Background for the Spheroidal Nuclear Model Proposal

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The conceptual developments on which my award is based occurred to me about 26 years ago in late 1949. I shall attempt, as accurately as I can remember, to reconstruct how I viewed the situation of the nuclear shell model and nonspherical nuclear shape at that time.

In a sense the subject began in 1910 when Ernest Rutherford's α particle scattering experiments (*1*) showed that the nuclear size is $\lesssim 10^{-12}$ cm in radius, although the atomic size is $\sim 10^{-8}$ cm. This led to Neils Bohr's 1913 theory (*2*) of the hydrogen atom in terms of quantized electron orbits about the nucleus. This was extended by many workers, especially via the Wilson-Sommerfeld quantization rule that $\int p_i dq_i = n_i h$ for each degree of freedom, where q_i and p_i are the generalized coordinates and momenta of an electron in its orbit about the nucleus. The proposal in 1925 by Uhlenbeck and Goudsmit (*3*) of the concept of spin ½ for the electron and the statement by Pauli (*4*) of the exclusion principle for electrons, later generalized to all spin ½ particles, led to an understanding of the periodic table of the elements, using the old quantum theory, in terms of filling electron shells.

The development of quantum mechanics in 1926 placed the subject on a proper foundation and led to an explosion of the development of atomic physics, as is evident from a perusal of the 1935 treatise by Condon and Shortley (*5*). In the case of the electron orbits or shells about the nucleus, the potential is dominated by the central coulomb attraction of the nucleus, thus permitting treatment of angular momentum as a good quantum number to a good approximation. The coulomb force law was completely known. For the nucleus, early attempts to treat it as composed of protons and electrons were unsatisfactory. When the neutron was discovered by Chadwick in 1932, the picture shifted to a nucleus composed of neutrons and protons bound by strong short-range forces. Measurements of nuclear spins soon established that the neutron and proton should probably be taken to have spin ½ and to obey Dirac theory and the Pauli exclusion principle, thus providing a basis for a nuclear shell model. My own detailed introduction to the subject was mainly provided by Bethe's massive review of nuclear physics (*6, 7*) in the 1936 and 1937 issues of *Reviews of Modern Physics*.

The subject of attempts at a nuclear shell model was reviewed by Bethe and Bacher (*6*). I was particularly familiar with the 1937 article by Feenberg and Phillips (*8*), "On the structure of light nuclei," where the Hartree method was used with a simplified assumed potential to investigate possible spin-orbit Russell-Saunders coupling states in filling the first $l = 1$ shell between ^{4}He and ^{16}O, to explain the behavior of ground and excited nuclear states, and so forth. A model of particles in a spherical box has the first 1*s* ($l = 0$) state filled by two neutrons (N) and two protons (Z) at ^{4}He. This nucleus is certainly exceptionally stable, having a binding energy of over 20 Mev for the last nucleon. The first *p* shell ($l = 1$) then begins, which is closed at ^{16}O. It is interesting that the mass $A = 5$ system is unable to bind the last nucleon and appears as a resonance for neutron or proton scattering on helium. The third shell holds the second *s* and the first *d* ($l = 2$) shell and is filled at ^{40}Ca ($Z = N = 20$), which is also unusually bound. It is the heaviest stable nucleus having $N = Z$. Beyond this the predicted shell closings disagreed with experiment. The basic force law between nucleons was poorly known.

Before 1940 it was known that the nuclear volume and total nuclear binding both increased roughly linearly with A, the number of nucleons. The range of the nuclear force between nucleons was known to be $\sim 2 \times 10^{-13}$ cm and to be deep enough to give the single bound *s* ground state for the deuteron when *n* and *p* spins were parallel, but not when they were antiparallel. A major question involved the reason for the "saturation" of nuclear forces—that is, why binding did not increase as $A(A - 1)$, the number of possible pairings with a "collapsed" nucleus having radius $\approx 10^{-13}$ cm. This was "answered" by Heisenberg, Wigner, Majorana, and others in an ad hoc fashion by assuming "exchange forces," which were attractive or repulsive depending on the wave function exchange properties. Only after 1950 did Jastrow introduce the concept of a short-range repulsion which is now accepted as the reason.

In 1935, Weizsacker introduced his semiempirical binding energy formula (*9*) including volume, surface, isotope, coulomb, and "odd-even" or pairing terms to explain the general trend of nuclear binding. The surface term noted that surface nucleons were less bound, giving a decrease in binding proportional to $A^{2/3}$ for the radius proportional to $A^{1/3}$. This gives less binding for light nuclei and partially explains why maximum stability occurs near ^{56}Fe. The isotope term is easily understood on a shell model basis or using a Fermi-Thomas statistical model. The number of filled space states increases as $(Z/2)$ or $(N/2)$ for protons and neutrons. For a given A, minimum kinetic energy occurs for $N = Z$. For $N > Z$, one must change $(N - Z)/2$ protons to neutrons of higher kinetic energy, with the average kinetic energy change per transferred nucleon proportional to $(N - Z)$, for a total kinetic energy increase proportional to $(N - Z)^2$. This favors $N = Z$ for stability. This is balanced by the coulomb repulsion energy of the protons, which is proportional to $Z(Z - 1)/R$. This favors having only neutrons. The stability balance for stable nuclei has an increasingly large fraction of the nucleons as neutrons as A becomes large. This term also gives reduced binding per nucleon beyond ^{56}Fe and leads to instability against α decay beyond $A \approx 208$ with not too long lifetimes for the ^{4}He fragment to penetrate the coulomb barrier. It was observed that even-N, even-Z (e,e) nuclei were unusually stable relative to odd, odd (o,o) nuclei, such that after ^{14}N the stable nuclei for even A were all (e,e), often having two stable even-Z values for each

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The author is professor of physics at Columbia University, New York 10027. This article is the lecture he delivered in Stockholm, Sweden, on 11 December 1975 when he received the Nobel Prize in Physics, a prize which he shared with Aage Bohr and Ben Mottelson. The article is published here with the permission of the Nobel Foundation and will also be included in the complete volume of *Les Prix Nobel en 1975* as well as in the series Nobel Lectures (in English) published by the Elsevier Publishing Company, Amsterdam and New York. The lectures by Drs. Bohr and Mottelson appeared, respectively, in the 16 and 23 July issues.

even $A \geq 36$. For odd A, there is almost always only one naturally occurring stable Z value, with (e,o) and (o,e) equally favored. This extra binding, $+\delta$ for (e,e), zero for A odd, and $-\delta$ for (o,o), has $\delta \sim 1$ to 3 Mev, decreasing as A increases approximately as $12A^{-1/2}$ Mev. [See Bethe and Bacher (*6*, p. 104).] It is also observed that the ground states of even-A nuclei have net spin zero, indicating a space pairing (potential energy) for strongest interaction to cancel the angular momentum contributions. Figure 1, from the Bohr-Mottelson text (*10*), plots the observed binding per nucleon for beta-stable nuclei against A, with a best-fit semiempirical curve for comparison. The deviations of the experimental bindings from the smooth curve give hints of shell structure effects.

In the early 1930's, the energy dependence of the interaction cross section for reactions involving neutrons or protons incident on nuclei was treated by what is now referred to as an optical model approach. The incident nucleon-nucleus interaction was treated using a smoothed interaction potential for the nucleon inside the nucleus. This model predicted "shape" resonances with huge resonance widths and spacings. Early experiments (*6, 7*) using slow neutrons revealed cross-section (compound nucleus) resonances for medium-heavy nuclei ~ 10 to 100 ev apart, with < 1 ev resonance widths. This led N. Bohr to suggest a liquid drop model (*11*) of the nucleus where the incoming nucleon, as for a molecule hitting a liquid drop, is absorbed near the surface and loses its identity. This is not necessarily incompatible with a shell model, since the shell model refers mainly to the lowest states of a set of fermions in the nuclear "container." However, when combined with the discouragingly poor fits with experiment of detailed shell model predictions (*8*), the situation around 1948 was one of great discouragement concerning a shell model approach.

In the first part of 1949, three groups presented different explanations of nuclear shell structure (*12*) in the same issue of *Physical Review*. Of these, that of Maria Mayer became the now accepted model. A similar proposal by J. H. D. Jensen and colleagues at the same time led to the Nobel Prize in Physics to Mayer and Jensen in 1963. From 1948 to about 1962, I taught a course in advanced nuclear physics for graduate students at Columbia. I was also as an experimental physicist, working on the completion of the Columbia University Nevis Synchrocyclotron, which first became operational in March 1950. During the 1949–1950

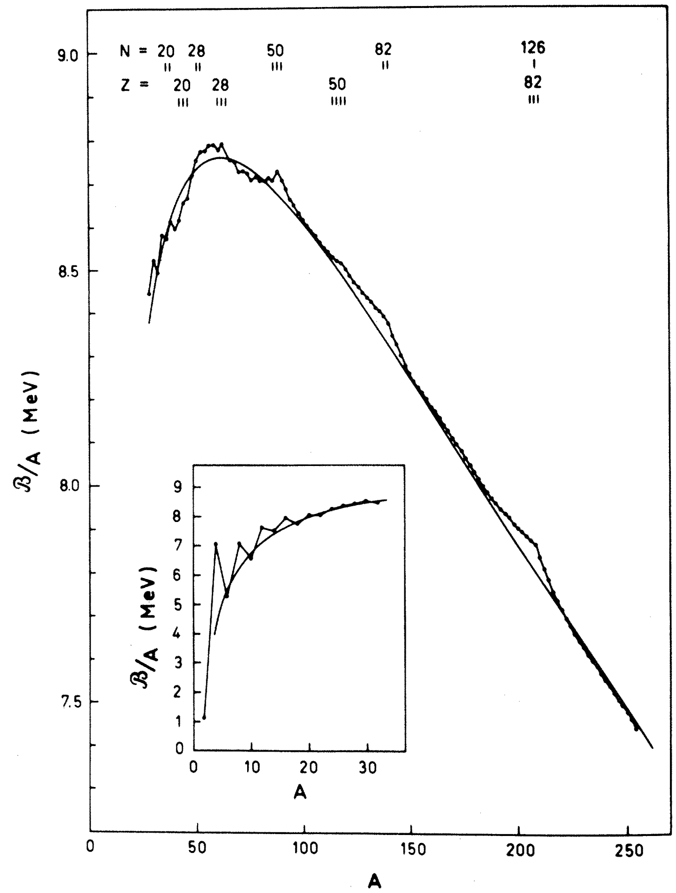
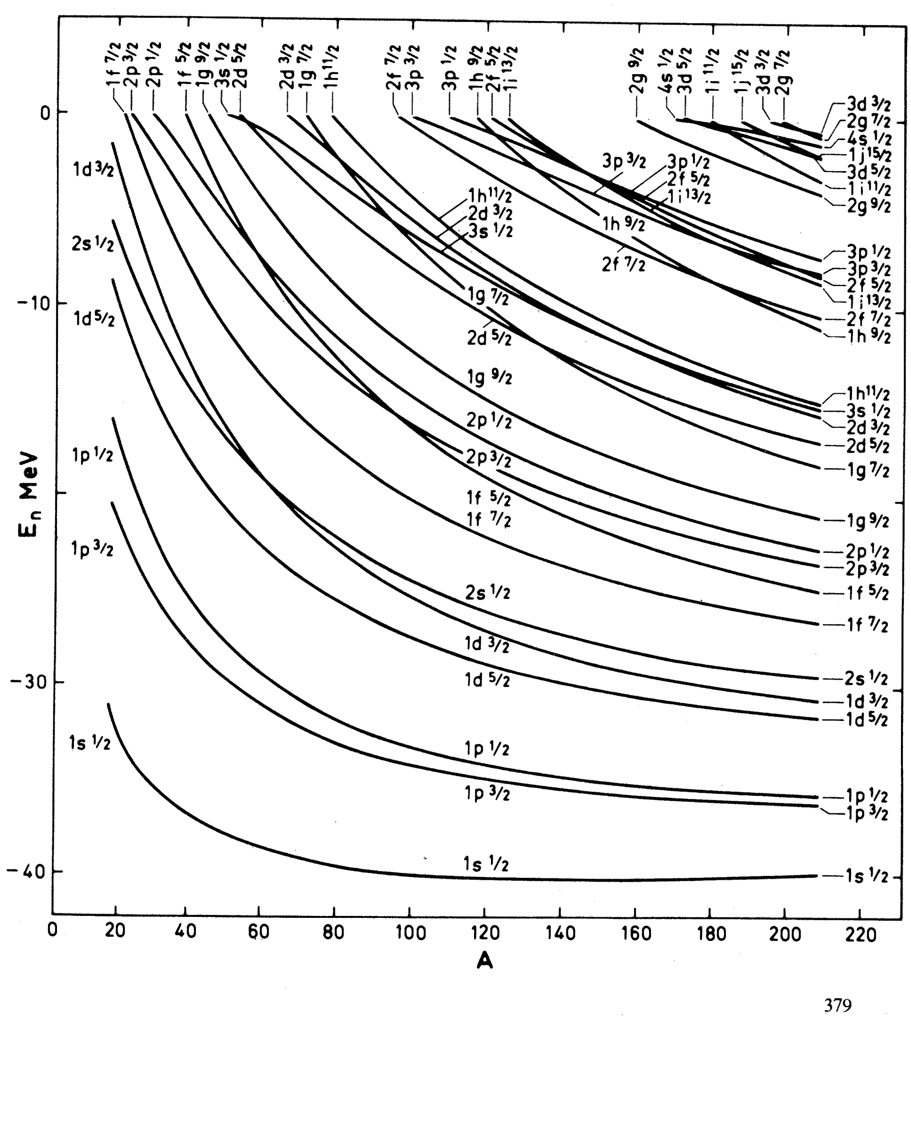


Fig. 1 (top). Average binding energy per nucleon is plotted for nuclei stable against β decay. It is compared with the semiempirical formula $B/A = [15.56 - 17.23\ A^{-1/3} - 23.28(N - Z)^2/A^2]$ Mev $- 3Z^2e^2/5R_cA$, with $R_c = 1.24A^{1/3}$ fm. [From (*10*), courtesy of W. A. Benjamin, Inc.] Fig. 2 (bottom). Energies of neutron orbits using a model of C. J. Veje. The least bound nucleons have energy ~−8 Mev, which is small compared with their potential or kinetic energies inside the nucleus. [From (*10*, vol. 1, p. 239), courtesy of W. A. Benjamin, Inc.]



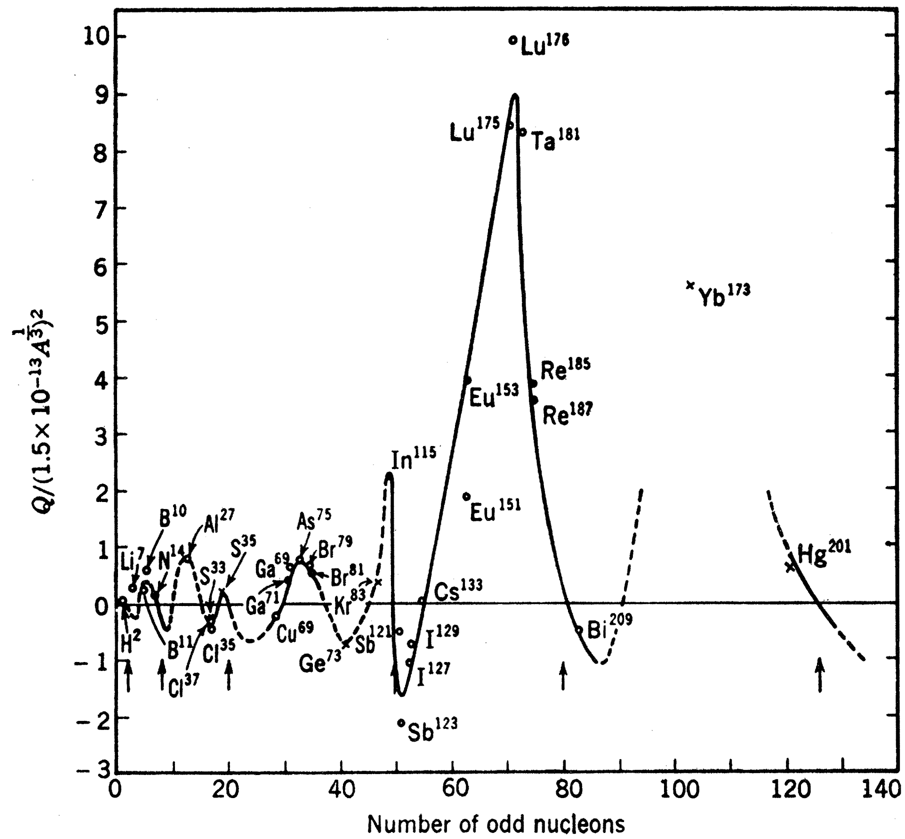


Fig. 3. Plot of $Q/(1.5\,A^{1/3} \times 10^{-13}\text{ cm})^2$ for known nuclear quadrupole moments, as presented by Townes, Foley, and Low (*14*).

academic year, I shared an office, Room 910 Pupin, with Aage Bohr, who was visiting Columbia that year. I was particularly excited about the Mayer shell model, which suddenly made understandable a vast amount of experimental data on spins, magnetic moments, isomeric states, β decay systematics, and the "magic numbers" at Z, N = 2, 8, 20 (28), 50, 82, and 126. I reviewed this material at a seminar at Columbia that year.

For over a year previously, I had felt that shell model aspects should have a large degree of validity for nuclei for the following reason. When one considers forming the nuclear wave functions, in $3A$-dimensional coordinate space, for A nucleons in a spherical box the size of the nucleus, the shell model states result in lowest kinetic energy. The effective potential energy and the shell model kinetic energy (for $r < R$) are both quite large compared with the net binding energy (~ 8 Mev) for the least bound nucleons. This is illustrated in Fig. 2 (*10*). The single-particle state energies versus A have as the "valence" nucleon that with E_n about −8 Mev. If one attempts to use Ψ functions wherein the spatial behavior

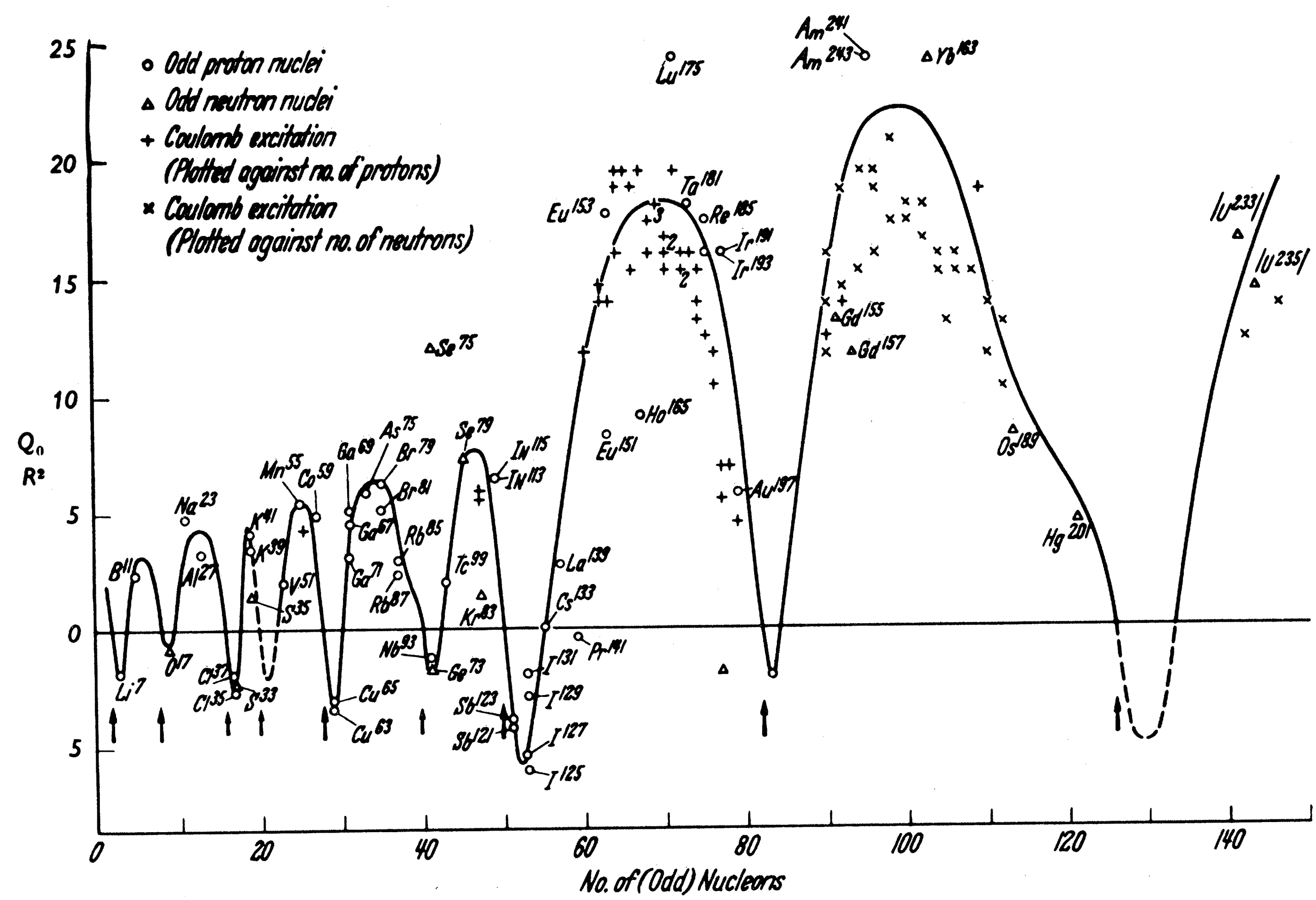


Fig. 4. Later plot of the intrinsic quadrupole moments, Q_0/R^2, prepared by Townes (*18*), using R = 1.2 $A^{1/3} \times 10^{-13}$ cm. This figure supersedes Fig. 3. It emphasizes the large size of the quadrupole moments relative to values $|Q_0/R^2| < 1$ expected for a spherical nucleus shell model.

for each nucleon is very different from that predicted by the shell model, the effect is equivalent to mixing in large amounts of higher energy states having compatible symmetry properties. This mixture of high-curvature Ψ states would greatly increase the $< T>$ for the least bound nucleons. I pictured the net Ψ function *not* as a pure Hartree product of single-particle Ψ functions, but as being nearly so for the long-wavelength Fourier aspects of the functions. The short-range nucleon-nucleon attractive force would lead to local distortions and clusterings in $3A$-dimensional space, such as of deuterons and of α particle structures, and so forth, but low-energy studies would emphasize the long-wavelength Fourier aspects which are suggested by the shell model. I was thus delighted by the success of the Mayer model. (I was not then aware of Jensen's work.) The N. Bohr liquid drop model for nuclear reactions and fission did not seem to me to contradict the shell model since the concept of scattering is meaningless for a many-fermion ground state, but not for an incident continuum-state particle, which is not inhibited by the Pauli principle from knocking bound nucleons to excited (unoccupied) states. The compound nucleus states emphasized by Bohr involved an eventual sharing of the excitation by many nucleons so ~ 10 ev level spacing for medium-A nuclei plus slow $l = 0$ neutrons could result. Since about 1941, I had been using the small Columbia cyclotron to carry out slow neutron time-of-flight spectroscopy studies in collaboration with W. W. Havens, Jr., and C. S. Wu, under J. R. Dunning. We were quite aware of the famous 1939 paper of N. Bohr and J. A. Wheeler on the theory of nuclear fission (*13*) which emphasized that excited nuclei need not be spherical.

In later 1949, C. H. Townes gave a colloquium presenting the results of a review by Townes, Foley, and Low (*14*) of the currently available experimental data on nuclear electrical quadrupole moments. The figure which they presented is shown in Fig. 3. The measured quadrupole moments are presented in the form $Q/(1.5 \times 10^{-13}A^{1/3}\text{cm})^2$. The trend shows a qualitative agreement with the Mayer-Jensen shell model, going to zero as one passes through closed neutron and proton shell numbers. For closed shell plus one extra high-l proton, the value of Q is negative as expected for a proton in an equatorial orbit. As nucleons are removed from a high-l closed shell, the value of Q becomes increasingly positive, reaching a maximum near where the l orbital is half filled, and subsequently decreasing. The problem expressed was that the value of Q/R^2, using $R = 1.5 \times 10^{-13}A^{1/3}$ cm, reaches 10 for ^{176}Lu, which is over 30 times what one might expect for spherical potential shell model wave functions coupled to give a 7^- state ($Z = 71$, $N = 105$, $\tau = 4 \times 10^{10}$ years). The rare earth nuclei particularly show much larger than expected Q values.

As Townes was talking, what seemed like the obvious simple explanation suggested itself to me. Although the Mayer shell model used single-particle wave functions based on a spherical potential, the Bohr-Wheeler fission paper showed that, if energetically favorable, the nucleus would distort to a spheroidal shape. For small values of the fractional difference β between the major and minor axes, for constant nuclear volume, the surface-area term increases as β^2, with

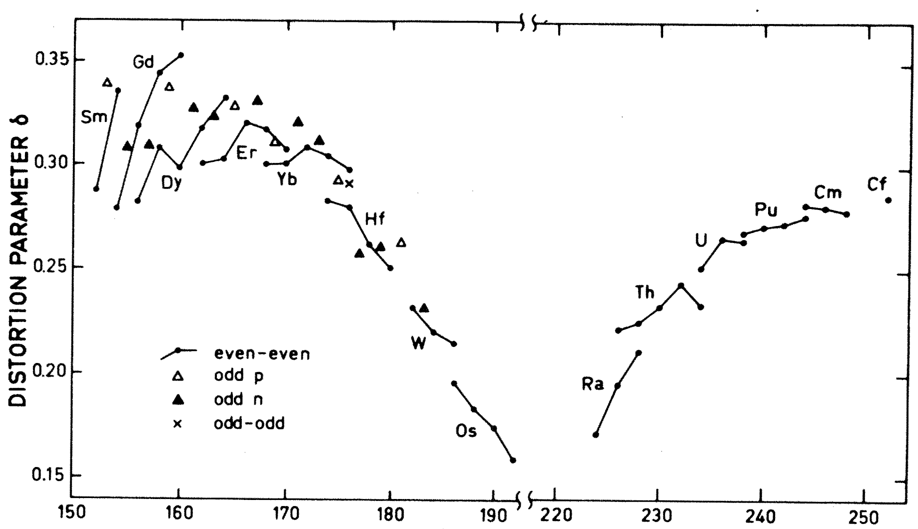


Fig. 5. Plot of the experimental distortion parameter δ (≈ β of this article) in the rare earth region and beyond mass ~ 220. [From (*10*, vol. 2)]

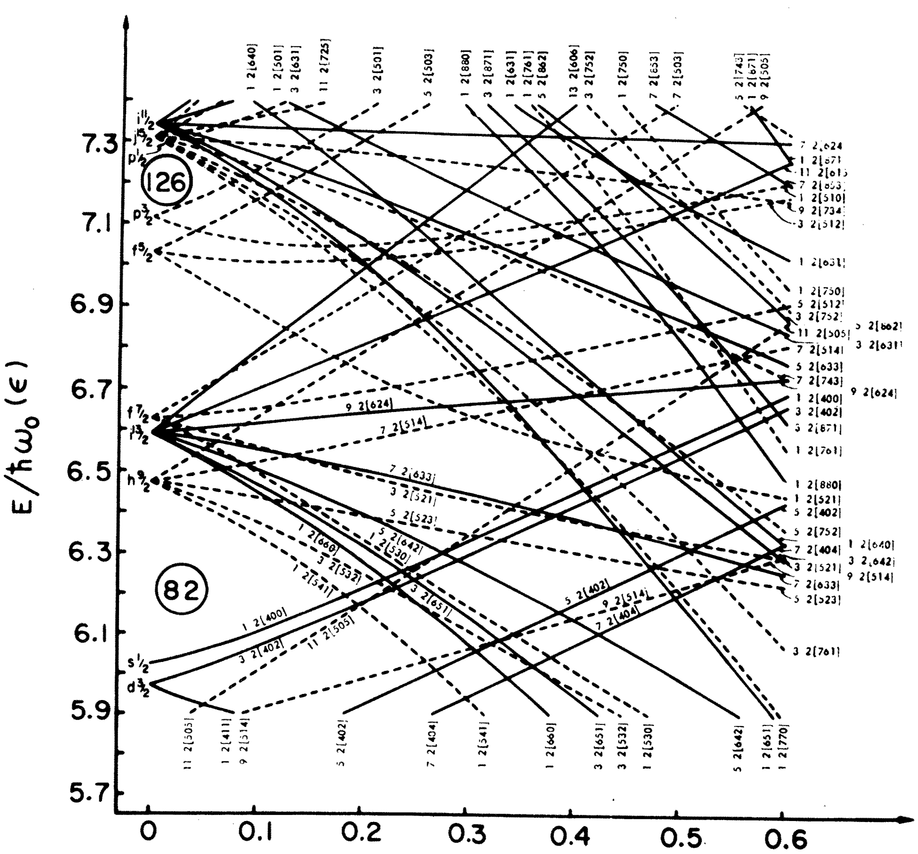


Fig. 6. Nilsson diagram of single-particle shell model proton states versus distortion for $Z > 82$.

the decrease in coulomb energy compensating in part (for high Z). My picture assumed constant well depth, but with a distortion where R increased to $(1 + 2\beta/3)R_0$ in the z direction and decreased to $(1 - \beta/3)R_0$ in the x and y directions (or to $R_0e^{2\beta/3}$ and $R_0e^{-\beta/3}$). If one uses trial Ψ functions which are identically distorted, the potential energy $<V>$ is the same, but the kinetic energy $<T_z> = (1 + 2\beta/3)^{-2}<T_z>_0$ and $<T_x>$ and $<T_y>$ become $(1 - \beta/3)^{-2}$ as large as before. For high-$|m|$ states, the orbits are nearly equatorial and $<T>$ is nearly proportional to R_x^{-2} or R_y^{-2}, with $<T_x>_0 \approx <T_y>_0 >> <T_z>_0$. This clearly favors β *negative*, or a bulge at the equator to disk (oblate) shape. Each 1 percent increase in equator radius (R_x and R_y) gives about 2 percent decrease in $<T>$, or $\delta T/T \approx + 2\beta/3$. For a closed shell, $<T_x>_0 = <T_y>_0 = <T_z>_0$ averaged over all l_z $(= m)$ for high l, so there is zero net linear term in the change in total kinetic energy with the distortion parameter β. For a high-l closed shell *minus* equatorial (high-$|m|$) orbitals, the net nuclear angular momentum is the negative of the contribution of the missing nucleons (holes) and the contribution to the kinetic energy term linear in β is equal and opposite to that of the missing equatorial orbit nucleons. The important point is that this yields a term linear in β favoring $|\beta| \neq 0$, while the restoring terms are quadratic in β. The expected equilibrium β is thus $\neq 0$, and is proportional to the coefficient of the linear term for not too large deviations of β from unity. This gives a prolate (cigar) shaped distortion.

The next step was to attempt a more quantitative evaluation of the β^2 restoring term. For this, I found the 1939 paper by Feenberg (*9, 15*) useful. He noted that the surface energy increased as $E_S = E_S{}^0[1 + (8/45)\beta^2 \ldots]$ and the coulomb energy decreased as $E_C = E_C{}^0[1 - (4/45)\beta^2 \ldots]$, which requires $F = 2E_S{}^0/E_C{}^0$ $(\approx 42.6\ A/Z^2) > 1$ for a net positive restoring β^2 term. This predicted zero net β^2 restoring term for $Z \sim 125$ for beta-stable nuclei (no resistance to fission). The net term was $\beta^2(2.74\ A^{2/3} - 0.054\ Z^2A^{-1/3})$ Mev. Using this value gave (*16*) $Q/R^2 = -11$ for a single high-l nucleon above closed shell for a fictitious case of $A \sim 176$. The picture, if anything, seemed capable of giving even larger Q/R^2 values than were observed experimentally.

For a prolate spheroidal potential, with the distortion axis in the z direction, the ϕ dependence of the single-particle Ψ for $l_z = m$ is still $e^{im\phi}$. However, l_x, l_y, and l^2 cannot be good quantum numbers. The core must somehow share the net angular momentum. This consideration helps when one considers the deviations of the observed magnetic moments from the Schmidt limits predicted by the simple shell model.

Aage Bohr pointed out to me at the time (*16*) that if the nucleus is a spheroid with an "intrinsic" quadrupole moment Q_0 relative to its distortion axis, and total angular momentum is I, the maximum "observed" Q is reduced by a factor $I(2I - 1)/(I + 1)(2I + 3) = 1/10, 2/7, 5/12$, and $28/55$ for $I = 1, 2, 3$, and 4. This emphasizes that $Q = 0$ for $I = 0$ or $1/2$, but Q_0 may not be zero. Bohr, Mottelson, and colleagues (*17*) subsequently treated the situation for coulomb excitation cross sections for low-lying rotational states. The excitation cross sections uniquely establish the intrinsic quadrupole moment Q_0 for the ground states of distorted even-even nuclei as well as for odd-A nuclei. Figure 4 was prepared by Townes around 1957 for a review article on measured quadrupole moments (*18*). The largest intrinsic quadrupole moments occur for the rare earth region before the double closed shell $Z = 82$, $N = 126$, and beyond $A \sim 230$, where even higher j single-particle states are involved. Figure 5 shows a recent plot from the just-released volume 2 of Bohr and Mottelson's *Nuclear Structure* (*10*). The distortion parameter δ is nearly the

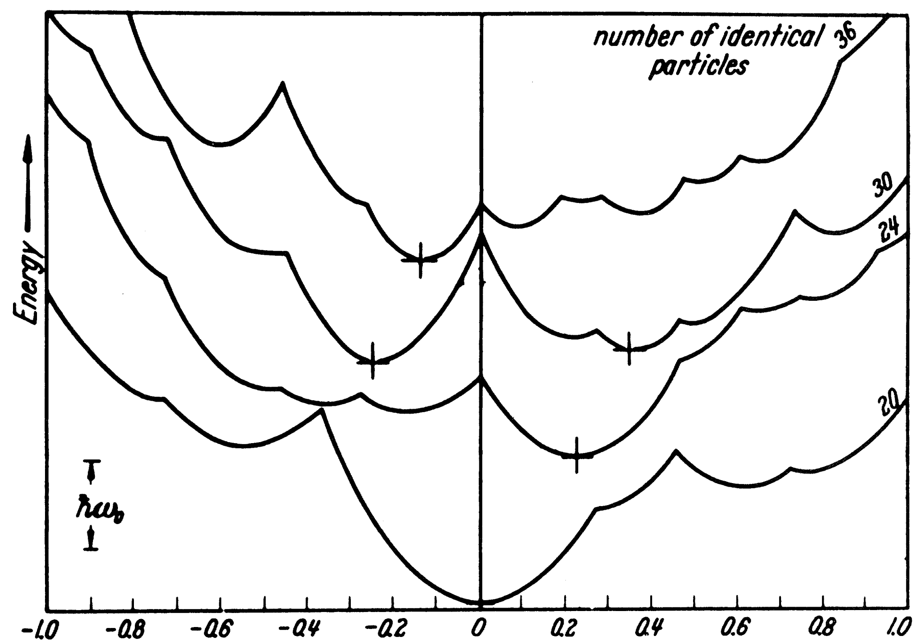


Fig. 7. Deformation potentials for various stages of shell filling–spheroidal harmonic oscillator binding potential. [From Moszkowski, figure 25 in (*20*)]

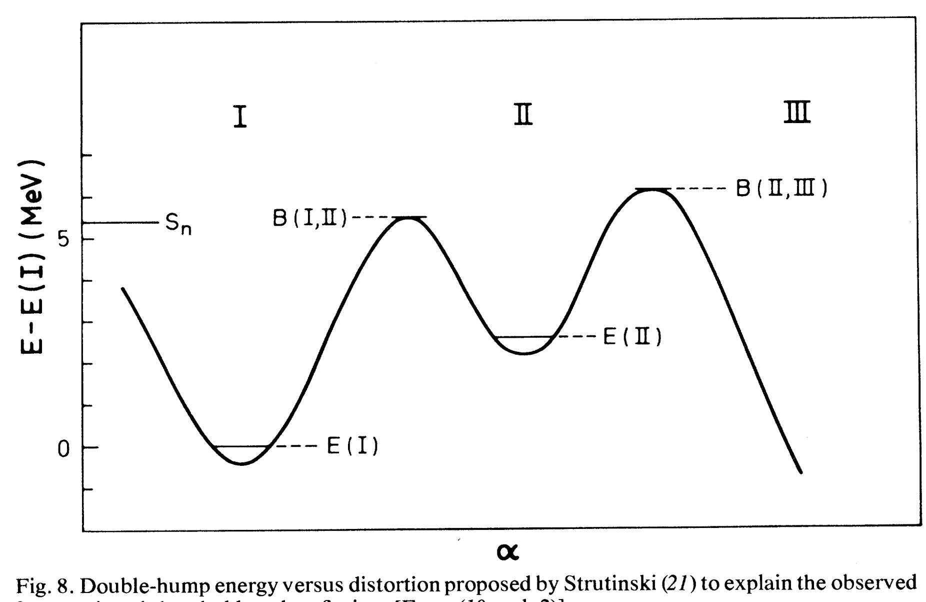


Fig. 8. Double-hump energy versus distortion proposed by Strutinski (*21*) to explain the observed features in subthreshold nuclear fission. [From (*10*, vol. 2)]

same as the parameter β discussed above. It is seen, as was evident from Townes' 1949 colloquium (*14*), that many nuclei deviate quite strongly from spherical shape so it does not make sense to use a spherical nuclear model in these regions of atomic size.

After Townes' colloquium, Bohr and I had many discussions of my concept. He was particularly interested in the dynamical aspects. The distortion bulge could in principle vibrate or move around to give the effect of rotational levels. The first result was his January 1951 paper (*19*), "On the quantization of angular momenta in heavy nuclei." The subsequent exploitation of the subject by Bohr, Mottelson, and their colleagues is now history and the main reason for our presence here at this time.

I should mention that the program of evaluating the energies of single-particle states in distorted nuclei was subsequently carried out in proper form by Mottelson and Nilsson and by Nilsson alone in the form of "Nilsson diagrams" such as in Fig. 6, which is for proton single-particle states beyond $Z = 82$ versus the distortion. They have also made detailed comparisons, with experimental values of the predicted distortions, and so forth, with generally excellent results (*10*). It has also been established that some nuclei have appreciable octupole electric moments and distortions, a generalization of the concept.

One interesting feature of the distorted nucleus shell model is that as the distortion increases, the net energy may go through a minimum and then increase until the energy of an initially higher-energy orbital, which decreases faster with deformation, crosses below the previous last filled orbital and subsequently becomes the defining least bound filled state. The net energy may then decrease and show a second minimum and so on when plotted against distortion. This is shown in Fig. 7, which is figure 25 of Moszkowski's review article (*20*). This effect seems to be present in subthreshold nuclear fission where the barrier shape has two minima, as shown in Fig. 8 (*10*, vol. 2, p. 633). This was suggested by Strutinski (*21*) in 1967.

There is one additional effect which I have not yet mentioned which favors spherical shape. If reference is made to the 1937 paper by Feenberg and Phillips (*8*) on the relative binding of different configurations having two or more $l = 1$ nucleons beyond the ^{4}He core, which are combined to form various total L and S (L-S coupling) states for a short-range attractive-only force, it is seen that the overlap is sensitive to how this is done. As an example, for $A = 6$, the two p nucleon wave functions take on the form $(x + iy)f(r)/2^{1/2}$, $(x - iy)f(r)/2^{1/2}$, and $zf(r)$. The combination $(x_1x_2 + y_1y_2 + z_1z_2)/3^{1/2}$ for $L = 0$ is more strongly bound than such choices as z_1z_2 or $(x_1x_2 + y_1y_2)/2^{1/2}$, which are favored by a spheroidal potential but do not correspond to an eigenstate of $\mathbf{L}^2$. Such an effect may inhibit the distortion for small distortions until the gain from the distortion is more overwhelming relative to such symmetry effects on the interaction potential energy.

Since 1950, I have been mainly concerned with experimental physics research using the Nevis Synchrocyclotron. I have been an admiring spectator of the developments of the theory by the Copenhagen group. My main other (experimental) contribution was in the muonic atom x-ray studies started with Val Fitch (*22*) in 1953, where we first established the smaller charge radii for nuclei. When I made my proposal for use of a spheroidal nuclear model (*16*), it seemed to be an obvious answer which would immediately be simultaneously suggested by all theorists in the field. I do not understand why it was not. I was also surprised and dismayed to hear one or more respected theorists announce in every nuclear physics conference which I attended through about 1955 some such comment as, "Although the nuclear shell model seems empirically to work very well, there is at present no theoretical justification as to why it should apply." Fortunately, such opinions are no longer expressed.

Although my consideration of the "forcing term" for spheroidal nuclear distortion considered the dependence of the single-particle kinetic energy on the distortion, I have never seen a description of my work elsewhere in those terms. A common equivalent phrasing is the "centrifugal force exerted on the barrier" by the orbit. Another method is to compute the increase in the potential energy interaction on distortion. This is equivalent, since for a single-particle eigenstate there is zero rate of change of energy with distortions of Ψ. Thus $\langle V \rangle$ and $\langle T \rangle$ must give equal but opposite contributions to the term linear in β.

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23. I wish to thank the *Physical Review*, W. A. Benjamin, Inc., and Springer-Verlag for permission to use the various figures.