The Fields Medals (I): Relating the Continuous and the Discrete

The highest award to which a mathematician can aspire is the Fields Medal, an award comparable in many respects to a Nobel Prize in the prestige it confers. J. C. Fields, who set up a trust for the gold medals that constitute the award, said only that they should be made "in recognition of work already done and as an encouragement for further achievements on the part of the recipient." This has been interpreted to mean that the medals should be given to young mathematicians (generally those under the age of 40), a tradition that has been closely followed since the first two medals were awarded in 1936. The Fields Medals are given out only every 4 years, at the quadrennial convening of the International Congress of Mathematicians. This year, Fields Medals were presented to Gregory A. Margoulis of the Soviet Union, Daniel Quillan of Massachusetts Institute of Technology, Charles Fefferman of Princeton University, and Pierre Deligne of the Institute des Hautes Etudes Scientifiques in France.

Gregory A. Margoulis was awarded the Fields medal for his pathbreaking contributions that shed light on a structure linking the two oldest and most fundamental concepts in mathematical thought—the continuous and the discrete. In particular, he discovered the interrelations between continuous mathematical structures known as Lie groups and their discrete substructures known as lattice subgroups.

About Margoulis' biography, sketchy information is available. He was born in Moscow in 1946. His father was a mathematician, and Gregory showed great interest in mathematics and chess. Although he became an outstanding chess player as a teen-ager, he gave up the game when he entered Moscow University. He received a candidate's degree, the equivalent of our Ph.D., at the university, his adviser being Professor Jakov G. Sinai. Margoulis has not yet received his doctor's degree, which is a prerequisite for appointment to a full professorship in the Soviet Union. He now occupies the position of research associate at the Institute for Transmission of Information in Moscow. He is married and has a 5-year-old son.

Photo by G. D. Mostow

Gregory A. Margoulis

Margoulis' first outstanding contribution was in solving a conjecture posed by Atle Selberg of the Institute of Advanced Study in Princeton. Selberg, a Norwegian who won a Fields medal in 1950, speculated about the nature of structures known as lattice subgroups of continuous groups. A lattice subgroup can be thought of as a scaffolding, such as the scaffolding of a building. Selberg conjectured that apart from some exceptions, all lattice subgroups are arithmetic, with the variables taking on only integral values. Thus these subgroups, which occupy a prominent role in algebra, geometry, complex variable theory, and number theory, and are examples of discrete phenomena, inherit their discreting from the integers.

In 1960, Selberg began an assault on the structure of these subgroups. Significant advances were made by others as well, including myself and André Weil of France and the Institute for Advanced Study. The analyses of the problem seemed to split in two directions: one direction involved non-cocompact lattices and the other involved cocompact lattices (Fig. 1).

In 1968 D. A. Kazhdan (now a professor at Harvard) and Margoulis made a brilliant breakthrough in the case of non-cocompact lattices in matrix groups. They proved the existence of a nontrivial unipotent element, which had been conjectured by Selberg. This is an element whose matrix has the form

$$\begin{matrix} 1 & * & *\cdots* \\ 0 & 1 & *\cdots* \\ 0 & 0 & 1\ *\cdot\cdot \\ \vdots & & \\ 0 & 0 & \cdots 1 \end{matrix}$$

with respect to some basis. From 1969 to 1974 Margoulis exhibited his extraordinary power in extracting from the existence of unipotent elements in non-cocompact lattices successively more and more deeply structured facts, until at last he proved Selberg's outstanding conjecture dealing with the arithmeticity of non-cocompact lattices in groups.

Parts of the latter development were

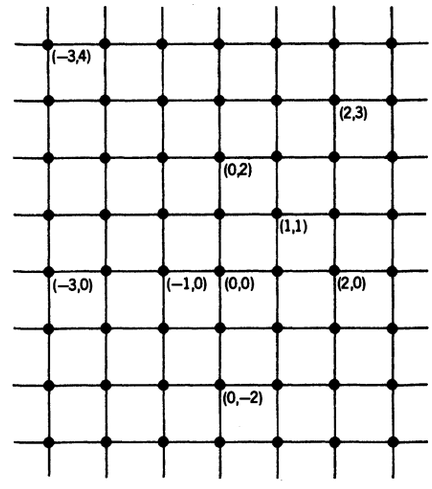

Fig. 1. A graph of $\mathbf{R}^2$. A multidimensional version of a continuous group with a discrete subgroup analogous to $\mathbf{Z}$ in $\mathbf{R}$ is $\mathbf{Z}^2$ in $\mathbf{R}^2$, or more generally $\mathbf{Z}^n$ in $\mathbf{R}^n$, where n is any whole number. For example, $\mathbf{R}^2$, the set of all ordered pairs (x, y) with x and y in $\mathbf{R}$, can be pictured as the set of all points in the plane of a sheet of graph paper, and $\mathbf{Z}^2$, the set of all (x,y) with x and y in $\mathbf{Z}$, can be pictured as the set of all points with integral coordinates—that is, the crossing points of the vertical and horizontal lines of the (infinitely extended) graph paper. The example $\Gamma = \mathbf{Z}^n$ and $G = \mathbf{R}^n$ satisfies one more significant condition—finite covolume. There is a region U of finite area if $G = \mathbf{R}^2$ or finite volume if $G = \mathbf{R}^n$ $(n \geq 3)$ such that $G = U + \Gamma$; that is, each g in G can be expressed as $g = u + \gamma$ with u in U and γ in Γ. One defines a lattice subgroup of a continuous group G as a *discrete* subgroup of finite covolume. If the subset U in the finite covolume condition can be taken to be a set of finite diameter, one calls the lattice cobounded (or more commonly, cocompact). In the example of $\mathbf{Z}^2$ in $\mathbf{R}^2$, $\mathbf{Z}^2$ is a cocompact lattice subgroup of $\mathbf{R}^2$.

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done later, independently, by M. S. Raghunathan of Tata Institute in Bombay after a comparably long analysis.

At this stage however, the case of cocompact lattices still loomed before the international community of researchers as a blank wall. In 1974, in a brilliant stroke, Margoulis realized how to scale that wall. In 1965, I had introduced a strategy for solving a related problem: if one starts with a particular lattice, is there only one group that could contain that lattice and is there only one possible location for that lattice in the group? In a project extending until 1973, I succeeded in proving that the answer is "Yes." According to Margoulis, my proof was of special interest because of its new conceptualization of the problem and because it introduced, for the first time, the use of ergodic theory in its analysis.

Then Margoulis took a bold step in his analysis of cocompact lattices. He took a lattice in one setting and considered its (possibly degenerate) image in another setting. He then used a mixture of algebra, analysis, and number theory to finally solve the problem of the structure of these lattices. Actually, his results apply not only to lattices in continuous group but more generally to lattices in other sorts of groups as well, specifically to lattices in algebraic groups over either **R** or Q_p for any prime number p.

When I lectured on Margoulis's results at Harvard in 1974, David Mumford, a Fields medalist in that year, entitled the talk "Recent breathtaking results of G. A. Margoulis." This unwonted adjective for a mathematical topic perhaps helps convey the electrifying excitement generated by Margoulis's result among the mathematicians of the world.

Unfortunately, Margoulis was not granted permission to travel to Helsinki to accept his Fields medal. In homage to his achievements, which were described by Jacques Tits of College de France at the award ceremony in Helsinki on 14 August 1978, the entire audience in Finlandia Hall rose to its feet, in a spontaneous gesture of admiration for the medalist who was so conspicuously absent.

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Antibodies (I): New Information About Gene Structure

Until recently, techniques for the direct examination of genes were not available. That situation has changed, however, as a result of the revolution in molecular biology that began early in this decade. It is now possible, for example, to pick out an individual gene from among the tens of thousands in the mammalian genome, then to use recombinant DNA techniques to manufacture enough copies of the gene to study and finally to determine the sequence of the nucleotides in that gene, and all in less than a year's time.

One of the more rewarding applications of the techniques has been to the study of antibody genes. Investigators are acquiring much new information about the numbers and arrangements of the genes in the mammalian genome. They have also determined the nucleotide sequences of at least four of these genes. The research has provided both a direct confirmation of a long-held hypothesis about the organization of antibody genes and some surprising new revelations about that organization. In addition, the gene studies are providing important clues to the solution of one of the long-standing problems of immunology—that is, how to account for the ability of a single animal to make as many as a million different antibodies.

What the DNA studies have confirmed about antibody gene arrangement is that there are separate genes coding for the variable and constant regions of antibody chains. An individual antibody molecule consists of four polypeptide chains—two identical light chains and two identical heavy chains (Fig. 1). Each of these polypeptide chains in turn consists of a variable region and a constant region. The variable regions of the light and heavy chains form the part of the antibody that combines with the appropriate antigen. Thus the variable regions must differ from antibody to antibody. But the amino acid sequence of the constant region is the same for all chains of the same type.

In 1965, William Dreyer of the California Institute of Technology and J. C. Bennett of the University of Alabama School of Medicine suggested "the two gene-one polypeptide theory" for the synthesis of antibody proteins. They proposed that two genes, one for the variable and one for the constant region, were needed for the production of a single antibody chain. Since then the evidence has been consistent with this hypothesis, but direct demonstration of the separate genes was not achieved until 1976, when Susumu Tonegawa and his colleagues at the Basel Institute for Immunology showed that the DNA segments coding for the constant and variable regions of mouse light chains are separate from one another in embryonic cells.

The surest piece of evidence for the separation came when Tonegawa and his colleagues, in collaboration with Walter Gilbert and Allan Maxam at Harvard University, determined the complete nucleotide sequence of a fragment of embryonic DNA carrying the gene for the variable region of a light chain. They found that beyond the codon (a sequence of three nucleotides specifying a particular amino acid) for amino acid 98 of the variable region, there was no agreement between the nucleotide sequence of the DNA and the amino acid sequence of any light chain. Thus, they concluded that the DNA beyond codon 98 did not specify any light chain structure and that the gene for the constant region could not be attached to this codon.

Although the genes for the variable and constant regions of an antibody protein might be separated from one another in embryonic cells, which do not make antibodies, immunologists assumed that the two genes would somehow get together in mature antibody-producing cells. And when the Basel workers looked at antibody gene patterns in a line of mature cells, they found that the genes coding for the variable and constant regions of the light chain produced by the cells appeared to be joined.

But on closer examination of the structure of the DNA encompassing both genes—genes Tonegawa and his colleagues thought would be connected directly to one another—the unexpected happened. They found that between the genes for the constant and variable regions of the light chains, the DNA contained a segment of about 1250 bases that was missing from the messenger RNA (mRNA) for the light chain in question. Thus, the DNA contained a nucleotide segment that could not be translated into protein structure since the sequence was

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