Speaking of Science

Mathematical Games: Are They Bona Fide Research?

So also the games in themselves merit to be studies and, if some penetrating mathematician meditated upon them, he would find many important results, for man has never shown more ingenuity than in his games.—BARON GOTTFRIED WILHELM VON LEIBNITZ, 29 July 1715

Mathematical games occupy a curious place in the scientific world. Interest in them flourishes, as witnessed by the popularity of the articles on games in *Scientific American*, but they are described as "frivolous" by some of their practitioners. Studies of these games are typically relegated to a position in the shadows of conventional mathematical research. However, the distinction between conventional mathematics and games is at times hard to make. Researchers are sometimes surprised to find that apparently frivolous bits of work develop into fine specimens of "serious" mathematics.

The appeal of games to mathematicians, according to Hugh Montgomery of the University of Michigan, is that both the thought processes involved and the gratifications derived are the same as those of serious mathematics. Nonetheless, some mathematicians steer clear of games altogether and others confess to feeling guilty about spending time on them. They tend to pass their results of studies of games through the mathematics community by word of mouth rather than to publish them.

Some insights into the sources of the distinction between games and serious mathematics arose from a recent conversation with John Horton Conway of Cambridge University, a man who has been described by his colleagues as "the uncrowned king of mathematical games." Conway was spending 2 weeks at Bell Laboratories in Murray Hill, New Jersey. The stated purpose of his visit was to discuss coding theory and other problems in discrete mathematics, but he was, as always, eager to play and to discuss games.

As is his custom when visiting the United States, Conway spent some time with Martin Gardner of *Scientific American*, telling him of his latest games and results of his studies of games. Gardner says he draws on these conversations, sometimes years later, when writing his monthly column.

Conway admits to spending most of his waking hours on games. "Games are very seductive," he says. His interest in games extends to all aspects of his life. For example, he viewed a mistake made by his programmable hand calculator—an HP65—as a game; from the mistake, he was able to deduce the machine's internal code, which was designed to be known only by its manufacturer. The mistake occurred when, one day, the machine misread a tape. Conway compared what the machine actually read to what it was supposed to have read and thereby deciphered the code. "It was like the Rosetta Stone," he says. Now he is able to play tricks with his calculator. He can do such things as punch in a string of instructions that deletes a command coded into the machine. This causes the calculator to freeze in a state in which it will no longer respond to any command.

Conway's interest in games first became manifest when he was an adolescent (he is now 39). At that time he became intrigued by knots. Although the tying and untying of various knots sounds like the sort of trick every Boy Scout should know, knots can be the basis of challenging puzzles and games. Topologists are also interested in problems involving knots, so Conway's first brush with games turned out to be related to a serious mathematical subject. But Conway says he does not consciously encourage his students to become involved with games for fear they will "end up doodling for the rest of their mathematical lives."

Although he considers games to be frivolous, Conway says he no longer feels guilty about spending time on them. His guilt feelings left him, he says, when he did some respectable research on more serious mathematical problems—research which assuaged his doubts about his mathematical abilities. Conway continues to do research on the long-standing mathematical problem of classifying finite groups, but he does not use games as a heuristic aid to develop insights into conventional mathematical problems. He believes that games only seldom yield results of interest to serious mathematicians. In fact, he says, the greatest surprise of his life came when a theory he developed to analyze strategies and advantages in games led to a description of numbers that is of interest to logicians.

In Conway's opinion, the simplicity of games may be a clue to what distinguishes them from serious mathematics. Even his game-inspired description of numbers is amazingly simple. According to Conway, the numbers are built up "in an atmosphere of complete and utter triviality" within a few pages of his book. The reader doesn't have to know what a number is or any facts about numbers, he says. Serious mathematics, on the other hand, has a certain depth of argument. This depth appeals to Conway and leads him to say that he would not feel intellectually satisfied if he spent all his time on games to the exclusion of serious mathematics. He explains that, "People have been beavering away for hundreds of years on problems in serious mathematics. They have developed clever analytical tools and very subtle arguments. The net effect is to build up a tremendous edifice that takes an enormous effort to understand." In contrast, games are simple to understand.

Andrew Gleason of Harvard University does not completely agree with Conway's analysis but points out that the line between serious and frivolous mathematics is very fuzzy. He believes that much work on games, and especially much of Conway's work, may well become part of the domain of serious mathematics. Although Gleason says he "has guilt feelings down inside" when he studies games, he attributes them more to a snob value associated with "real" mathematics than to the uselessness of games.

Ronald Graham of Bell Laboratories points out that it is possible to view games from a positive perspective that is independent of any concern for their applicability to the rest of mathematics. He says that games keep more people happily occupied thinking along mathematical lines than does serious mathematics. Martin Gardner, who believes that games are particularly useful in teaching mathematics, says "The best way to interest children in mathematics is to give them problems that are fun to work on." The world of games provides space for pleasurable thought. It also provides, in its occasional applications, the excitement of surprise.—GINA BARI KOLATA