

trouble to reach some obscure corner of London" in order to hear a rock group called "The Citations."

Some critics feel that this intense preoccupation with the stuff of citations belies a drive for the corporate dollar, of which Garfield has quite a few at stake. As one reviewer of his *Essays* put it, "Garfield promotes his products; Garfield announces his plans for editorial change; Garfield evaluates his products for the information community; Garfield introduces his associates and vouches for their character and integrity; Garfield

acknowledges the adulation his publications have received among users. Garfield's gross is his intense preoccupation with his corporate welfare. It is never so labeled, but no label is needed."

But it is, after all, Garfield's corporate self-interest, coupled with his subtle insights into the information needs of the scientific community, that has put such revolutionary tools into the hands of scientists around the globe. And, as *Science* found in Philadelphia, the craving for corporate growth has not been at the

expense of a certain style. The company's fleet of chauffeur-driven cars, for example, includes a Cadillac, a Lincoln, a Jaguar, and, until 2 years ago, at which point Garfield gave it to his son, a Checker.

—WILLIAM J. BROAD

Erratum: In the report "Children absorb tris-BP flame retardant from sleepwear: urine contains the mutagenic metabolite, 2-3-dibromopropanol" by A. Blum *et al.* (15 September 1978, p. 1020), the unit of measure for dibromopropanol in Table 1, column 3, should have been nanograms, rather than milligrams, per milliliter. In reference 28, sentence 2, the word "sells" should have been "formerly sold." We apologize for this error to Apex Chemical Company, Inc., which discontinued sale of Fyrol flame retardant, for use in children's sleepwear.

RESEARCH NEWS

Computer Science: Surprisingly Fast Algorithms

Mathematicians have traditionally been more concerned with showing that solutions to problems exist than with determining what the solutions are. The advent of the computer changed this situation by making it possible to at least think about computing solutions to complex problems. But it has become increasingly clear that there is a big difference between a solution that can be computed in theory and one that can be computed in practice. Often the straightforward way of solving a problem involves so many operations that, even for moderately sized problems, the solution is effectively noncomputable.

From this concern with finding computable solutions to problems came the idea of developing "fast" algorithms that require fewer steps than those currently in use. The payoffs from fast algorithms can be enormous. For example, the development of the fast Fourier transform (FFT) completely changed whole areas of science, such as crystallography, by making possible computations that were previously infeasible. Similarly, entire issues of engineering journals have been devoted to applications of the FFT.

Recently, computer scientists have discovered new fast algorithms for manipulating polynomials and power series. (A polynomial is an expression of the form $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$, where the a 's are constants. The integer n is the "degree" of the polynomial. A power series is a polynomial extended to include an infinite number of terms.) Since operations on power series are so common, the new algorithms could be of great practical importance.

The new algorithms are also interesting from a mathematical point of

view. According to Alan Borodin of the University of Toronto, they are completely nonintuitive and so are "very, very surprising." For example, one result is that any power of a polynomial can be computed as quickly as squaring the polynomial.

Discoveries of these fast algorithms began in 1972, when M. Sieveking of the University of Zurich found a new, rapid way to compute the first N terms of the reciprocal of a power series. Then H. T. Kung of Carnegie-Mellon University noticed that Sieveking's method is actually a well-known procedure called Newton iteration. Shortly after Kung made this observation, John Lipson of the University of Toronto and, independently, Kung and Joseph Traub of Carnegie-Mellon determined that Newton iteration can be applied to solve power series equations, thus leading to a number of fast algorithms.

The idea behind Newton iteration is to approximate the power series solution to a particular equation with a polynomial consisting of the initial terms of the power series that is the actual solution. At each step of the Newton iteration, more terms are added to the polynomial approximation. Traub explains that when Newton iteration is applied to power series equations, the iteration always converges and the number of correct terms at least doubles at each step.

Using Newton iteration, Kung showed that the first N terms of the reciprocal of a power series can be computed as quickly as multiplying two N th degree polynomials. The fastest known method for polynomial multiplication is the FFT, which requires $N \log N$ operations.

Dividing a power series by a power se-

ries can be thought of as a multiplication problem in which one power series is multiplied by the reciprocal of another. When division is represented in this way, Kung's result about reciprocals leads to the conclusion that the first N terms of the quotient of two power series can be computed in no more than $N \log N$ steps. Thus division is no harder than multiplication.

Kung and Traub recently generalized the result about power series to apply to all algebraic functions, including such complicated functions as the reciprocal of a power series. These algebraic functions, which arise in many areas of mathematics, are the solutions of polynomial equations in which the coefficients of the variables are themselves polynomials. The textbook method of computing algebraic functions is by comparison of coefficients—a method that can be extremely slow. For example, if the algebraic function satisfies a polynomial of degree n , computation of the first N terms of the polynomial by comparison of coefficients requires as many as N^n operations. With new algorithms, the first N terms of any algebraic function can be computed with the same number of operations that are needed to multiply two N th degree polynomials, or $N \log N$. They established this result by showing that the first N terms of any "regular" algebraic function can be computed rapidly by iteration. (A regular function is of a certain form that makes the calculation of its coefficients straightforward.) Then they showed that any algebraic function can easily be converted to a regular function.

Still another application of this new approach to manipulating power series is