previous experimental results are plotted in Fig. 2. These limits correspond to 1 standard deviation, that is, a confidence level of 67 percent. For 95 percent confidence levels, each limit must be adjusted upward by a factor of 3, as indicated on the plot. In addition, we have taken the calculations of Zeldovich *et al.* (*8*) and, after scaling them appropriately with energy, plotted the predicted q/p ratios. For several regions corresponding to integral values of Z and m, the intensity from known ions (such as $^2H_2^+$ and $^{14}N^{2+}$) was sufficiently high to require the removal of the silicon detectors from the beam. As a result there are regions, generally less than 1 percent wide in mass, in which we were unable to search for quarks. These regions are indicated in Fig. 2 by short vertical bars. The existence of these beams served the useful purpose of proving that the cyclotron was still in tune and that if there were quarks at concentrations greater than the limits we have placed, they would have been observed.

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Territorial Division: The Least-Time Constraint Behind the Formation of Subnational Boundaries

Abstract. *Nations usually locate their smaller administrative subdivisions in regions of highest population density. This report derives a precise form of the size-density relationship from the general assumption that social structures evolve in such a way as to minimize the total time expended by society in their operation. The result is confirmed empirically.*

All modern societies are subdivided into sets of primary political divisions (for example, states, counties, departments). Where societies exhibit internal variation in population density, the smaller territorial units tend to be located in the more densely settled regions (*1*). This negative relationship between size and density can be derived from the general assumption that social structures evolve under the constraint of minimizing the total societal time expended in their operation.

Territorial subdivision results from the necessity for people to travel between dispersed residences and some central place (for example, a county seat) under limiting conditions of time (the 24-hour day) and time-saving technology (the average velocity of the means of transportation). If territorial divisions are too large, portions of the population will not be able to interact with a center. If divisions are too small, the cost of maintaining the centers would be unnecessarily high, assuming there were enough local resources to maintain them at all. The theoretical derivation will develop equivalencies between these opposing cost factors and societal time expenditure, determine the condition under which total time expenditure would be a minimum, and show that the negative size-density relationship follows from this condition.

Imagine an undifferentiated plane which is to be divided into territorial units, each containing a center designed to serve the population associated with it. Select an imaginary unit and call its area A and its population P. Now let S represent the average travel distance to the center, given the distribution of the population within the unit. This average distance, divided by the velocity of the means of transportation v, gives us the average travel time expended by the population in using the center.

Maintenance of the center and provision of its services to the population will require a further time expenditure, both in direct man-hours of work and in the form of indirect costs paid by the population to support such work. If we let h represent the time cost of maintaining the center, divide this cost by the total population, and add the result to the term S/v, we obtain the expression

$$T = S/v + h/P \tag{1}$$

where T is the average societal time expended in using and maintaining the service center of the territorial unit.

Since our task is to find the area which will minimize average time expenditure, we must introduce A in both right-hand terms of Eq. 1. Simple dimensional analysis (*2*) suggests that the average distance S will be proportional to the square root of the area A, regardless of the shape of the territorial unit. Thus, with g as the constant of proportionality, we have the substitution $S = g\sqrt{A}$ for the first term. The constant has been evaluated for certain regular polygons which occur frequently in the study of spatial relationships (*3*); its exact value will not be essential in the present derivation. The definition of density, D, as population per unit area ($D = P/A$) permits substitution of AD for P in the second term of Eq. 1 to yield

$$T = g\sqrt{A}/v + h/AD \tag{2}$$

from which we obtain the derivative

$$dT/dA = g/(2v\sqrt{A}) - h/A^2D \tag{3}$$

which, set equal to zero and solved for A, gives us

$$A = (2vh/gD)^{2/3} \tag{4}$$

as the condition under which T will be a minimum (the second derivative of Eq. 2 can be shown to be greater than zero).

Holding v, h, and g constant, we can obtain a linear form of Eq. 4, relating areal size to density,

$$\log A = K - 2/3(\log D) \tag{5}$$

where K is the log of $2vh/g$ to the two-thirds power. Equation 5 readily lends itself to empirical test with a least-squares estimator to determine the slope relating log-size to log-density.

Such an analysis has been carried out for 98 modern nations (*4*). While the slopes for individual nations vary somewhat around the expected $-2/3$ value (and in some cases the number of subdivisions within a nation was too small to permit adequate statistical test), the aggregated 1764 political subdivisions did yield a regression slope between log-size and log-density of −0.66, a result which very clearly conforms to the theoretical expectation developed here. It would thus seem that the general assumption, that social structures evolve in such a way as to minimize the societal time which must be expended for their operation, is sufficient to account for the observed empirical relation between size and density.

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3. The value of g for all n-sided regular polygons, including the circle as a limiting case, can be determined as follows: Let the centroid of the n-gon be the origin of the polar axis bisecting one of its central angles, forming the angle ϕ ($= \pi/n$) with an adjacent radius R of its circumcircle. The axis, radius, and n-gon edge will form a triangle with area equal to $\frac{1}{2}R^2\cos\phi\sin\phi$. The average distance to the center of the n-gon will then be given by

$$E(r) = \frac{2}{R^2\cos\phi\sin\phi}\int_0^{\phi}\int_0^{\psi} r^2\,dr\,d\Theta$$

where $E(r)$ is the expected value of the distance between the origin and a random point (r, Θ) within the triangle, and $\psi = R\cos\phi/\cos\Theta$, the upper limit of r given Θ. Evaluating the integral gives

$$E(r) = \frac{R}{3}\left(1 + \frac{\ln(\sec\phi + \tan\phi)}{\sec\phi\tan\phi}\right)$$

which, divided by the square root of the area of the n-gon, gives the value of g. In practice, the only n-gons which attract the attention of regional scientists are the triangle, square, hexagon, and circle; the first three permit tight packing in lattice structures and the fourth is useful in ideal constructions around a central point. The value of g is 0.4037 for the triangle, 0.3826 for the square, 0.3772 for the hexagon, and 0.3761 for the circle.
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Head Capsule Transmission of Long-Wavelength Light in the Curculionidae

Abstract. *The head capsule of many weevils contains a region that transmits only far-red and near-infrared light. In the alfalfa weevil,* Hypera postica, *this extraocular cutoff filter evidently works in conjunction with the compound eyes, enabling the insect to use visual cues in locating or recognizing its host plant.*

Most members of the insect family Curculionidae, commonly known as weevils, possess a unique morphological characteristic which seems to be part of a newly discovered receptor system sensitive to long-wavelength light. In these insects all or part of the head capsule acts as an optical filter, which transmits only far-red and near-infrared radiation (650 to 900 nm) to the underlying tissues. Receptors sensitive to this portion of the electromagnetic spectrum are virtually unknown among arthropods (*1*).

Fig. 1. Transmission of light through the head capsule of an alfalfa weevil as recorded on infrared-sensitive film.

The head capsule does not appear capable of transmitting any light when an intact weevil is examined by reflected light, and it is probably for this reason that the structure has, until now, been undetected. The cephalic skeleton is heavily sclerotized, usually dark in color, and covered by a stubble of acuminate scales. But if the head is removed and illuminated from behind with a tiny spot of white light, its translucent region will assume a deep red hue (Fig. 1).

The head capsule filter was first discovered in the alfalfa weevil, *Hypera postica* (Gyllenhal), where it forms a large crescent encompassing the vertex and genae of the head. It extends forward to the level of the ocular sutures at the top of both compound eyes, and narrows to a point on each side of the head near the subgenae. The rest of the head, including the frons and the snout, is nearly opaque. All other members of the genus *Hypera* have similar structures, and a survey of 39 other species representing 28 subfamilies revealed 25 species with substantial red transmission and an additional 12 species with only moderate absorption. In a few cases, notably *Anthonomus grandis* Boheman (cotton-boll weevil), the entire head, including the beak, the frons, and the subgenae, is translucent to red light. A few other insects may perceive light which passes through the head (*2*), but in every case the cephalic exoskeleton is lightly pigmented and transmits a wide range of visible light.

In the alfalfa weevil, the distinctive pigmentation (*3*) absorbs about 90 percent of incident light at 590 nm, but only about 10 percent at 640 nm. Most other species examined show a similar cutoff range; however, the Curculioninae and the Anthominae transmit a little more yellow (90 percent absorption at 530 nm and 10 percent at 600 nm), and the white pine weevil (*Pissodes strobi* Peck) is translucent over a wide range of visible light (*4*).

In some of these species there is a remarkable correlation between wavelengths transmitted through the head capsule and the phototactic action spectrum. Under controlled conditions, the starved alfalfa weevils and cotton-boll weevils are attracted to green light (500 to 550 nm) and to wavelengths in the far-red and near-infrared region. However, when the vertex of each weevil's head capsule is covered with an opaque lacquer, the response to red light is almost completely eliminated (*5*). In fact, most weevils which have a red head capsule filter are attracted to some extent by wavelengths in the range 700 to 900 nm. Those which lack the filter, such as the black vine weevil, *Brachyrhinus sulcatus* (Fab.), do not respond to wavelengths greater than 630 nm. Apparently light passing through the red filter influences positive phototaxis by changing the sensitivity of the compound eyes or by stimulating an extraocular receptor.

Since the cephalic filter is so wide-

Table 1. Response of alfalfa weevils in the behavioral bioassay. The decrease in the visual response to alfalfa in the treated groups was evaluated by the χ^2 test with 1 degree of freedom.

Treatment	Number attracted		χ^2
	Alfalfa	Control	
None	438	162	
Clear shellac	215	71	0.49*
Lampblack and shellac	161	108	14.90†

*Not significant. †Significant at the 99 percent level.