two relations. In part I he developed at length an analogy for lines of force in terms of fluid flow through a resistive medium, to make the conduction picture for forces as intuitively lucid as the traditional action at a distance description. In part II he redeveloped the reciprocity of electric currents and magnetic lines; and here, at the crucial juncture where his former theorem 2 had been inadequate, he showed how a new intensity could be defined, consistent with the mathematics of continuous flow, that would complete the missing symmetry of his two former theorems.

Theorem 1, Ampère's law, stated that the sum of current quantity through any surface is equal to the sum of magnetic intensity around the bounding edge. Mathematically, Maxwell now realized, that relation depended only on the current forming a continuous closed loop [$\nabla \cdot \mathbf{Q}_e = 0$]. Theorem 2 was an attempt at a reciprocal equation for magnetic quantity through any surface, which should have been related to a longitudinal property of current around the edge, but it could not be current intensity as usually defined. Maxwell simply postulated that an intensity of the required kind should exist, since the only mathematical condition for it was that the magnetic quantity behaved as a continuous flow [$\nabla \cdot \mathbf{Q}_m = 0$]. The analogical conduction description of forces in part I, therefore, as applied to closed flow circuits of both magnetic force and electric current [$\nabla \cdot \mathbf{Q}_m = 0$; $\nabla \cdot \mathbf{Q}_e = 0$], guaranteed the possibility of reciprocity in the mutually embracing curves of part II. Adopting a long-standing label of Faraday's for a supposed electromagnetic condition of matter, an electrotonic state, Maxwell called his new invention the electrotonic intensity. It was not yet a physical state, but it could nevertheless be imagined physically (*9*, p. 205):

> We may conceive of the electro-tonic state at any point of space as a quantity determinate in magnitude and direction, and we may represent the electro-tonic condition of a portion of space by any mechanical system which has at every point some quantity, which may be a velocity, a displacement, or a force, whose direction and magnitude correspond to those of the supposed electro-tonic state. This representation involves no physical theory, it is only a kind of artificial notation.

With the seemingly small addition of this new intensity, Maxwell completed mathematically his earlier attempt at reciprocal description of current lines and magnetic lines in electromagnetic space.

(We would say today that he had invented the vector potential, as Maxwell himself later called the electrotonic state.) In order to capture the accomplishment, he returned again to a set of simple laws, with two laws now for conduction symmetry and two for reciprocity (*9*, p. 206):

Law I. The entire electro-tonic intensity [$\mathbf{I}_0$] round the boundary of [any] surface measures the quantity of magnetic induction which passes through that surface, or, in other words, the number of lines of magnetic force which pass through that surface. [Compare the former theorem 2.]

$$\left[\oint \mathbf{I}_0 \cdot d\boldsymbol{\lambda} = \iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma} \text{ or } \nabla \times \mathbf{I}_0 = \mathbf{Q}_m\right]$$

Law II. The magnetic intensity at any point is connected with the quantity of magnetic induction by a set of linear equations, called the equations of conduction [$\mathbf{Q}_m = k_m\mathbf{I}_m$].

Law III. The entire magnetic intensity round the boundary of any surface measures the quantity of electric current which passes through that surface. [Compare the former theorem 1.]

$$\left[\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} = \iint \mathbf{Q}_e \cdot d\boldsymbol{\sigma} \text{ or } \nabla \times \mathbf{I}_m = \mathbf{Q}_e\right]$$

Law IV. The quantity and intensity of electric currents are connected by a system of equations of conduction [$\mathbf{Q}_e = k_e\mathbf{I}_e$].

Although these four laws express the embracing curves, they are completely general and do not depend on following lines of force themselves around closed loops. Any embracing loops imagined in a space of currents and magnetic lines will yield the same relations as a physical current loop and a definite magnetic line. The theorems represent the structure of a constant electromagnetic field in any region, no matter how small.

Note that the conduction equations and the reciprocity equations express two different sorts of symmetry. The electrotonic intensity stands by itself in law I without relation to either conduction equation. In that sense the descriptions of electric and magnetic lines are not entirely symmetrical. The reader knowledgeable in electromagnetic theory will note further that the symmetries expressed here are not at all those familiar from the present-day Maxwell's equations, and since they apply only to the steady state, they have nothing to do with electromagnetic induction (*12*). Induction became the centerpiece of Maxwell's theory only at a later stage of development. It did have a prominent place even here, however, although not in the symmetries.

The phenomenon of a current being produced in a closed conductor when the magnetic quantity through the loop changed—electromagnetic induction—was Faraday's first major discovery and has always been his most famous one. By 1850 no credible electrical theory could fail to subsume it. Maxwell, therefore, from the beginning of his work in 1854, had been concerned to describe current induction along with the general relations of electric and magnetic curves. Loosely following Faraday, he had expressed the effect originally as follows (*13*):

The electromotive force along any line [the driving intensity tending to produce a current along the line] is measured by the number of lines of [magnetic force] which that line cuts in unit of time. Hence the electromotive force round a given circuit depends on the decrease of the number of lines which pass through it in unit of time.

$$\left[\oint \mathbf{I}_e \cdot d\boldsymbol{\lambda} \propto -\frac{\partial}{\partial t} \iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}\right]$$

There was a problem with this description, however, and one recognized by Faraday long before Maxwell. How could the magnetic quantity merely passing through the loop affect the electromotive force at the loop? Would that not be action at a distance all over again? "It is natural," observed Maxwell in 1855, "to suppose that a force of this kind, which depends on a change in the number of lines, is due to a change of state which is [merely] measured by the number of these lines" (*9*, p. 187). That was Faraday's reasoning in originally proposing the electrotonic state. A wire in a magnetic field would supposedly be in the peculiar state, and when the field was removed, the collapse of the state would appear as an induced current. Maxwell's new mathematical expression for an electrotonic state fit the requirement precisely. It provided "the means of avoiding the consideration of the quantity of magnetic induction which *passes through* the circuit" (*9*, p. 203). Law VI, following those quoted above, expressed the relevant relation (*14*):

Law VI. The electro-motive force on any element of conductor is measured by the instantaneous rate of change of the electro-tonic intensity on that element, whether in magnitude or direction.

$$\left[\mathbf{I}_e = -\frac{\partial \mathbf{I}_0}{\partial t}\right]$$

Applied to a closed circuit, that meant "the electro-motive force in a closed conductor is measured by the rate of change of the entire electrotonic intensity round the circuit referred to unit of time" (*9*, p. 207).

$$\left[\oint \mathbf{I}_e \cdot d\boldsymbol{\lambda} = -\frac{\partial}{\partial t} \oint \mathbf{I}_0 \cdot d\boldsymbol{\lambda}\right]$$

Only the electrotonic intensity in the circuit itself was now in direct action.

Electromagnetic Propagation and Light

With the introduction of an electrotonic state, Maxwell had attained what seemed to him at the time a unified geometrical, or structural, description of electromagnetism, where the structural emphasis was on the steady-state relations of currents and magnetic forces. But the attainment brought a series of new problems, including the physical nature of the electrotonic state; how it was transmitted from its origin in a magnet or current through nonconducting spaces; and how, if it arrived at a secondary conducting circuit, it induced a current there. For Maxwell, following the trend of British mathematical natural philosophy, these problems suggested an answer based on a mechanical model of the luminiferous ether. That new focus would shift dramatically the direction of his thinking, and would change ultimately even the content symbolized by the mutually embracing curves. The image itself, however, would maintain its symbolic form and guiding role.

To see the problem a little more clearly, and in one of the ways Maxwell saw it (*15, 16*), consider two conducting loops, one connected to a battery and containing a switch that is to be closed at the beginning of a thought experiment. According to Maxwell's analysis of 1855, developed above, the initial increase of current would produce an increasing electrotonic state everywhere in the surrounding space which, when it reached the second circuit, would appear as an induced current. The process could be described macroscopically as resulting from an increasing linkage of magnetic lines through the two circuits, but that image conveyed no sense at all, not even structurally, either of the physical nature of the electrotonic state or of its mode of propagation through the nonconducting space between two circuits. The image of linkage, furthermore, did not apply to the peculiar case of induction in open circuits, where the effect could only be an electrostatic tension between the open ends of the conductor. How then were electrostatic force and electrotonic intensity related?

Maxwell had been concerned about these problems in his 1855 paper, where he had expressed the hope of providing later a mechanical representation of the electrotonic state. He had even supplied some formal mathematics applicable to open circuits, but had explicitly limited his application to closed conducting loops, with the excuse that "we know little of the magnetic effects of any currents which are not closed." Even while

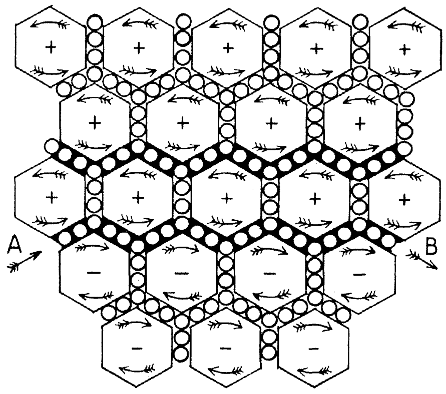


Fig. 6. Maxwell's representation (corrected slightly) of an electromagnetic ether in and surrounding a current-carrying conductor. The section is transverse through the vortex cells but longitudinal through the current, which proceeds from *A* to *B* as an actual translation of idler particles. By tangential action the cells adjacent to the current are set in vortex rotation, which in turn exerts a tangential action on the next adjacent idler particles. If free to translate, the latter particles will roll along between the rows of cells and will constitute an induced current; if not free, they will rotate in place, exerting a tangential action on the next adjacent cells and setting them in vortex rotation. Thus, the magnetic lines (linear vortices) will propagate through space.

thus distinguishing open and closed circuits, he had nevertheless believed for some time, following Faraday, that electrostatic induction (tension) and current conduction were the same, differing only in degree of resistivity of the medium: "Thus the analogy between statical electricity and fluid motion turns out more perfect than we might have supposed, for there [in statics] the induction goes on by conduction just as in current electricity, but the quantity conducted is insensible owing to the great resistance of the dielectrics" (*17*). Could it be, then, that electrostatic induction took place in closed loops; that an incomplete circuit for currents was actually closed by static induction in the open space? Maxwell apparently did not see the problem in quite that explicit form in 1855, or doubtless he would have carried it further, but many of the elements of such a conception were lying in readiness for a burst of new vision, and were surely a powerful motivation for its eventual appearance. First, however, he would have to connect the problem of magnetic propagation between electric circuits with electrostatic action, while simultaneously both distinguishing current conduction from electrostatic induction and equating their magnetic effects. That complex of relations would form a new basis of interpretation for the mutually embracing curves, this time as a structural image for the physical process of propagation itself.

Maxwell's flurry of fresh insights arose in 1861, when he made good his promised mechanical representation of magnetic lines, currents, and the electrotonic state. In "On physical lines of force" (*18*), he presented his notorious vortex model of the magnetic ether, using the diagram shown in Fig. 6. Each line of force was the axis of a vortex filament. For adjacent vortices to have the same sense of rotation about their axes, thereby representing lines of force in the same direction, they were supposed to be contained in cellular regions separated by "idler" particles, the particles being in rolling contact with the cells. These idler particles constituted electricity. They were free, in all substances, to rotate in place between rotating cells, but in conductors they could also translate between cells, with more or less resistance. A stream of them would constitute a current which, because it exerted a tangential action on adjacent cells, would set them in rotation as vortices, or magnetic lines, surrounding the current. The next adjacent idler particles would then also be set rolling by tangential action of these first vortex cells (constituting electromotive force) and if the particles could not translate (being in a nonconductor) they would set the next adjacent cells in rotation, and so on out into space in the vicinity of the initiating current. Lines of magnetic force would thereby be formed surrounding the current at larger and larger radii. Whenever the propagating effects reached a closed conductor the idler particles would simply translate while rolling against the next adjacent cells, thereby constituting an induced current in the opposite direction from the original. Any resistance to translation, however, would eventually set those next adjacent cells in rotation, and the induced current would be stopped, thus reproducing the observed phenomena of electromagnetic induction.

Considering this strange machinery as a heuristic model, valuable for showing the possibility of a mechanical theory, Maxwell proceeded to show that it reproduced exactly the required relations between currents, magnetic force, and the electrotonic state. And the electrotonic state had now a simple mechanical interpretation as rotational momentum of the vortices. One problem remained, however, in the initial description: what exactly was the mechanism of tangential action that transmitted rotation from idler particles to

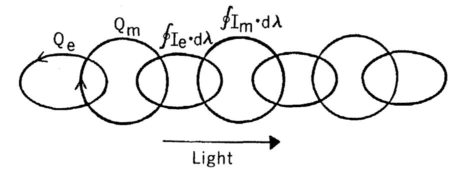


Fig. 7. The mutually embracing curves reconstituted as the basis of propagation of electromagnetic effects. Curve $\mathbf{Q}_e$ represents an actual line of current that is increasing in strength and curve $\mathbf{Q}_m$ represents an actual magnetic line, also increasing in strength. The remaining curves are imaginary lines around which electric and magnetic intensities are summed.

cells, and from the exterior of cells through their interior? The vortices had to be fluid in order to produce the correct dynamic behavior of magnetic lines, but they could not be perfectly fluid because then no tangential action could occur in their interiors. Neither could the connection between idler particles and cell walls be perfectly rigid in normally nonconducting space, for then the starting and stopping of induced currents would be instantaneous, to say nothing of the problem of motion of normal bodies through such a system. To avoid these problems, apparently, as well as to avoid dissipation of energy in a merely viscous medium, Maxwell proposed in a later installment that the cells were elastic and that they interacted elastically with the idler particles (*19*). Some such mechanism would explain conveniently propagation from point to point throughout the ether. It would do more, however, and that was soon to be the basis for transformation of the imagery we have been discussing. It would provide an explanation of electrostatic effects in nonconductors and in open circuits.

Maxwell by now realized that static induction could not be simply weak conduction, for equally good insulators showed widely varying capacities for static induction. But if static electric charge were merely elastic displacement of idler particles, whereas currents involved extended motion, the difficulty would be resolved. The displacement, $\mathbf{Q}_e'$, while it was occurring, would constitute a brief current, a displacement current, $\partial \mathbf{Q}_e'/\partial t$. And just as the effect of a changing magnetic field, when propagated to a closed conductor, would be an induced current, so the process of propagation itself would take place by successive induction of displacement currents. Each such induction by a magnetic vortex would reinduce a second vortex and so on through space. With that famous invention, the essentials were complete for Maxwell's remarkable synthesis of all contemporary electricity and magnetism. To set the capstone, he showed that the propagation of magnetism by electric displacement would occur at the known velocity of light. His electromagnetic theory of light was immediate: "We can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena*" (*16*, p. 500).

The Mutual Embrace Reconstituted

We have seen that Maxwell's initial goal in employing the mutually embracing curves was a macroscopic description, in the steady-state case, of the relation between a magnetic field and its associated electric currents. Only secondarily was electromagnetic induction a consideration. If the constant-field description led naturally to an explanation of current induction, it was, at that level of analysis, satisfactory; and the image of closed linked rings beautifully fit the criterion. Nevertheless, the problem of providing a mechanical vortex description of a changing field, and of the communication of its effects, had refocused Maxwell's attention on induction as the fundamental aspect of a field description. Any change in the magnetic field would propagate by successive, and reciprocal, inductions of displacement currents and magnetic lines.

Use of the term reciprocal introduces the last stage of Maxwell's analysis that will be considered here. Dynamic reciprocity had always been the foundation, for both Faraday and Maxwell, of the mutual embrace of electric currents and magnetic lines, but a new sense of that reciprocity now occurred to Maxwell and has since been taken as the symbol of his system. Any changing magnetic quantity would induce a net electromotive force (summed electric intensity) in any curve surrounding it. The response at any point in the medium to the electric intensity would be a displacement current, or electric quantity [$\mathbf{Q}'_e = k'_e\mathbf{I}_e$], obeying continuity of flow [$\nabla \cdot \mathbf{Q}'_e = 0$]. That changing electric quantity, similarly, would induce a net magnetic intensity in any curve surrounding it, and the response at any point in the medium would be again a continuously flowing magnetic quantity [$\mathbf{Q}_m = k_m\mathbf{I}_m$; $\nabla \cdot \mathbf{Q}_m = 0$]. As the process repeated itself the initial disturbance would propagate in all directions in such a way as to preserve the reciprocal relation between electric and magnetic sums over arbitrary closed curves. Thus the mathematical relation of mutual embrace would itself propagate through space. This might be represented as the chain of Fig. 7, although it is important to recognize that the links in the chain could represent actual lines of force only in very special circumstances. Since any two linked curves could be imagined as small as one pleased, the mutual embrace provided a structural representation for the overall process of propagation from point to point throughout the field. With that view, the mechanical model became less immediately significant. At the geometrical level mechanisms in the medium gave way to structure in the field, and the electrotonic state concomitantly sacrificed its prominent place to the displacement current.

Table 2. Comparison of Maxwell's equations in the absence of sources in 1868 and at present.

1868	Present
$\mathbf{Q}'_e = k'_e\mathbf{I}_e$	$\mathbf{D} = \epsilon\mathbf{E}$
$\mathbf{Q}_m = k_m\mathbf{I}_m$	$\mathbf{B} = \mu\mathbf{H}$
$\oint \mathbf{I}_e \cdot d\boldsymbol{\lambda} = -\frac{\partial}{\partial t}\iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}$	$\oint \mathbf{E} \cdot d\boldsymbol{\lambda} = -\frac{\partial}{\partial t}\iint \mathbf{B} \cdot d\boldsymbol{\sigma}$
or $\nabla \times \mathbf{I}_e = -\frac{\partial \mathbf{Q}_m}{\partial t}$	or $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} = 4\pi\frac{\partial}{\partial t}\iint \mathbf{Q}'_e \cdot d\boldsymbol{\sigma}$	$\oint \mathbf{H} \cdot d\boldsymbol{\lambda} = 4\pi\frac{\partial}{\partial t}\iint \mathbf{D} \cdot d\boldsymbol{\sigma}$
or $\nabla \times \mathbf{I}_m = 4\pi\frac{\partial \mathbf{Q}'_e}{\partial t}$	or $\nabla \times \mathbf{H} = 4\pi\frac{\partial \mathbf{D}}{\partial t}$
$\nabla \cdot \mathbf{Q}'_e = 0$	$\nabla \cdot \mathbf{D} = 0$
$\nabla \cdot \mathbf{Q}_m = 0$	$\nabla \cdot \mathbf{B} = 0$

True to his style, Maxwell encapsulated his new conception in a set of simple theorems, which once again defined the reciprocity of electric and magnetic action. They are most clearly set out in the form of embracing curves in an 1868 "Note on the electromagnetic theory of light" (*20*). The new theorems should be compared directly with the laws stated above in order to capture the transformation in content. Law III appears here virtually unchanged, but for the explicit term embracing:

> Theorem A. If a closed curve be drawn embracing an electric current, then the integral of the magnetic intensity taken round the closed curve is equal to the current multiplied by 4π.

$$\left[\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} = 4\pi \iint \mathbf{Q}_e \cdot d\boldsymbol{\sigma}\right]$$

The intent, however, of the old theorem is radically altered, for Maxwell is now not so much concerned with current as with displacement current.

> Theorem D. When the electric displacement increases or diminishes, the effect is equivalent to that of an electric current in the positive or negative direction.

$$\left|\frac{\partial \mathbf{Q}'_e}{\partial t} \text{ replaces } \mathbf{Q}_e\right|$$

In other words, if the total electric displacement through any closed curve varies, the rate of change will measure the total magnetic intensity around the curve.

$$\left[\oint \mathbf{I}_m \cdot d\boldsymbol{\lambda} = 4\pi\frac{\partial}{\partial t}\iint \mathbf{Q}'_e \cdot d\boldsymbol{\sigma}\right]$$

The relation reciprocal to this is no longer the old law I, connecting magnetic quantity and electrotonic intensity in the steady state, but law VI, on induction, and in the circuital form that had been secondary:

> Theorem B. If a conducting circuit [following theorem D, any closed curve in a dielectric] embraces a number of lines of magnetic force [magnetic quantity], and if, from any cause whatever, the number of lines is diminished, an electromotive force will act round the circuit, the total amount of which will be equal to the decrement of the number of lines of magnetic force in unit of time.

$$\left[\oint \mathbf{I}_e \cdot d\boldsymbol{\lambda} = -\frac{\partial}{\partial t}\iint \mathbf{Q}_m \cdot d\boldsymbol{\sigma}\right]$$

The mutually embracing curves have been here reconstituted as mutually inducing curves of magnetism and electric displacement. Two additional equations, of conduction, relate magnetic quantity to magnetic intensity, and electric displacement (quantity) to electromotive force (intensity). Thus the reciprocity of the new curve is complete in a way in which the earlier image was not, for the quantities and intensities of the conduction relations are the same as those reciprocating in the induction relations.

That is the tight symmetry familiar in our present Maxwell's equations in the absence of charges or currents, as shown in Table 2.

Conclusion

We have followed the evolution of a single image through mutliple stages in Maxwell's conceptualization of the electromagnetic field. The mutually embracing curves served to anchor his ideas, from start to finish, on a geometrically solid framework to which all speculation could be tied. At the same time the structure itself suggested new paths of investigation—through the flow analogy, for example, and through reciprocal dynamics. A consistent mathematical formulation for the electrotonic state was the first result, with strong hints of more to come. The extensions emerged as the displacement current and the concomitant picture of propagation of electromagnetic effects through a medium, all firm ground for an electromagnetic theory of light. But the new perceptions, reflected back on the old imagery, required its reformulation. With propagation at the center of attention, the mutual embrace of electricity and magnetism was reconstituted as mutual induction—still a mutual embrace, but productive now of offspring.

A somewhat different summary of Maxwell's work arises from reflecting on the usual reconstruction of his insights. It has often been said that Maxwell's contribution to electromagnetic theory consisted fundamentally in adding the displacement current [$4\pi\, \partial \mathbf{D}/\partial t$] to Ampère's law [$\nabla \times \mathbf{H} = 4\pi \mathbf{j}$] and that he was motivated either by the need to preserve charge conservation in open circuits or by his perception of a need to complete the symmetry with Faraday's law of induction [$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$]. There is a sense in which each of these claims is true, but as the analysis above has shown, they miss entirely the historical core of Maxwell's thinking. He did add the displacement current, but far more fundamental was his expression of Ampère's law with a quantity-intensity distinction built in. He was concerned with charge conservation in open circuits, but before that problem could even arise explicitly he had to distinguish between conduction and static induction. He was deeply concerned with the symmetry between electricity and magnetism, but his original symmetry did not involve electromagnetic induction at all; instead he followed Faraday's dynamic reciprocity between a constant current and its associated constant magnetic force. Completing the symmetry of these mutually embracing curves produced a second form of Ampère's law and an electrotonic state [mathematically our vector potential $\nabla \times \mathbf{A} = \mathbf{B}$]. The displacement current arose not from symmetry considerations but from Maxwell's attempt to elaborate mechanically the meaning of the electrotonic state, especially for open circuits and for the transmission of magnetic effects to produce current induction. Only with the displacement current in full view did Maxwell reformulate his conception of mutual embrace on the basis of electromagnetic induction rather than steady-state currents. Thus the present-day symmetry of Maxwell's equations derived from the role of the displacement current, rather than the displacement current arising from symmetry.

References and Notes

1. For a discussion of the intransigence of the Laplacians see J. Herivel, *Joseph Fourier: The Man and the Physicist* (Clarendon, Oxford, 1975), especially pp. 153-159.
2. The context directly relevant here is discussed in M. N. Wise, thesis, Princeton University (1977); University Microfilms 77-14, 252, pp. 25-32.
3. T. S. Kuhn in *Critical Problems in the History of Science* [M. Clagett, Ed. (Univ. of Wisconsin Press, Madison, 1969), pp. 321-356] has given a lucid survey of interconversion, and other problems suggesting energy conservation.
4. M. Faraday, *Experimental Researches in Electricity* (three volumes, London, 1834, 1844, 1855), vol. 3, paragraph 3268; the diagram accompanies paragraph 3265.
5. From a letter of 13 November 1854 reprinted in "The origins of Clerk Maxwell's electric ideas, as described in familiar letters to William Thomson," *Proc. Cambridge Philos. Soc.* **32**, 702 (1936).
6. J. C. Maxwell, "On the mathematical classification of physical quantities," *Proc. London Math. Soc.* **3** (1870); reprinted in (*7*), vol. 2, pp. 257-266; see p. 261.
7. ———, *The Scientific Papers of James Clerk Maxwell* (Cambridge Univ. Press, Cambridge, 1890).
8. This simplified version of Maxwell's theorem underestimates the complexity of temporal evolution in his thought. He had not distinguished physically, at this stage, between quantity and intensity. He had distinguished only two geometric aspects, surface and linear, of magnetic force conceived as a condition of space, magnetic polarization. This geometrical mode of viewing physical entities was very deep in Maxwell's perception generally.
9. J. C. Maxwell, "On Faraday's lines of force: Parts I and II," *Trans. Cambridge Philos. Soc.* **10** (1856).
10. See (*7*), vol. 1, pp. 184, 194, and 194n, where "embracing" is twice italicized. Not only here but repeatedly in later papers Maxwell employed the metaphor of "embrace."
11. Letter of 13 September 1855, in (*5*), p. 712.
12. Much the same point has been made by C. W. F. Everitt in his excellent article on Maxwell for the *Dictionary of Scientific Biography* (Scribner's, New York, 1974), vol. 9, pp. 206-207; reproduced with additions as *James Clerk Maxwell: Physicist and Natural Philosopher* (Scribner's, New York, 1975), pp. 90-93.
13. Letter of 13 November 1854, in (*5*), p. 703.
14. I have omitted law V, on the force between currents, in order to avoid an extended discussion of Maxwell's energy considerations, which are not directly essential for the image of embracing curves.
15. See J. C. Maxwell, "On physical lines of force: Parts I and II," *Philos. Mag.* **21** (1861), figure 3; (*7*), vol. 1, p. 488 and the description beginning on p. 477.
16. Also see "On physical lines of force: Part III," *Philos. Mag.* **23** (1862); (*7*), vol. 1, pp. 489-491.
17. This statement is from part I of (*9*), p. 181, where it is largely unexamined; the preceding phrase is from part II of (*9*), p. 196, which was written several months later. Maxwell may thus have begun in part II to see some difficulties with his earlier thoughts, but having once seen current conduction and static induction as the same, he was all the more likely to invent a new conception to fulfill the old vision.
18. (*15*), pp. 451-488; the vortex description is on p. 477. Maxwell's diagram requires a slight correction as pointed out in Everitt's book (*12*), pp. 95-96.
19. (*16*), pp. 489-502. A fourth part concluded the work.
20. Appended to "On a method of making a direct comparison of electrostatic with electromagnetic force," *Philos. Trans. R. Soc. London* **158** (1868); (*7*), vol. 2, pp. 125-143, theorems quoted from pp. 138-139.
21. I thank J. G. Burke and E. S. Abers for helpful comments and C.W.F. Everitt for a significant and detailed critique.