Form and Origin of the Parry Arcs

Computer simulation shows how arcs of light above and below the sun are produced.

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During his voyage in search of a northwest passage in 1819 and 1820, Parry recorded his observations of flora, fauna, native people, geology, hydrology, and meteorology (*1*). He made sketches and gave detailed descriptions of a number of sun halo displays. Among other things, he described an arc above the sun that bears his name: the Parry arc. Subsequent sightings of the Parry arc are not very common, but a number have been reported in the literature, and a few good photographs of the effect have been published (*2*).

Parry arcs are explained in terms of light passing through the alternate side faces of a hexagonal-prism ice crystal, as illustrated in Fig. 1. We call such a crystal, whose length is great compared with its width, a pencil crystal, after its resemblance to a common wooden pencil. Crystals of this shape are known to occur in the atmosphere (*3*). Three of the side faces of the hexagonal prism have been extended in Fig. 1 to illustrate that a light ray, passing through the crystal as shown, is deviated as if it passed through a 60° prism of ice. The deviation of a light ray from the sun resulting from such a passage depends on the orientation of the ice crystal, but the minimum angle of deviation is about 22°. To see a ray of light from the sun which has been deviated by 22°, one must look in a direction 22° away from the direction of the sun, as illustrated in Fig. 2. The result of such refraction by a large number of pencil-shaped ice crystals with random orientations is a halo around the sun, with the angular radius of the inner edge equal to 22°. Pencil-shaped crystals with lengths greater than 20 micrometers, however, do not tend to assume random orientations as they fall through still air, but tend to become oriented with their long axes horizontal (*4–6*). The distribution of crystal orientations that results from this effect is illustrated in Fig. 3, where the ice crystals have nearly horizontal axes but random rotations about the axes and random orientations of the axes in the horizontal plane. Sunlight passing through such a distribution of ice crystals gives rise to a circumscribed halo (*7*) which, for sun elevations less than about 40°, takes the form of an upper and a lower tangent arc—arcs tangent to the 22° halo directly above and below the sun.

It has been generally thought that Parry arcs result from sunlight passing through pencil-shaped ice crystals whose orientations are even more restricted than those producing the circumscribed halo. It has been suggested that not only is the crystal axis confined in a horizontal plane, but rotation around the axis is restricted so that a pair of the faces (top and bottom) remain nearly horizontal (Fig. 4a) or, perhaps, so that a pair of the faces (side faces) remain nearly vertical (Fig. 4b). The aerodynamic forces for either of these orientations would seem to be quite small, and there is some disagreement about which orientation would be the most stable. Tricker (*5*) suggests that an ice crystal of the form shown in Fig. 4c would give the desired orientation effect, although we know of no direct evidence for the existence of such crystal forms. It is possible that crystals responsible for the effects we discuss in this article are the arms of a snowflake-type crystal, where the plane of the entire flake determines the orientation. Such a flake, in the appropriate size range, would tend to fall with its plane horizontal. Or the crystal may be part of a cluster of crystals (*3, 8*) nucleating from the same seed. The resulting orientation would be determined by the geometry of the cluster.

In this article we explore the nature of the arcs that would result from sunlight passing through crystals oriented as shown in Fig. 4, a and b. By comparing computer simulations with photographs, we will try to deduce which (if either) of these distributions appears in nature. If we can explain the appearance (both the shape and intensity distribution) of Parry arcs at one sun elevation, we should then be able to use the same model to predict the appearance of the arcs for other sun elevations where the resulting sky effects may not have been identified as Parry arcs.

Computer Simulation Method

The basic calculation in this investigation is determining the direction of a light ray after it passes through an ice crystal, given the elevation of the sun and the orientation of the crystal. In principle, this is a simple problem; we need only apply the law of refraction (Snell's law or Descartes' law, depending on your national origin) twice, once when the ray enters the crystal and once when it exits. In practice, the resulting general expressions are quite complicated. The procedure we follow is to pick a crystal orientation and, for a particular sun elevation, calculate the direction of the refracted ray. We then consider at what direction in the sky that crystal would have to be located to send the light to the observer's eye (see Fig. 2). The crystal is located on a plane perpendicular to the line between the observer and the sun, and its position coordinates are fed to a plotter, which plots a spot. The simulation process repeats this procedure many times, each time giving the crystal another of the orientations we expect to find in the skyful of ice crystals.

To correctly represent the intensity distribution of the resulting effect, we must take account of several intensity factors. There are reflection losses at the entrance and exit faces of the crystal, which are calculated exactly by using the Fresnel expressions for unpolarized light (*9*). (Total internal reflection gives a transmission factor of zero.)

The cross-sectional area of the light beam incident on the first crystal face gives another relative intensity factor. This area varies as the cosine of the

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angle of incidence on that face. In general, however, not all of the light that comes in the entrance face strikes the exit face inside the crystal, so we calculate the fraction of the cross section of incident light that corresponds to light which gets through the exit face. The only approximation made in considering this factor is to neglect the effect of the ends of the crystal; that is, our expressions would be exact in the limit of a crystal whose length is very great compared to its diameter (*10*) (an infinite aspect ratio). All of these factors yield an overall relative-intensity factor which lies between 0 and 1.

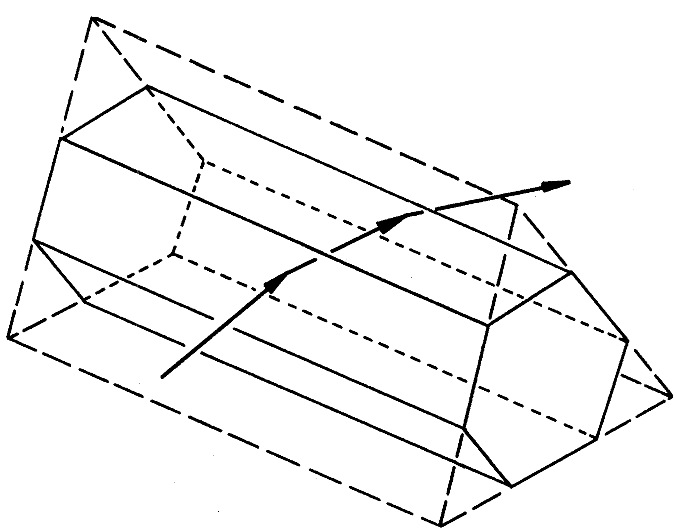
Before plotting a point we compare the calculated intensity factor with a random number between 0 and 1 and plot the point only if the intensity factor is greater than the random number. Following this procedure, for example, only one-tenth of the spots associated with rays having a relative intensity factor of 0.1 would be plotted. Thus, on the final spot diagram the spot density gives a measure of intensity.

In general, the orientation of a crystal is determined by specifying three angles. In the case treated here, one of these angles is fixed (by restricting the axis to the horizontal plane), the second is allowed to vary through 360° (rotation of the axis in the horizontal plane), and the third is allowed to change over a restricted range (rotation about the crystal axis) for the Parry arc simulations. The angles are determined independently by a random-number-generator subroutine, which gives random angle values within the desired limits.

All of the results shown here are for the sun considered as a point source. Since the sun actually has an angular diameter of about ½°, our simulations should be smeared over this angle, but this would not significantly alter their appearance. The simulations are done for only one wavelength, that of red light, for which we use an index of refraction for ice of 1.309.

Circumscribed Halo

Ice crystal orientations that give rise to Parry arcs are subsets of the orientations that give rise to circumscribed halos (or tangent arcs). To distinguish Parry arcs from circumscribed halos, it is important to examine both effects. Figure 5a shows the simulation for circumscribed halos for sun elevations ranging from 0° to 70°. Each of these plots resulted from the trial of 50,000 crystal orientations, although typically the intensity factors reduced the number of points plotted to about 3,000. The shapes and intensity distributions of these simula-



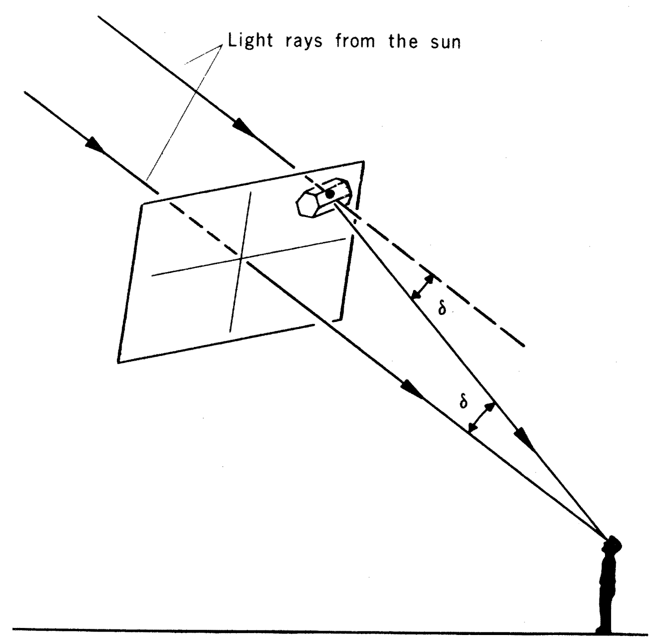
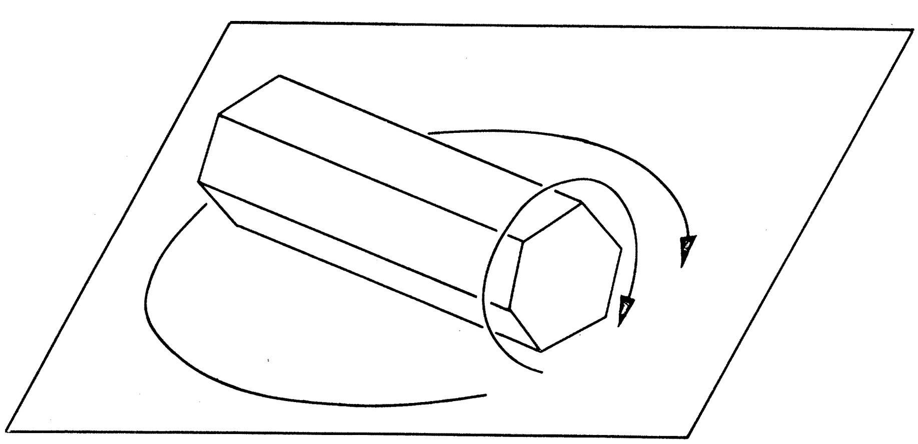


Fig. 1 (left). Path of a light ray through a hexagonal ice crystal. Three crystal faces are extended to show that the refraction is equivalent to that of a 60° prism. Fig. 2 (right). Observer viewing a ray of light refracted by an ice crystal.



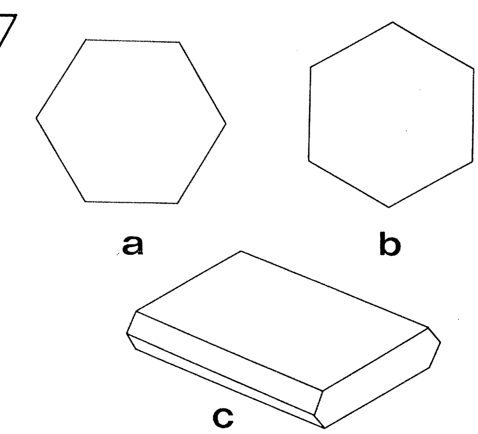


Fig. 3 (left). The orientations of pencil crystals that give rise to the circumscribed halo. Fig. 4 (right). (a and b) Two possible orientations of pencil crystals. (c) A suggested ice-crystal form.

tions agree quite well with many observations, giving us confidence that we understand the basic process that gives rise to upper and lower tangent arcs and circumscribed halos.

Parry Arcs

The arcs simulated from the ice crystal orientations in Fig. 4, a and b, are shown in Fig. 5, b and c. We call the results from the distribution of Fig. 4a Parry arcs, and those from Fig. 4b alternate Parry arcs. The fact that both of these simulations are shown does not mean that both distributions are found in na-

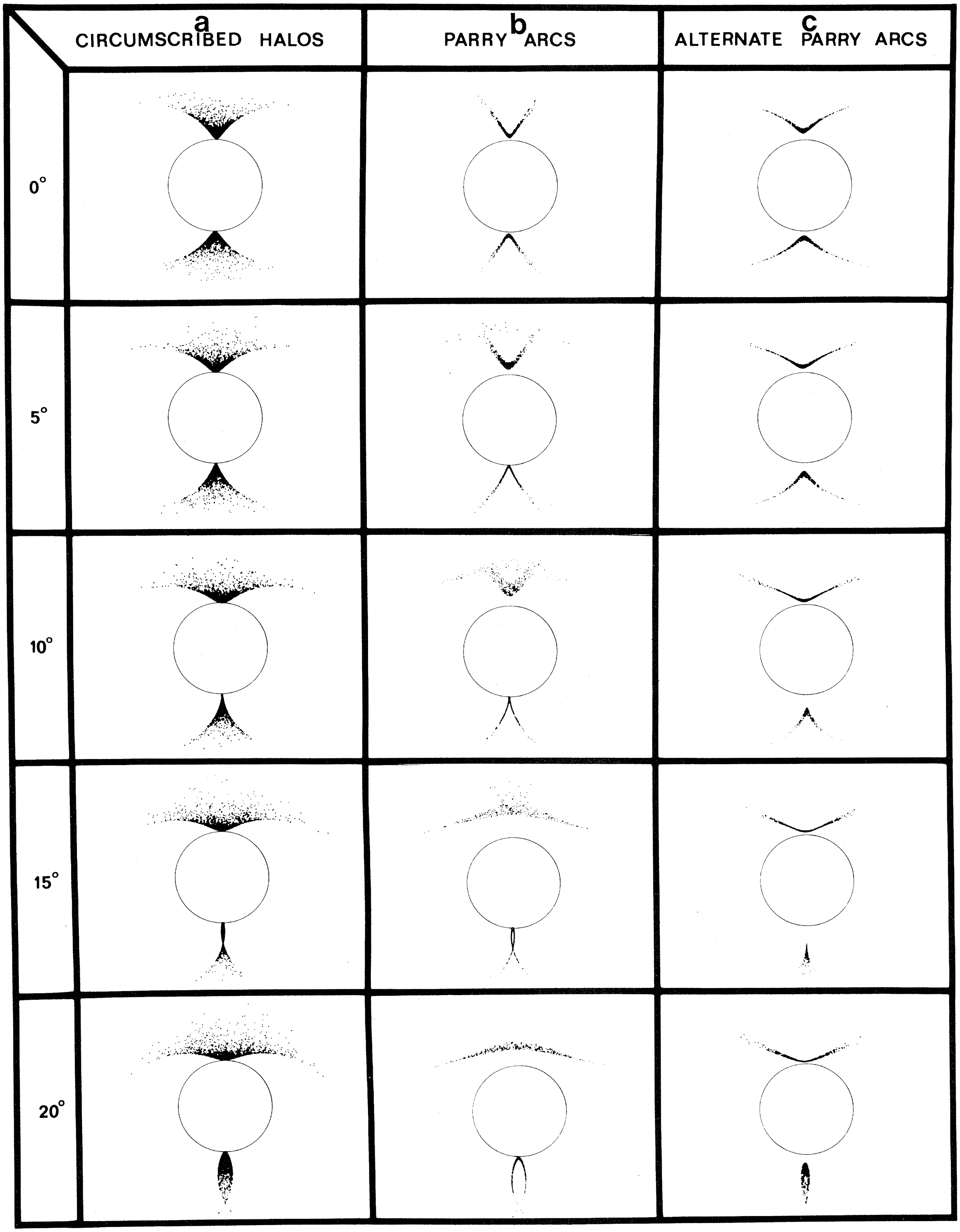
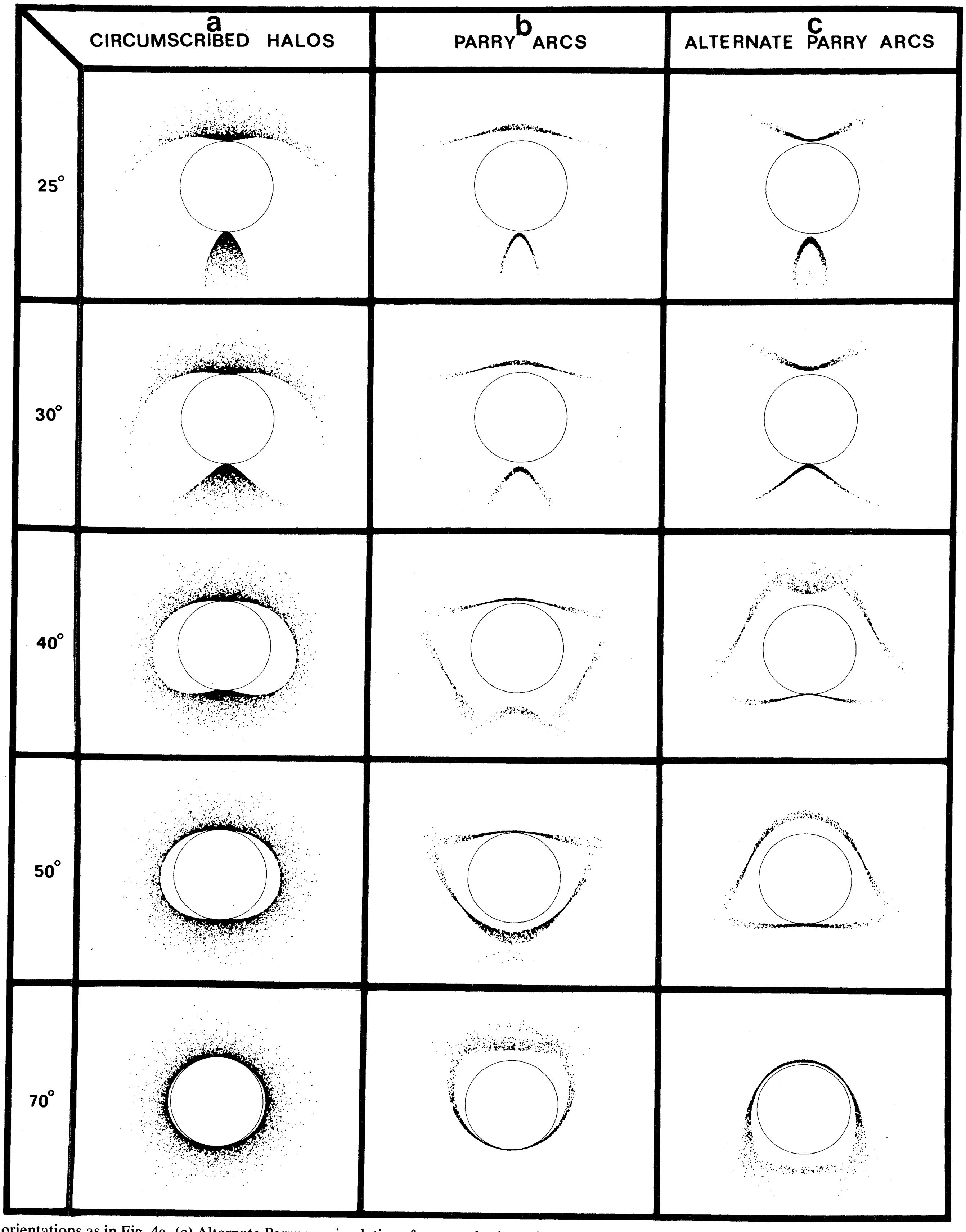


Fig. 5. (a) Tangent arc and circumscribed halo simulations. (b) Parry arc simulations for crystal

ture; one of the objects of the investigation is to determine which distribution does exist.

The Parry arc simulations (Fig. 5b) result when the top face of the crystal is tilted ± 3° from the horizontal, with the crystal axis remaining exactly horizontal. With this distribution of orientations there can be two Parry arcs above and two below the sun, although not all will occur for all elevations of the sun. The formation of the arcs above the sun (upper Parry arcs) is illustrated in Fig. 6a. They are labeled as types 1 and 2, as suggested by Goldie (*11*). Putnins (*12*)

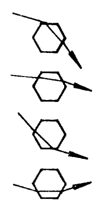


orientations as in Fig. 4a. (c) Alternate Parry arc simulations for crystal orientations as in Fig. 4b.

numbered the faces of the crystal as shown in Fig. 6c and identified a ray by giving its entrance and exit faces, so that the type 1 and type 2 arcs in Putnins' notation would be the 1-3 and 6-2 arcs (*13*). In the simulation, the arc seen at low sun elevation (approximately 0° to 15°) would be the type 2 arc and that seen at higher elevation (approximately 10° to 90°) the type 1 arc. We would expect to see them both appear together only for a sun elevation of 10° to 15°. Similarly, the two lower Parry arcs would be described as type 3 and type 4 in Goldie's notation and as 6-4 and 5-3 in Putnins' system.

Each of the four Parry arcs has been simulated with the same number of trial orientations (3000) so their relative apparent intensities should be significant.

It seems to us that the naming of the various Parry arcs is not very satisfactory. Most of the optical effects of the sky are named after their appearance—for example, circumscribed halo, upper tangent arc, rainbow, circumzenithal arc, and so forth. The exceptions to this system are the effects named after people, such as the Parry arcs or the arcs of Lowitz. Since the name Parry arc is generally accepted in the literature, it would seem desirable to keep it. We would prefer, however, to identify the various forms of Parry arcs by names that describe their appearance, rather than by an arbitrary designation that has no intuitive connection either with the effects as they are observed or with the mechanism producing them. Looking at the simulation of the arcs (Fig. 5b), we see that the type 1 arcs are always concave toward the sun and the type 2 arcs are always convex toward the sun. A similar distinction can be made for the type 3 and 4 arcs. We propose to identify



Model	Goldie	Putnins	This article
	Type 1	1 - 3	Upper suncave
	Type 2	6 - 2	Upper sunvex
	Type 3	6 - 4	Lower suncave
	Type 4	5 - 3	Lower sunvex

Fig. 7. Various systems of nomenclature for the four Parry arcs.

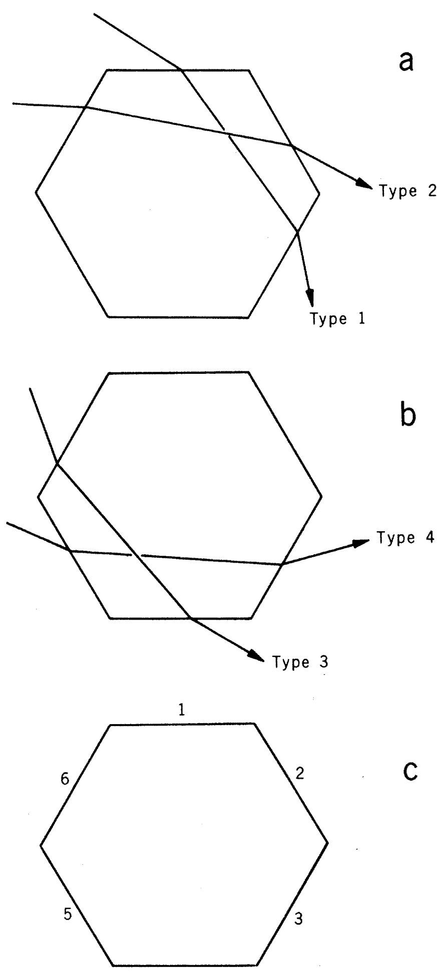
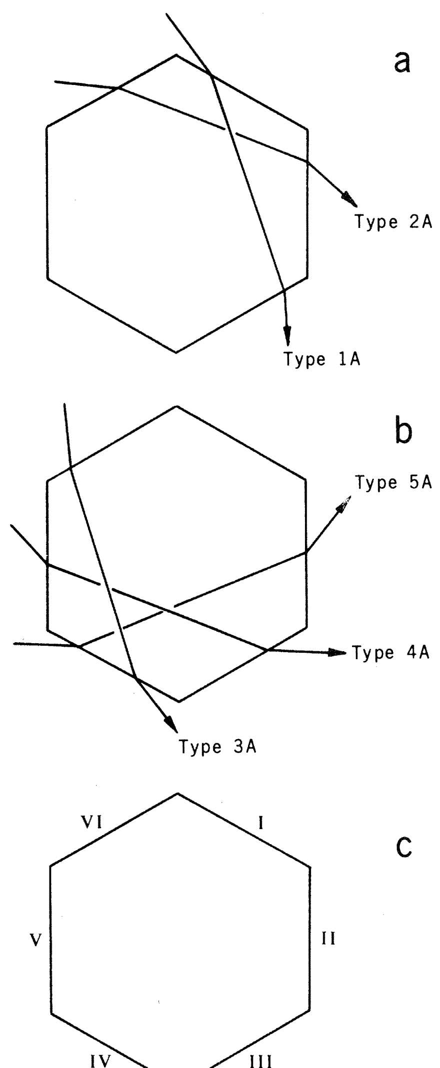


Fig. 6. Rays that produce the four Parry arcs.



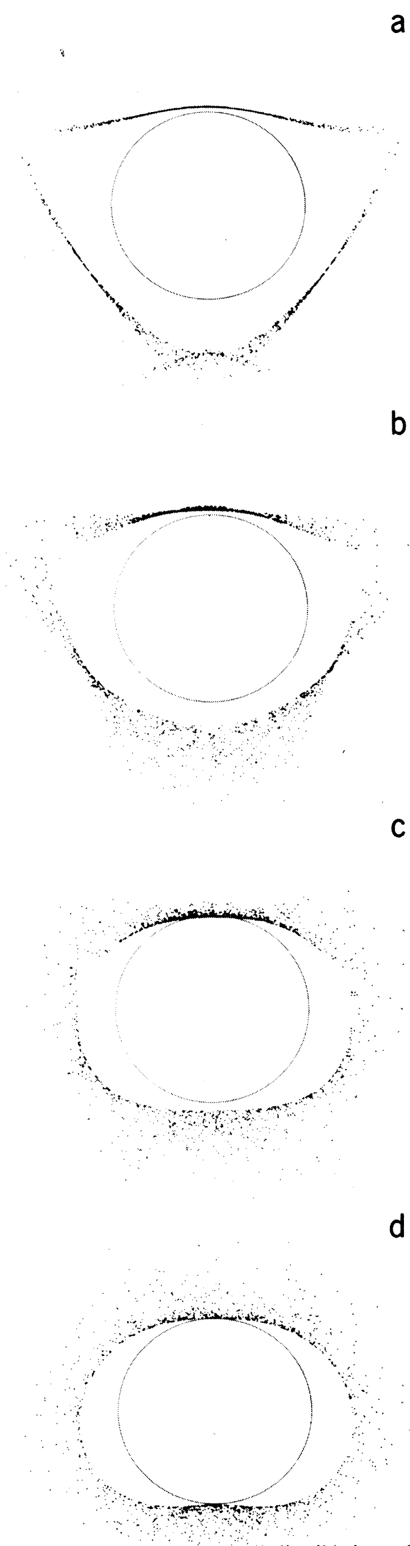


Fig. 8 (left). Rays that produce the five alternate Parry arcs. Fig. 9 (right). Simulations showing the change from Parry arcs to the circumscribed halo as the orientation of the crystals becomes poorer. In all distributions the crystal axes are horizontal. The top face of the crystal deviates from the horizontal position by up to 1° (a), 6° (b), 16° (c), and 30° (d). Simulation is for a sun elevation of 45°.

the arcs by their position above or below the sun and by their curvatures, calling them suncave arcs or sunvex arcs as they are either concave toward the sun or convex toward the sun. For example, in Fig. 5b for the sun elevation of 25° the upper arc would be called the upper Parry suncave arc and the lower arc the lower Parry sunvex arc. Figure 7 gives the description of the various arcs according to the three systems of nomenclature.

Alternate Parry Arcs

It has been suggested (*6, 12*) that pencil crystals might be oriented with their axes horizontal but with two side faces vertical (as in Fig. 4b). The arcs that would result from such orientations are shown in Fig. 8, a and b; there are two possible arcs above the sun and three below the sun. In Fig. 8, a and b, the rays are numbered by analogy with Goldie's designation, the A being added to indicate an alternate possible form of the arc. Putnins' description is illustrated in Fig. 8c; for example, the type 2A arc would be the VI-II arc.

The simulation of the alternate Parry arcs is shown in Fig. 5c for crystals having horizontal axes and a pair of side faces vertical (± 3°).

Discussion

Some photographs of arcs that we have seen identified as Parry arcs appear, by comparison with our simulation, to be upper or lower tangent arcs. Since the crystal orientations that lead to Parry arcs are a restricted set of the orientations that produce tangent arcs (or circumscribed halos), varying degrees of orientation can yield intermediate forms between Parry arcs and tangent arcs (or circumscribed halos). The Parry arc simulations of Fig. 5b result from maximum tilts of the top face ± 3° from the horizontal. Figure 9 illustrates the effect of this maximum tilt angle on the shape of the arc. As the maximum tilt angle increases to ± 30°, the Parry arc turns into the circumscribed halo.

Figure 10 is a photograph taken at Halley Bay in Antarctica. The picture was taken during the Antarctic night and shows several interesting effects associated with light from the moon. Of specific interest to us here are the two arcs above the 22° halo for the moon at an elevation of about 21°. By reference to the simulations of Fig. 5, it appears that the upper arc is the upper Parry suncave arc and the other is the upper tangent arc. This is typical of several photographs or reports we have seen for the sun at moderate elevations, where the Parry arc appears along with the tangent arc. It is possible that two sets of crystals coexist in the same region of the sky, giving rise to the different arcs, but it seems more likely that the effects arise from different layers of ice crystals located at different altitudes.

It is sometime difficult to distinguish between tangent arcs and Parry arcs without the intensity information provided by the simulations. For example, Fig. 11b shows a brilliant arc below the sun observed by Jayaweera and Wendler (*14*) for the sun at an elevation of 25°. There should be a difference in position between the upper edge of the lower tangent arc and the lower Parry sunvex arc of only about 1.5°. On the basis of position measurement, the observers identified the arc as a Parry arc. Figure 11 shows two simulations done to the same scale as the photograph (*15*). Figure 11a is of a lower sunvex Parry arc produced by pencil crystals with the top face horizontal (± 5°). The simulation of Fig. 11c is of the lower tangent arc (maximum tilt equal to ± 30°). It appears that the actual effect is somewhere between these two extremes, but closer to the lower tangent arc; that is, there is only a slight degree of ice-crystal orientation with respect to rotation around the horizontal axes.

A similar case (but one that leads to the opposite conclusion) is the photograph in Fig. 12b; the sun elevation here was determined to be 1.7°. We assumed that the upper sunvex arc in this picture was a tangent arc until we did the Parry arc simulations. Figure 12 also shows two simulations for this solar altitude. Figure 12a gives the result of a Parry arc distribution with maximum tilt of ± 3° and Fig. 12c gives the tangent arc simulation (*15*). The shapes and intensity distributions indicate that the arc in Fig. 12b is a Parry arc.

These examples, and all the others we have seen so far, indicate that Parry arcs are produced by ice crystals with horizontal axes and a pair of faces nearly horizontal, as illustrated in Fig. 4a. In most cases, predictions based on the two models (Fig. 4, a and b) are sufficiently

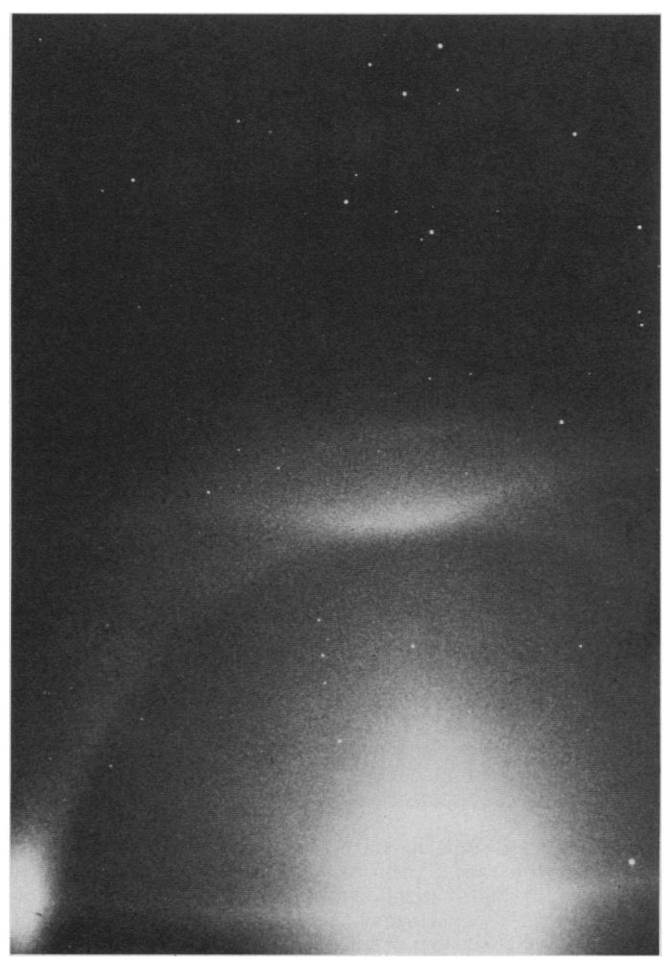
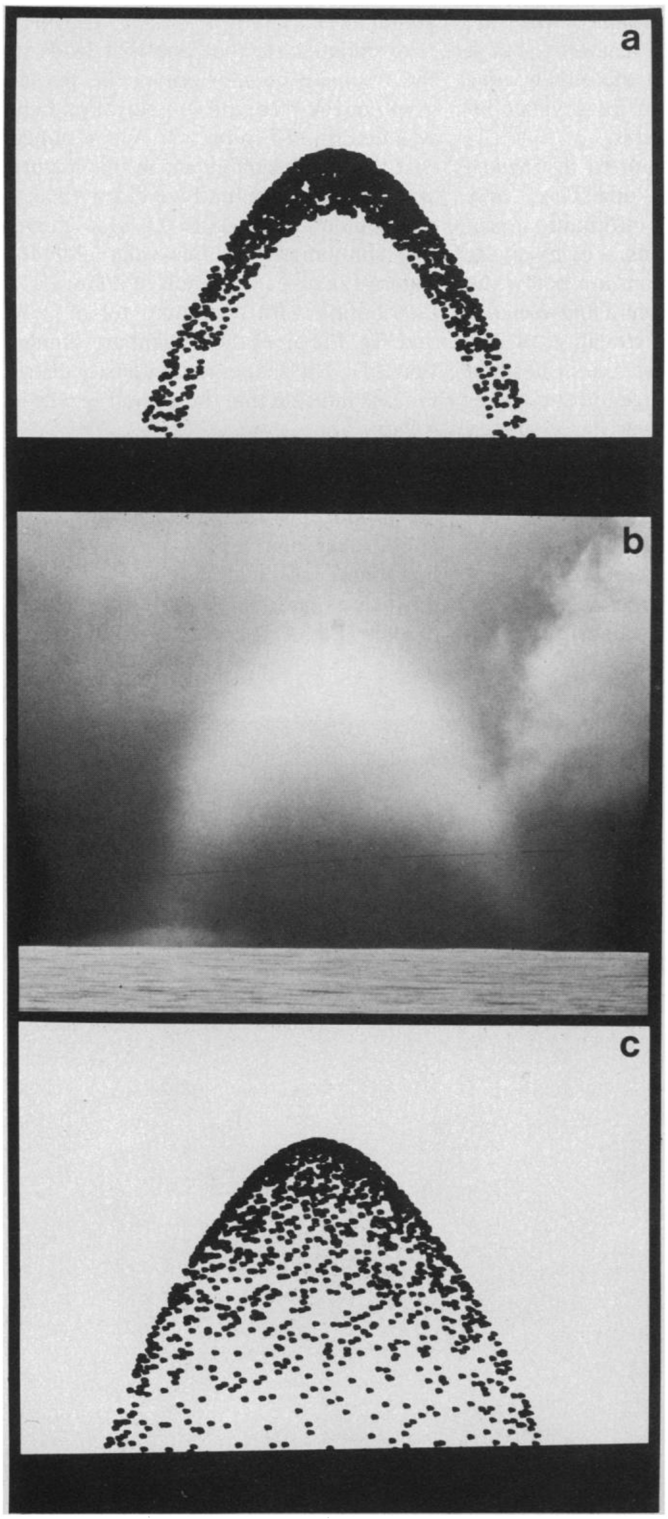


Fig. 10. Photograph showing the moon, the 22° halo, the upper tangent arc to the 22° halo, and the upper Parry suncave arc. Also shown is the parhelic arc, a parhelion, and a trace of the circumzenithal arc. (A purist in terminology would identify these as lunar effects by referring to a mooncave arc, parselenic arc, and so forth. It would seem to be an unnecessary complication.) [Photograph by C. Bienkowski]

different to distinguish between them in examining photographs of the effects. Observers who photograph such effects are urged to record the time and location, since from this information we can determine the sun elevation.

The simulations show some effects that we have not found reported, at least not identified as Parry arcs. For example, at a solar elevation of 40°, the lower suncave arc divides into two arcs at the sides of the 22° halo. It seems possible that these arcs may, at times, be confused with the arcs of Lowitz (*5*). When the sun is at an elevation of 60° to 70° an interesting "wedding ring" effect is predicted, but to our knowledge it has not been observed. We hope that these predictions will stimulate awareness, which will result in new observations.



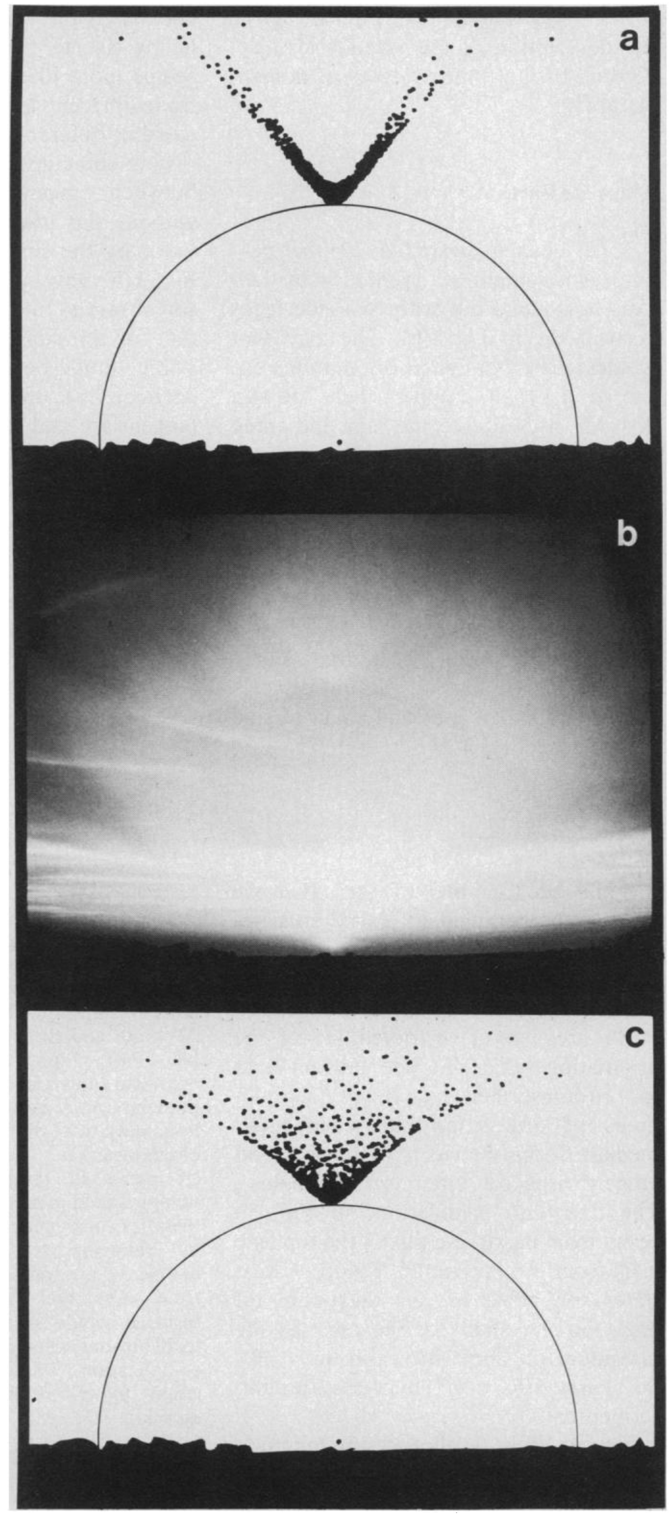


Fig. 11 (left). Arc below the sun. (a) Simulation of Parry arc resulting from pencil crystals with horizontal axes and top face horizontal (± 5°). (b) Photograph of arc and 22° halo (*14*). (c) Simulation of lower tangent arc resulting from pencil crystals with horizontal axes but no other selective orientation. Fig. 12 (right). Arc above the sun. (a) Simulation of Parry arc resulting from pencil crystals with horizontal axes and top face horizontal (± 3°). (b) Photograph of arc and 22° halo. (c) Simulation of upper tangent arc resulting from pencil crystals with horizontal axes but no other selective orientation. [Photograph by R. Greenler]

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15. In all the simulations discussed before this the reference plane shown in Fig. 2 was oriented perpendicular to the line of sight to the sun; that is, the simulations were sun-centered and would match the perspective of photographs in which the sun was centered in the picture. The simulations of Fig. 11 are made with the reference plane perpendicular to a line of sight to a point just below the arc, which is in the center of the photograph, to match the perspective of the photograph. Similarly, the simulations of Fig. 12 are made to match the perspective of the photograph in that figure.

16. One of the authors (J.R.M.) received summer support from the National Science Foundation's Undergraduate Research Program. Computer support was provided by the graduate school of the University of Wisconsin–Milwaukee, and publication costs were supported by grant DDP 76-18593 from the National Science Foundation.

Economic Models in Ecology

The economics of resource allocation provide a framework for viewing ecological processes.

David J. Rapport and James E. Turner

Ecological processes have traditionally been studied from several vantage points. One approach focuses on energy flows through ecological communities from primary producers to consumers at higher trophic levels (*1*). Another approach considers species interactions in terms of population dynamics (*2*). A third explores the geographical distribution of species and the relationship between species diversity and area (*3*).

None of these approaches, however, explicitly address what some (*4, 5*) have regarded as one of the central problems of ecology—the ways in which scarce resources are allocated among alternative uses and users. This question is, of course, fundamental to economic thinking (more specifically to microeconomic theory) and it is for this reason that we have recently seen the introduction of essentially economic models and modes of thought in ecology (*6–21*). In some cases economic models and concepts have been transferred directly across disciplinary boundaries (*5, 7, 10–14, 16–18*), while in other instances ecologists have rediscovered economic principles in an ecological context (*6, 8, 9, 15, 19, 20, 21*).

These developments have occurred in a number of diverse areas of ecology, including models of optimal foraging (*6–8, 11, 12, 15, 16, 21*), reproduction strategies (*9, 12, 19, 20*), territoriality (*10*), altruism (*20*), and social caste systems (*17*). Viewed as a group these and other recent contributions may lay the foundations for an approach to ecology in terms of an economics of natural communities. In this article we review how economic analysis has contributed to our understanding of ecology and show how a comprehensive framework for economic analysis of ecological phenomena may emerge.

That economic principles are relevant to the study of ecology is by no means a new idea. H. G. Wells, Julian Huxley, and G. P. Wells (*22*) in their treatise *The Science of Life* defined ecology as biological economics or an extension of economics to the whole world of life. For these authors, economics is "the science of social subsistence, of needs and their satisfactions of work and wealth. It tries to elucidate the relations of producer, dealer, and consumer in the human community and show how the whole system carries on. Ecology broadens out this inquiry into a general study of the give and take, the effort, accumulation and consumption in every province of life" (*22*, p. 961).

In the history of science, biological-economic analogies have played a significant role. Malthus (*23*) borrowed from "the laws of natural increase in the animal and vegetable kingdom" in forecasting a dismal economic future for mankind. Darwin (*24*), as is well known, received a critical inspiration for formulating his theory of evolution by means of natural selection from a reading of Malthus's essay on population. It occurred to Darwin that not only man, but all other species too, are engaged in a struggle for existence owing to their requirement for limited resources, and that those species that evolved ways to use resources more efficiently would be favored in their struggle for survival.

Dissatisfied with the predominance of mechanical analogies in economic thinking, the economist Alfred Marshall (*25*), writing at the turn of this century, insisted that the Darwinian concept of natural selection is also the most important economic principle, and he frequently asserted that, as economics became a mature science, biological analogies would displace mechanical analogies. Some years later John Maynard Keynes (*26*) made the observation that the Darwinian "principle of survival of the fittest could be regarded as a vast generalization of Ricardian economics."

Several other examples of biological-economic analogies may be cited (*27*), but among the most colorful was Adam Smith's frustrated attempt to extend the invisible hand to the economy of nature

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