level outlined in the proposal to NSF would take another quarter-million dollars. Xuong estimates that during the 6 years from conception to operation of the protein crystallography system at San Diego, he has spent $450,000, although he adds that it would be considerably cheaper to duplicate. For this reason, it is unlikely that x-ray equipment with two-dimensional detectors will be available in every laboratory. But experiments such as protein crystallography are already very expensive, and an optimistic assessment by Kretsinger at Virginia is that, in 10 years, manufacturers may be able to sell diffractometers equipped with two-dimensional multiwire proportional counters for less than the price of conventional diffractometers now, allowing for inflation.

—ARTHUR L. ROBINSON

Information Theory: A Surprising Proof

When a long-standing mathematical problem is solved, the proof is usually complicated and technical, if not voluminous. Many of these proofs can be understood only by researchers in narrow subspecialties of mathematics. Mathematicians are taken aback, then, by a recent solution to a well-known problem in information theory. The problem was first posed by Claude Shannon, the founder of information theory, more than 20 years ago. The solution, devised by Lászlo Lovász of the József Attila University at Szeged, Hungary, is short (only a few typed pages), simple enough to be easily understood by most mathematicians, and ingenious.

Many who have seen Lovász's solution to Shannon's problem agree with Ronald Graham of Bell Laboratories in Murray Hill, New Jersey, who says the proof is "remarkable" but that he is not surprised that Lovász is the one who came up with it. Although only 28, Lovász has already made quite a name for himself by solving, within a short period, several notable problems. According to Bruce Rothschild of the University of California at Los Angeles (UCLA), Lovász seems to work only on the hard problems.

The information theory problem Lovász solved is that of measuring the rate at which information can be sent, with no possibility of error, over noisy channels. These are channels in which one signal may be confused with another when the signals are received. Information theorists sometimes assume that not all signals transmitted over noisy channels can be confused with each other. In these idealized situations, two signals may be confused but a third may be so different from either of them that it is never confused with them.

When some of the signals sent over a noisy channel cannot be mutually confounded, Shannon noted a way to transmit error-free messages at a maximum rate. The trick is to send blocks of signals, rather than individual ones, and use nonconfoundable signals in the design of the blocks. In that way, no block can be confused with another. It turns out that, in each case, there is an upper limit to the amount of information that can be sent in this way. This maximum—known as the Shannon capacity—has, except in a few simple cases, proved impossible to determine exactly.

When studying sets of confoundable signals, mathematicians often represent the systems as graphs. Each point on such a graph denotes a signal, and two points are connected by a line if the signals they represent can be confused. The problem of determining the Shannon capacity of a system whose graph is a pentagon (Fig. 1) has been described by David Cantor of UCLA as "the first nontrivial case of the problem of finding Shannon capacities." He says that the problem has proved a continuing challenge to mathematicians, and some of the brightest people he knows have worked on it. This is the problem that Lovász solved.

In order to find the Shannon capacity of the pentagon, Lovász transformed the problem from one in information theory into one in geometry. Other investigators had previously used methods of linear programming to estimate upper bounds for the problem. Lovász's method not only provides upper bounds at least as accurate as those given by linear programming, but also, in the case of the pentagon, provides an upper bound that is the same as a lower bound known previously. Thus Lovász's method gives the exact value of the Shannon capacity. Moreover, Lovász's proof is so simple, Cantor says, that if someone had devised it 20 years ago, mathematicians would have dismissed the problem as a trivial one. Nonetheless, Edgar Gilbert, also of Bell Laboratories, points out that Lovász's inspiration for his proof seems unprecedented. "What he did took me by surprise," Gilbert says.

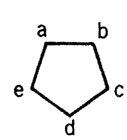

Fig. 1. A graph of the problem Lovász solved. Points *a*, *b*, *c*, *d*, and *e* refer to distinct signals. Points connected by a line can be confused with each other. Thus *a* and *b* can be confused, but *a* and *d* cannot. If one-letter messages are sent, only two signals, such as *a* and *d*, can be sent in one time period with no danger of confusion. Thus, in two time periods, four different messages can be sent with no errors. If two-letter messages are sent (constituting blocks of two signals), there are five distinct nonconfoundable messages that can be sent in two time periods. These messages are *aa*, *bc*, *ce*, *db*, and *ed*.

Although Lovász's proof is just beginning to circulate among mathematicians, some are already applying his method to other information theory problems. For example, Lawrence Shepp and Andrew Odlyzko of Bell Laboratories in Murray Hill and, independently, Howard Rumsey, Eugene Rodemich, and Robert McEliece at the Jet Propulsion Laboratory in Pasadena, California, have applied it to a problem centering on error-correcting codes. These are codes in which a certain amount of redundancy is introduced, thereby allowing those receiving a message to detect errors. Odlyzko says that "it is possible that Lovász's method can be pushed still further" in applications to information theory problems. Lovász himself says that he is applying his method to other problems similar to the one he solved.

Most investigators familiar with Lovász's solution say it has no immediate practical import because, in practice, there is no such thing as two signals that can never be confused with each other. Thus, most codes are designed so that the probability of errors is small, but not zero. Even Shannon describes the problem of finding Shannon capacities as a "conceptual problem." However, he says that many important scientific problems were first posed as conceptual problems, and he hopes that Lovász's method will lead to solutions for a whole class of problems. In any event, he says, "I am interested and happy to see this whole problem of the pentagon finally laid to rest."—GINA BARI KOLATA