PLUTO, is a thoroughly tested instrument, having been one of two detectors used on the DORIS storage ring. The two others will need several weeks or possibly months of debugging before being ready for full-time data taking. The fourth and fifth detectors are under construction and are expected to be in place by the coming spring.

PEP, coming on line a year behind PETRA, may get scooped on some major new discoveries (Fig. 3). Researchers at SLAC will have two already proven detectors (Mark II and DELCO) in place by the October 1979 opening date in addition to one all-new instrument that will need to be broken in. Three other new detectors will be ready about 6 months later.

A search for new elementary particles containing heavier quarks could be one of the earliest of findings of this type. With the experience of the J/psi in hand, physicists now believe that an increase in the ratio of the probability of producing hadrons to the probability of producing muon pairs is a signal of a new quark. Since an increase in this ratio is a threshold process, even if the resonance should be missed, observation of such an increase would tell experimenters to remeasure at lower, perhaps only cursorily scanned, energies. But, points out Wolfgang Panofsky, director of SLAC, other effects also can contribute to an increase in this ratio, thus requiring precision measurements. Even when a storage ring is operating at full performance, it takes hundreds of hours to gather data for one point on a high-precision energy scan.

Thus, given the necessity of breaking in PETRA and of debugging its new detectors, it could be some time, say observers, before results of experiments requiring such high-precision scans are forthcoming. At present, DESY physicists are experiencing difficulty in packing large numbers of electrons and positrons into their respective bunches, a problem that goes under the general heading of beam instability. Such problems have been encountered and solved in other machines in the past, and DESY's Voss emphasizes that PETRA is well within its planned timetable for storing high electron-position currents at high energies. Nonetheless, until beam instability effects are overcome, many types of experiments will not be possible.

Since beam instabilities are a fact of life in storage rings, physicists operating PEP may well run into similar difficulties in a year's time. If they are lucky and avoid such problems, or if they are able to incorporate directly the solutions that their German counterparts come up with, experimenters using PEP may not find themselves so far behind after all. In any case, all observers agree that there is more than enough "physics" for both machines to lead long and productive lives.—ARTHUR L. ROBINSON

Fields Medals (III): A Broad Attack on Analysis Problems

Charles Louis Fefferman was born on 19 April 1949 in Washington, D.C. His remarkable development was that of a child prodigy. At the age of 14 he entered college at the University of Maryland, where understanding professors guided his education. He went to graduate school at Princeton University, where he received the Ph.D. when he was 20 years old. At the age of 22 he became a full professor at the University of Chicago (the youngest full professor at a U.S. college), and a year later he moved to Princeton University, where he is now working. His outstanding achievements were recognized by a number of awards before he received the Fields Medal. In 1971 he received the Salem Prize and in 1976 the Waterman Award of the National Science Foundation.

Fefferman works on various aspects of analysis, such as harmonic analysis, partial differential equations, and several complex variables. These are relatively old subjects and the problems in these fields are notoriously complicated and difficult. One of Fefferman's major contributions is the solution of a problem in the theory of functions of several complex variables. In this theory, one studies mappings given by holomorphic (complex analytic) functions of several variables and asks which regions in the space of n complex variables can be mapped into each other—that is, are biholomorphically equivalent. In contrast to the situation in the theory of one complex variable, simply connected regions are generally not equivalent in two or more variables. For example, it has been known for a long time that the inside of a sphere $|z_1|^2 + |z_2|^2 < 1$ is not biholomorphically equivalent to a "bicylinder" $|z_1| < 1$, $|z_2| < 1$. In short, regions are much more "rigid" under biholomorphic mappings in two or more variables than they are in one variable. Usually one requires these regions to be "pseudoconvex."

Now it is natural to expect that a biholomorphic mapping that takes one such pseudoconvex region with a smooth boundary into another will also be smooth up to the boundary. This problem of the boundary smoothness is as basic as it is simple to state. Nevertheless, it is a very difficult problem on which several prominent mathematicians worked without success. This difficulty stems from the rigidity mentioned above which makes ineffective methods that are applicable to one complex variable. In 1974 Fefferman succeeded in proving the theorem on boundary smoothness. This makes it now possible to study the mapping on this boundary. His paper was a very difficult one, and various researchers have tried to simplify the proof, thus far without success. A number of papers have appeared in which Fefferman's theorem is used. Without question, this result is important to the theory of several complex variables.

Related to this work is a later discovery about the boundaries of such biholomorphically equivalent regions. By extending these odd-dimensional boundaries by one dimension, he was able to introduce an indefinite metric (Lorentz metric) that changes only by a factor under biholomorphic transformations. In particular, the null geodesics—the analogs of light rays in relativity theory—form an invariant set of curves on the boundary. These curves had been discovered before but by a completely different method. Thus Fefferman unexpectedly introduced the differential geometric concept of an indefinite metric into complex function theory. Many of these concepts are still so new that one cannot predict where they will lead.

Fefferman's earlier studies were with harmonic analysis, a subject that deals with questions related to Fourier integrals and Fourier series. Problems in this area are very delicate and difficult, and progress on them is correspondingly slow. For example, it was known for a very long time that even for continuous functions Fourier series need not converge everywhere. This led to the need for other convergence concepts, and every student of mathematics now learns that the Fourier series of a function which, when squared, is integrable (L^2 function) converges in the space of L^2 functions. This result was proved as early as 1907, but the question of wheth-

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er the Fourier series of such an L^2 function converges almost everywhere—that is, everywhere but a "small" exceptional set—remained a nagging one. This question was finally answered, in the affirmative, only in 1966 by the Swedish mathematician L. Carleson.

Convergence questions for Fourier series are usually studied for functions of one variable, and it is tempting to believe that functions of several variables behave similarly as far as their Fourier series are concerned. This is by no means the case, as Fefferman discovered.

Consider, for example, the Fourier series of a function of two variables, $f(x,y)$, of period 2π in x and in y. It is a double series

$$\sum c_{mn}\, e^{i(mx + ny)}$$

where m and n run over all integers. To study the convergence it is natural to restrict the summation to all integers inside a rectangular $|m| \leq M$, $|n| \leq N$, and then study the convergence of the corresponding trigonometric polynomial

$$p_{MN} = \sum_{|m| \leq M}\sum_{|n| \leq N} c_{mn}\, e^{i(mx + ny)}$$

as M and N tend to ∞. Fefferman showed that for rather simple continuous functions the p_{MN} need not converge almost everywhere even if the ratios M/N and N/M are bounded. This result came as a complete surprise. Remarkably, if $N = M$—that is, in the case of a square—they do converge almost everywhere. Another unexpected fact concerns the trigonometric polynomials $p_R(x, y)$ obtained by summing the Fourier series over the integer pairs belonging to the disk

$$n^2 + m^2 < R$$

It is well known that for an L^2 function $f(x,y)$ the p_R tend to f in the space of L^2 functions, but Fefferman found that the corresponding statement is false for functions in L^p, $1 < p < \infty$, if $p \neq 2$. (These are functions that, when raised to the pth power, are integrable.)

For the proof of these results Fefferman employed new ideas and related them to an intriguing geometric problem, called Kakeya's needle problem: What is the smallest area of a region in the plane in which a needle (line segment) of length 1 can be continuously turned by 360° in such a way that at the end of the motion the needle occupies the original position? Both translations and rotations are allowed. The first guess is a circle of radius 1/2 and area $\pi/4$. But one can do even better with a convex region, namely the equilateral triangle of area $1/\sqrt{3}$. It came as a great surprise when Besicowitch in 1928 showed that there are regions of arbitrarily small area with this property; these regions are not convex but very "spiky." Somebody trying to turn a long beam in a dense forest will appreciate this problem and its solution. Fefferman brought this geometric fact to bear on the convergence question of two-dimensional Fourier series.

The examples discussed above give only a superficial idea of Fefferman's deep investigations on Fourier series. The mathematically versed reader will be interested in the sweeping survey article which Fefferman presented at the International Congress of Mathematicians in Vancouver, British Columbia, in 1974.

Another of Fefferman's discoveries deals with functions of "bounded mean oscillations" (BMO functions). To describe the role of these functions, it is necessary to introduce some concepts. In functional analysis one works with various function spaces, called Banach spaces. An example of a Banach space is the periodic functions $f(x)$ of period 2π for which the integral

$$\int_0^{2\pi} |f(x)|^p dx$$

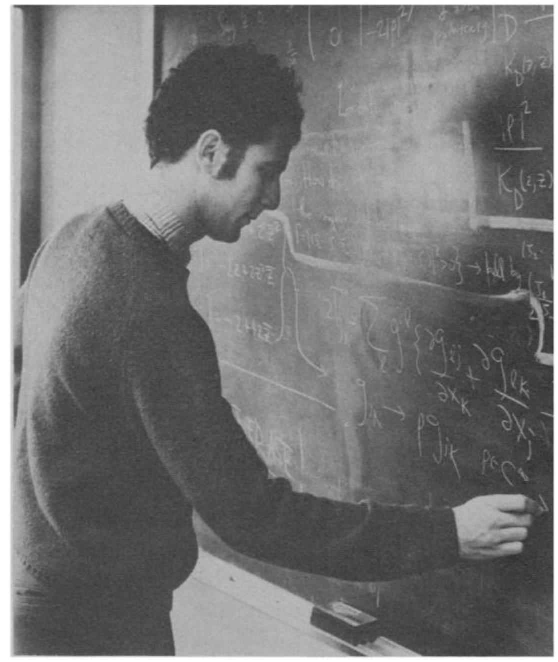
is finite. For $p \geq 1$ these functions form the elements of the L^p spaces. Closely related are the Hardy spaces H^p, which consist of the L^p functions that are boundary values of functions that are analytic inside the circle $|z| < 1$.

There is a general abstract procedure by which one associates with any Banach space another Banach space called a dual space. It has been known for a long time that the dual of L^p is L^q, where $q = p/(p - 1)$. Similarly, the dual space of H^p is H^q, provided $p > 1$. Although the dual space of L^1 can, in a rather elementary way, be characterized as the space L^∞ of all bounded measurable functions, it remained a mystery to describe the dual space of H^1. By analogy to L^1 one might expect that this space consists of the boundary values of bounded analytic functions; this guess turns out to be false.

In 1971 Fefferman cleared up this mystery; he showed that the elements of this space are actually the same BMO functions that 10 years earlier had been discovered by F. John in his fundamental studies motivated by elasticity theory. The definition of these BMO functions is somewhat complicated. They are functions $f(x)$ for which the mean oscillation

$$\frac{1}{b - a}\int_a^b |f(x) - A_{ab}| dx$$

from a constant average A_{ab} over an interval $a \leq x \leq b$ is bounded for all possible intervals. These functions need not be bounded; a typical example of an unbounded BMO function is $f(x) = \log|x|$. However, these functions are large only on relatively small sets. It was this property that was important in John's work. He considered elastic materials subject to small strains and showed that, as one would expect, they suffer small displacements. The derivatives of these displacements may be unbounded, but they are large only in small sets. In fact, the derivatives are BMO functions—for which Fefferman found a completely different description, and thus added them to the arsenal of function spaces.



Charles Fefferman [Photo by John Simpson]

There are a number of other areas to which Fefferman has contributed in an essential way, such as the theory of partial differential equations. Here he used deep ideas of Fourier analysis to settle some difficult problems about the existence of solutions of differential equations. In this as in his other work one notices a general trait: not to focus narrowly on a special problem, but to view it and attack it on a broad front. He connects problems of complex analysis with those of differential geometry, such as the geodesics that entered complex analysis, in a most startling way. He combined Fourier analysis with partial differential equations in a new way. His versatility and ability to bring together and master different branches of mathematics may well be one reason for the remarkable progress of Fefferman's work.

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