the gas version—which is generically related to the Rover program that developed a nuclear rocket engine for which there was no apparent practical use—the total amount of fissile material in the core might barely be enough to make one bomb. Such a reactor scheme, according to William Kitterman at ERDA, might be the only way to make the benefits of nuclear power available to any nation, regardless of its intentions, without also transferring a sufficient quantity of material with which to develop a nuclear arsenal. The homogeneous reactor proposal is a radical one—it would mean throwing out virtually all the nuclear technology that is now used.

The set of alternatives that have come to the surface during the last year of heightened public debate over proliferation is no doubt incomplete, and much of the analysis of the benefits of various proposals is in flux. Not the least of the problems is to decide where the proliferation line should be drawn, since there are now four or five different sets of criteria being used to distinguish where the risk of proliferation becomes too great. But the fact that reactors have never before been designed with proliferation considerations in mind is beginning to permeate the consciousness of many people, and "the technical questions are so fascinating that they are getting the community hooked," says Ted Taylor at Princeton University. What is becoming clear, says another well-respected nuclear engineer, is that there are a great many alternatives between the light water reactor, which "wastes uranium like crazy," and the liquid metal fast breeder, which is probably the most proliferation-prone technology yet conceived.

—WILLIAM D. METZ

This is the second of two articles.

Catastrophe Theory: The Emperor Has No Clothes

Catastrophe theory is one of the few areas of mathematics research that have surfaced from the mathematics community and caught the fancy of the press and the general public. The theory and its application were the subject of the first article on mathematics published in *Newsweek* in at least 7 years, were subsequently the subject of a *Scientific American* article, and have been praised by a number of mathematicians as well as by many investigators in other fields of science, including the social sciences. This attention, however, may have been premature, according to a number of eminent mathematicians who are harshly critical of catastrophe theory models.

The originator of catastrophe theory, René Thom of the Institut des Hautes Etudes Scientifiques in France, says in *New Scientist*, "Catastrophe theory is not a 'scientific theory'; it is a language and as with ordinary language every author will use it to his own taste and with his own 'style.' " But mathematicians stress that models based on catastrophe theory make use of a well-established body of mathematics, that was itself developed by Thom and others. This mathematics constitutes a study of how solutions to a system of equations vary when certain parameters that appear in the equations are perturbed. When the parameters vary, the solutions can be pictured as jumping from one value to the next. These jumps, or discontinuities, are said to be "catastrophes."

Catastrophe theorists believe that many discontinuous phenomena, such as the crash of a stock market or the sudden attack of an angry and frightened dog, lend themselves to descriptions in terms of these models. Catastrophe theory has been applied, during the past 6 years, in an enormous number of disciplines, including physics, biology, sociology, political science, economics, linguistics, and psychology. A number of investigators are now trying to use these models to study practical problems. For example, some psychologists in England are using a catastrophe theory model of the behavior of prisoners to advise prison authorities and to make decisions about ways to prevent prison riots.

Perhaps most disturbing to the critics are the claims made by the proponents of catastrophe theory models. Thom, for example, says that these models will provide such insight that, in the future "only mathematicians will have the right to be intelligent." The theory has been lauded as "an intellectual revolution in mathematics—the most important development since calculus." E. Christopher Zeeman of the University of Warwick in England, one of the chief publicists for catastrophe theory models, writes in *Scientific American*: "A mathematical method [catastrophe theory] for dealing with discontinuous phenomena has only recently been developed." In the opinion of many mathematicians, this statement ignores numerous developments, such as quantum mechanics, bifurcation theory, shock wave theory, and thresholds. Proponents of catastrophe theory models constantly stress their applicability to biology and the social sciences—fields in which other mathematical models have so far been only minimally successful.

Partly in response to the extraordinary publicity these models have generated and the extravagant claims made for their applicability to practical problems, some mathematicians have been examining the theory more closely. They have concluded that many of the statements and claims about the models are exaggerated, even irresponsible, and that the mathematical reasoning behind them is often sloppy or blatantly wrong.

Models Are Criticized

The current attack on catastrophe theory models is being led by Héctor Sussmann of Rutgers University in New Brunswick, New Jersey, and his associate Raphael Zahler (who made a very harsh criticism of catastrophe theory models of the propagation of nerve impulses). Sussmann, Zahler, and their supporters stress that they are criticizing not the mathematics underlying catastrophe theory but rather the models themselves. The heart of the criticism, according to John Guckenheimer of the University of California at Santa Cruz, is that Zeeman and Thom, the principal developers of these models, have a "real reluctance to get their hands dirty with the scientific details of the applications. Even more amazing to some is their cavalier attitude toward mathematics. Thom is hardly interested in proving theorems and Zeeman has used the work 'theorem' more loosely than any other mathematician I know."

Sussmann, focusing on the models of Zeeman, makes the feelings articulated by Guckenheimer more specific. Zeeman's models are the most numerous, the most widely known, and the most extensively criticized. Zeeman, contacted about Sussmann's harsh criticisms, said he was unfamiliar with the details of the criticisms; when they were described, he gave no direct or specific rebuttals to any of them.

One persistent problem in these models, Sussmann says, is that Zeeman plays on the propensity of readers to define

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words in such a way as to make a statement correct. Zeeman sometimes uses a word in a proof in several different ways, whereas if one definition of the word were adhered to, the proof would make no sense. For example, Zeeman uses the word "continuity" twice in an embryology model, with two different meanings. In a model of aggression, he uses two meanings for the word "attack."

A second problem is that, all too often, statements are "proved" true by assertion, or on the basis of what are "silly reasons," Sussmann says. In Zeeman's embryology paper, for example, a frontier, or boundary between two kinds of tissue, is said to stabilize and this is "proved" by Zeeman's assertion that it does. But Sussmann and Zahler contend that it need not stabilize; in fact, they have found an example that, Sussmann says, is consistent with all of Zeeman's hypotheses and choices of meaning but for which the frontier does not stabilize. In the same paper, Sussmann reports, Zeeman "proves" that the frontier moves if nothing exceptional happens by providing a ridiculous reason. After eliminating what he considers to be irrelevant mathematical jargon, Sussmann translated Zeeman's "proof" into English as follows: "If the frontier did not move, that would be exceptional. Hence it moves, Q.E.D."

Another serious problem with Zeeman's models, according to Sussmann, involves a misuse of the mathematics behind catastrophe theory. Sussmann says that Zeeman implicitly (or explicitly, in the case of his *Scientific American* article) assumes that it is possible to deduce the shape of the entire critical set of a family of functions from knowledge of its behavior at some points. This assumption is crucial to Zeeman's models, and Sussmann says, "it is the only step that involves nontrivial mathematics. . . . What bothers me is that Zeeman is enough of a mathematician to know that what he is saying is false. Why does he say it then?"

In view of this unconventional method of reasoning, many mathematicians find it not too surprising that these models lead to untenable conclusions. These conclusions, however, are ignored by proponents of catastrophe theory. According to Sussmann, Zeeman's stock market model implies that a purely speculative market will never crash. His model of aggression leads to the conclusion that if a dog that is not afraid is made progressively angrier, the dog either will not attack or will "attack gradually." Which of the two is predicted depends on how the word "attack" is defined. Sussmann points out that the prediction that the dog will not attack is clearly false, and the prediction that it will attack gradually is meaningless. In his opinion, the reason these models often lead to such absurd conclusions is that "the catastrophe theory being used has nothing to do with the phenomena being studied, and there is no reason to expect that it might."

Sussmann did not set out to be the principal critic of catastrophe theory models, but he fell into this role almost by accident and was buoyed on by an overwhelming response from mathematicians. He began by investigating catastrophe theory and its applications in order to conduct a seminar at Rutgers. He was subsequently invited to give a talk on catastrophe theory at last year's biennial meeting of the Philosophy of Science Association. By that time, Sussmann had become highly critical of the applications of catastrophe theory and fascinated with the sociology of its growing popularity. He said what he thought at the meeting and was soon being invited to speak to mathematicians at universities throughout the country.

People who have heard Sussmann speak say he receives an enthusiastic response. "Sussmann convinces everyone," says Joseph B. Keller of the Courant Institute of Mathematical Sciences of New York University (Keller is spending this year at Stanford University). "Even people who supported the applications of catastrophe theory walk out of Sussmann's lecture saying, 'How could anyone believe that?' "

A list of those who have expressed verbal or written support for Sussmann's position reads like a who's who in American mathematics. It includes Stephen Smale of the University of California at Berkeley, a recipient of the Fields Medal—the most prestigious award in mathematics. Other supporters are a well-known mathematician at MIT (who wishes to remain anonymous) who says, "It's about time someone told us the emperor has no clothes," and Mark Kac of Rockefeller University, who says that the applications discussed in the *Scientific American* article "represent the height of scientific irresponsibility." Kac finds the applications to the social sciences "exaggerated and not wholly honest."

Why Were the Models Hailed?

If the models are so poorly justified it seems reasonable to ask why so many scientists became so enthusiastic about them. Keller believes that one reason some mathematicians found these models so attractive is that "They offer the hope of doing something useful without knowing anything but mathematics." After 30 years of teaching applied mathematics, Keller is convinced that most mathematicians are held back from applying their work by their lack of knowledge of the field of application; catastrophe theory models offer the delusion that such knowledge is unnecessary.

Jonathan Rosenhead of the London School of Economics speculated in *New Scientist* that social scientists may be attracted to catastrophe theory because "the mathematical apparatus manages to impart a scientific authority which the social theory along could not bear."

Other critics believe that scientists are seduced into thinking they understand models based on catastrophe theory because they are presented with an abundance of pictures and an absence of equations. The scientists assume that the mathematics underlying these models can lead to significant new insights into the phenomena modeled. Expositions of these models generally include allusions to deep mathematical results, proved elsewhere. Thus, scientists are led to believe that the appropriateness of catastrophe theory to a model of the phenomena is fully justified.

Many mathematicians now say they find the difficulties with the catastrophe theory models easy to appreciate. Even if some were overanxious to see these models succeed, the fact remains that the catastrophe models have been, for years, nearly immune to criticism. Sussmann is intrigued by this phenomenon and has suggested some reasons for it.

It is generally agreed that social scientists and others in fields in which catastrophe theory is applied hesitate to criticize the models because they are intimidated by the mathematics. Sussmann believes that Thom, at least, does not intentionally try to use mathematics to intimidate, but that his words have that effect. For example, in writing for linguists, Thom used such terms as "catastrophe sets," "manifolds of finite codimension," and "inverse transformations."

Mathematicians will not usually admit to being intimidated by such jargon, but most say they are unfamiliar with Thom's mathematics; this is not the mathematics that applied mathematicians tend to use. Because Thom's mathematical work is considered outstanding (he won the Fields Medal, for example) and Zeeman is also a respected mathematician, mathematicians were reluctant to question their work.

Sussmann also points out that the sheer number and magnitude of the claims made by the advocates of catastrophe theory make criticism difficult. Advocates claim applications to so many fields that potential critics are overwhelmed. And they say things such as, "The method has the potential for describing the evolution of forms in all aspects of nature, and hence it embodies a theory of great generality." People may be dubious of such statements, but the audacity of the claims tends to make them think that something must be there. In truth, says Keller, "there have been zero concrete accomplishments."

The catastrophe theorists also hinder their critics by the way they phrase their claims, Sussmann points out. They deal in "ifs," "coulds," and "maybes." For example, Zeeman writes that "catastrophe theory could thus provide a mathematical language for the hitherto 'inexact' sciences." To counter this claim, the critic must show that catastrophe theory could not provide such a language—a task that is extremely difficult, if not impossible. By such means, catastrophe theorists put the burden of proof on their critics.

Another difficulty in criticizing applications of catastrophe theory, according to Sussmann, is the vagueness of the theorems and proofs. When they do not define terms, do not specifically prove claims, and are unclear about what they are actually doing, catastrophe theorists set up a smoke screen. They can later claim that their critics do not really understand their work.

Guckenheimer fears that the current criticism of catastrophe theory models may force mathematicians to divide into camps and may result in a loss of the content and insight that might be gained from the mathematics behind catastrophe theory. He says that "There is a real possibility that catastrophe theory will blossom into a discipline unto itself under Zeeman's charismatic leadership. Every imagined discontinuity will be fit by a cusp surface with the proper slapdash ad hoc assumptions. If this happens, the important features of catastrophe theory may well get lost."

Keller has a somewhat different opinion of the possible effects of the current criticism. He feels that Sussmann is doing a service to society by pointing out the problems with these models since "overblown claims about the possibility of doing something with mathematics have a whiplash effect. Afterwards people think mathematics cannot be usefully applied." Although it is too late to nip the claims for catastrophe theory in the bud, Keller and others hope that the catastrophe theory craze will now began to wane.—GINA BARI KOLATA