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The Arctic Mirage and the Early North Atlantic

H. L. Sawatzky and W. H. Lehn

The desert mirage, or *fata morgana*, is an almost universally familiar phenomenon. It occurs when air overlying a warm surface is heated by conduction, which induces temperature and therefore density gradients of a magnitude sufficient to alter the "normal" optical properties of the air. Refraction, and mirroring at significant surfaces of discontinuity, cause the path of light passing from an image to an observer to become not a straight line, but a complex of changes in direction, with the result that the image is optically displaced, vertically and often also laterally, from its actual location. False lakes, floating mountains, inverted images, and the like are commonly experienced examples of the desert mirage.

There is an equivalent but essentially inverse effect, much less frequently experienced, yet nevertheless not rare in the middle and higher latitudes. This is the arctic mirage or, using the Icelandic word for the phenomenon, the *hillingar* effect, in which the observed image is optically displaced, in a vertical direction only, from its real location. We propose that the arctic mirage may have been a significant factor in the development of certain early historical concepts of the nature of the world, and possibly in exploration and discovery.

Nature of the Arctic Mirage

The arctic mirage occurs when air rests on a pronouncedly colder surface. This situation establishes (i) a higher vapor pressure in the immediately superincumbent air than at the surface; (ii) temperature inversion conditions which impart a high degree of stability; and (iii) a vertical temperature gradient which imparts particular refractive properties to the affected air (*1–3*). Light passing from an image to an observer, within a band of air described by these conditions, is continuously refracted in an arc with the earth on its concave side. The result, over large, relatively featureless surfaces, is to permit the passage of light from an object to an observer over distances greater than the normal distance to the horizon (see Fig. 1) (*4, 5*).

Although warm air advected over a colder surface may cause the phenomenon, the key to the occurrence of the arctic mirage in its clearest expression are large, relatively stationary high-pressure cells. Subsidence within such air masses results in adiabatic heating, at a uniform rate of approximately 1°C per 100 meters. As the air is simultaneously cooled by radiation and by conduction from the colder surface, a temperature inversion, whose depth and magnitude are determined by the relative motionlessness of the air mass and the temperature characteristics of the surface and superincumbent air, comes into being. The higher vapor pressure prevailing in the air inhibits the escape of moisture from the surface, and hence the formation of fog or mist, leaving the almost vapor-free air extremely transparent. The high degree of atmospheric stability that characterizes the situation further facilitates the direct passage of light, while the positive upward temperature gradient produces an exaggerated negative upward density gradient, which substantially enhances the normal refractive capability of the air (see Fig. 2).

Given a sufficiently steep vertical temperature (density) gradient, the refractive capability of the air may become great enough to equal or exceed the curvature of the earth, which to the observer is thus optically rendered flat or concave upward, with the horizon receding toward infinity, or, in practical terms, toward the limits from which light can, by virtue of the transparency of the air, pass from an image to an observer. The threshold condition for achieving a refractive effect matching the curvature of the earth is established by a temperature increase with height of approximately 0.11°C per meter. A higher gradient causes the surface to appear saucer-shaped. Observations of vertical temperature gradients as great as 0.5°C per meter, under conditions of strong inversion, are on record (*6*). However, much weaker inversions, which can have vertical profiles 300 meters or more deep, still have a greater refractive capability than normal air, and may therefore raise ele-

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vated elements of the landscape—mountainsides, cliffed coastlines, and so forth—into the field of view by overcoming the normal obstruction to a direct line of sight posed by the intervening curvature of the earth's surface (*7*). High-latitude water and ice surfaces admirably meet all conditions for the occurrence of the arctic mirage.

Arctic Mirage in Folk Tradition

Rural people of the regions rimming the North and Baltic seas and the North Atlantic have a long tradition of knowledge of the arctic mirage and of interpretation of the information it may convey from beyond the normal horizon. Seafaring folk are especially familiar with it, even today, and, as in the case of Icelanders in their prairie and lakeshore settlements in Canada, their awareness of the *hillingar* phenomenon has been perpetuated through continuing encounters with it. There are, indeed, other regions in which suitable atmospheric and surface conditions may be anticipated with some frequency in winter and early spring, and may, in fact, occur at any time of year. Examples are the basin of glacial Lake Agassiz and other areas of equivalent terrain in North America, and the Russian steppe. The mid-19th-century German folk author Theodor Storm matter-of-factly describes such a phenomenon in coastal Friesland (*8*): "It was late September. In her room Trin Jans, almost ninety, lay dying. She had asked to be raised upon her pillows, and her gaze swept the landscape beyond the small leaded windows. Against the sky *a less dense layer of air must have overlain a denser* [italics added], for, projected over the rim of the dike, there appeared the shimmering, silvery expanse of the sea, lifting to the horizon, bathing the room with its brightness."

David Thompson, in the narrative description of his explorations in western North America, gives the following account (*9*, pp. 121–122): "What is called Mirage is common on all these Lakes, but frequently [is] *simply an elevation of the woods and shores that bound the horizon* [italics added]. The . . . Mirage is seen in the latter part of February and the month of March, the weather clear, the wind calm, or light; the thermometer from ten above to twelve degrees below zero [Fahrenheit], the time about ten in the morning. . . . While the Mirage is in full action, the scenery is so clear and vivid, the illusion so strong, as to perplex the Hunter and the Traveller; it appears more like the power of magic, than the play of Nature. . . . [The Indians] said it was Manito Korso; the work of a Manito . . . ."

In the Agassiz basin, under the influence of the arctic mirage, distances to the horizon of more than 65 kilometers are not rare. The most persuasive evidence, however, comes to us from the realm of the North Atlantic itself. On 17 July 1939, Captain John Bartlett, from his ship the *Effie M. Morrissey*, at a position between 63°38′N, 33°42′W and 63°42′N, 33°32′W, clearly observed and identified the outlines of the Snaefells Jökull (1430 meters), more than 500 kilometers to the northeast on the west coast of Iceland (*10, 11*).

Early Historical Concepts

Considering the way in which people who know it appear to accept it and derive information from it, the arctic mirage suggests a logical basis of conjecture about the origins of several important aspects of the medieval interpretation of the nature of the world—of the regions on the approaches to the "rim" of the world, and of the character of the world itself. It also suggests clues to the basis of the progressive—and possibly systematic—advance from Europe to Iceland and the New World five centuries and more before the "Age of Discovery."

The Celts and the Norse may well have contributed more than is commonly supposed to the body of legend concerning the nature of the world. In the existing legends—many of them familiar to us from Greek and Roman mythology—the world was flat or saucer-shaped. Upon its disk rested the World Island, shading off near its edge into the dim region called Thule (or Ultima Thule), fronting on the all-encircling ocean, beyond which was the Abyss. Since all streams flowed eventually to the ocean, it was reasoned that its waters must be carried by subterranean channels, whose entrances were marked by deadly whirlpools, back to the land, there to reemerge in streams and rivers, completing the cycle. It has been commonly assumed that these concepts were formulated on the basis of observations within the Mediterranean basin (the whirlpools of Charybdis and Scylla, for example). Fridtjof Nansen (*12*, vol. 2, p. 151), early in the 20th century, postulated the transference of these concepts from the Mediterranean to northwestern Europe, by osmosis of folk legend from people to people, ultimately to enter the oral folk literature of the Celts and the Norse. However, we question the likelihood of absorption into the folk literature of non-

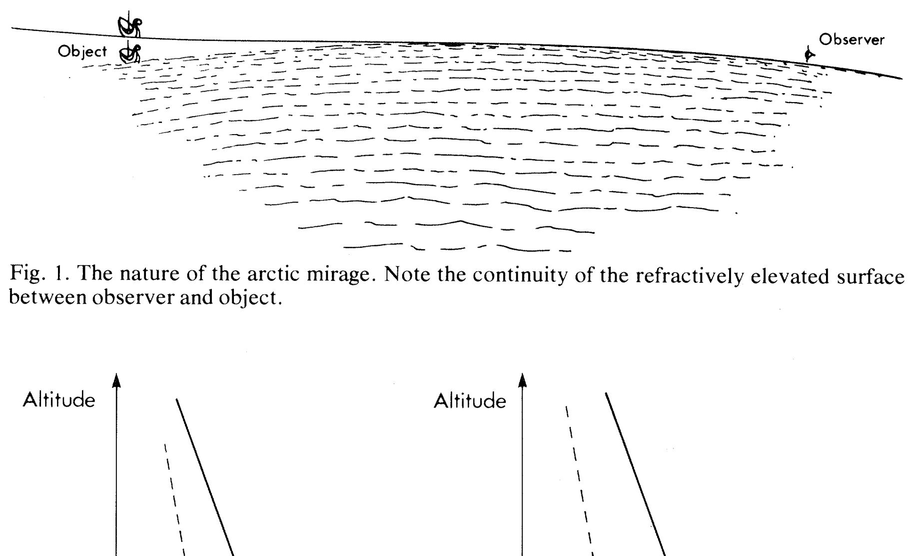

Fig. 1. The nature of the arctic mirage. Note the continuity of the refractively elevated surface between observer and object.

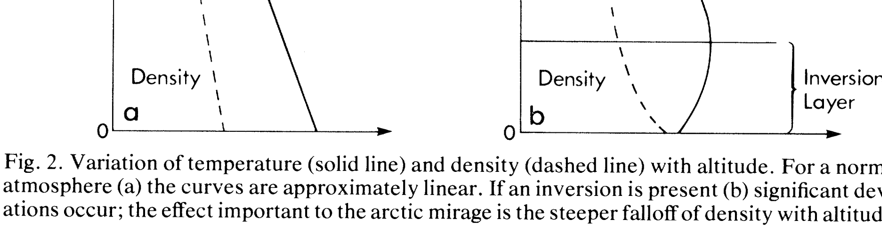

Fig. 2. Variation of temperature (solid line) and density (dashed line) with altitude. For a normal atmosphere (a) the curves are approximately linear. If an inversion is present (b) significant deviations occur; the effect important to the arctic mirage is the steeper falloff of density with altitude.

literate peoples of basic concepts of the nature of the world not corroborated by their own cumulative experience. It appears considerably more probable that early Mediterranean travelers to northwestern Europe, like Pytheas of Massalia (Marseilles), who visited "Britain, Thule and the land from which amber comes" about 334 B.C., could have returned with concepts which entered the written lore of their people. When northwestern Europe came to have a written literature, elements borrowed from Mediterranean cultures were "relocated." The mid-13th-century *Historia Norwegiae* had "Charybdis, Scylla, and unavoidable whirlpools" located in the north (*12*, vol. 2, p. 151).

In any event, it is not at all necessary to postulate an osmosis of legend, for the limits of the world known to the Celts and Norse were, in fact, as their legends described them. When the atmosphere was clear enough to see over long distances, the arctic mirage was likely to cause the surface of the world to extend outward toward its "edge" at dead level or at a slight incline. Indeed, it seems possible that reports of observations of this sort, from the regions approaching what was believed to be the rim of the world, may have had much to do with the failure of the theory of the spherical earth to win acceptance for many centuries. The concept of the spherical earth, advanced by Greek astronomers in the third century B.C., appears to have subsequently dominated among Greek and Roman philosophers. The Church, on the basis of a literal interpretation of Scripture, held tenaciously to the opposite view until it was finally refuted by the accumulation of astronomical and terrestrial evidence in the 15th and 16th centuries. During medieval time, as Christianity advanced through northwestern Europe, it was the Church, and not the ancient philosophers, which held sway over men's minds and corroborated their perception of the earth as being flat (*13*, vol. 1, pp. 89–98).

Having placed the Celtic and Norse explorers within a framework of natural phenomena that provide a reasonable basis for their ideas about the nature of the world while being consonant with overlapping legends from other cultures, let us briefly consider some linguistic evidence. Nansen devoted much effort and enlisted the help of noted linguists in an attempt to discover the roots and origins of the word Thule, descriptive of the farthest land, on the shores of the sea surrounding the World Island. The only expression he found in any European language or dialect that was technically compatible and also contained the idea or suggestion of land in any form was the Celtic *tel*, whose meaning is "raise" or "raise oneself," and is the root of the Irish *telach* and *tulach*, both meaning "a height or mound" (*12*, vol. 1, p. 59; *14*). Thule, then, the upraising land, at the limits of the flat or saucer-shaped world . . . ? Nansen, presumably because it would have conflicted with his theory of the osmosis of legend from the Mediterranean northward, dismissed the Celtic root as having no possible connection with Thule. We believe otherwise.

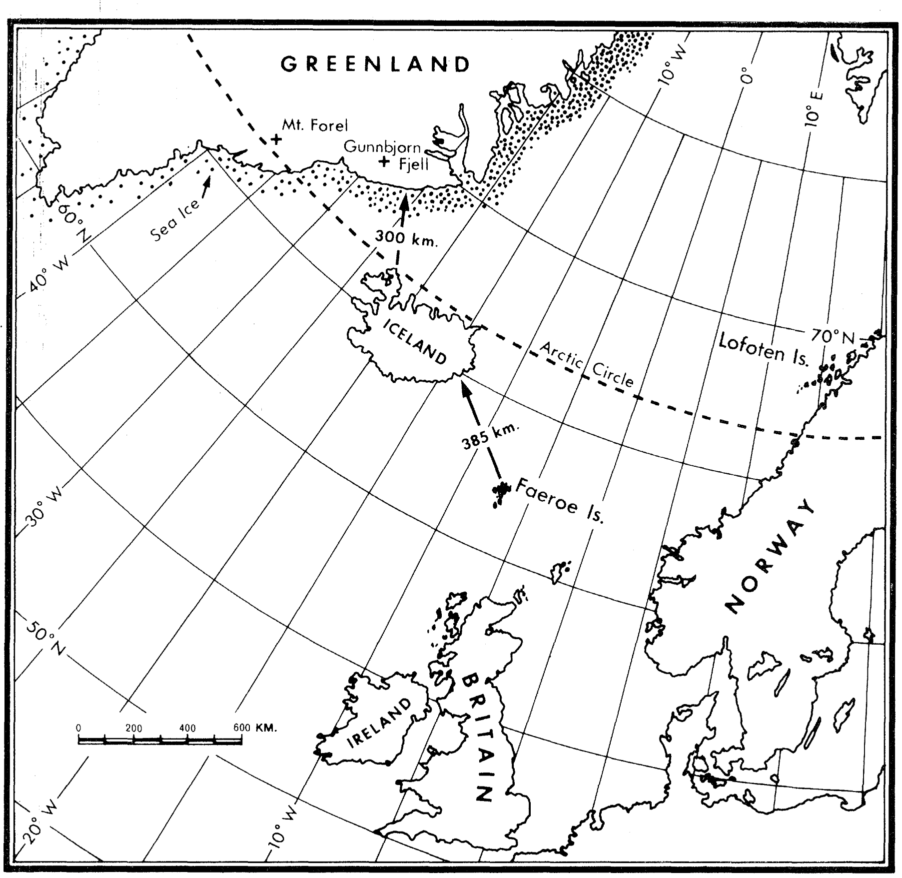

Fig. 3. The North Atlantic realm. Minimum overwater distances are indicated between the Faroe Islands and Iceland and between Iceland and Greenland.

The Maelstrom

The waters between Europe and Greenland were reported by medieval mariners to be filled with treacherous whirlpools, vortices, or "sea-fences" (*hafgerdingar*) (*12*, vol. 2, p. 244; *15*). The medieval Greenlandic priest Ivar Bárdsson described "the many whirlpools that there lie all over the sea" in northern waters (*16*). The 13th-century *King's Mirror*, as well as describing *hafgerdingar*, presented sober, amazingly accurate accounts of the large marine mammals found in the North Atlantic, together with descriptions of the sea ice and its drift (*12*, vol. 2, pp. 155 and 243).

Various explanations, beyond the transference of legend, have been advanced for the established broad awareness of and preoccupation with the presence of these much-feared phenomena, believed capable of sending ships to certain destruction. The powerful eddy current in the vicinity of Mosken in the Lofoten Islands (see Fig. 3), also called Moskenström or Maelstrom, is little more satisfactory as an explanation of the prevalence of this legend in the regions facing the North Atlantic than are Charybdis and Scylla, for the Lofoten Islands, near 70°N latitude off the coast of Norway, lay on no shipping lane and therefore came under the purview of very few people, if any (*17*).

Theories about the *hafgerdingar* based on speculation about possible encounters with submarine earthquakes do not provide a satisfactory explanation, either. Consider that, during medieval times, most years saw four or fewer ships voyaging between Europe and Iceland (*18, 19*). Even fewer visited Greenland. The probable frequency of any of those few ships being fortuitously present at the time of an earthquake, even in the seismically active zone about Iceland, is therefore very small. The odds against such experiences occurring with sufficient frequency for the observed phenomena to become firmly established in legend are too great for this possibility to be taken seriously.

It is not necessary, however, to advance such theories as described above in order to arrive at an explanation for the origins and wide dissemination of the *hafgerdingar* legend. Consider first that merchant vessels were in medieval times manned, not by hired crews, but by coadventuring traders, acting independently of one another in all other circumstances (*18*, pp. 322–328). It is therefore likely that a relatively large number of different traders visited Iceland—and Greenland—in this way. The total combined opportunity to experience an encounter with the much-feared vortices or whirlpools of legend, and for the subsequent telling of it, must have been large indeed. For they did exist, and still do, albeit as optical illusions only, and the high losses to misadventure among trading ships in those times were certainly adequate to keep the supposition alive, in the almost certain absence of other evidence, that any vessel which failed to return from a voyage in those waters had in all likelihood been inexorably drawn to its doom in such a vortex. Consider also that seldom was a voyage to Iceland or Greenland undertaken outside the period between late spring and early fall, when the North Atlantic is less storm-racked than at other times (*20*). This is the season when atmospheric conditions favor the occurrence of the arctic mirage.

At sea, in the absence of normal terrestrial aids to orientation, the visual impact of the arctic mirage is much more profound even than on land, and the observer has an overwhelming impression of being already below the lip of a genuine, if shallow, vortex. This impression, caused by the foreshortening of the refraction-induced slope in all directions, in effect "staircases" the visible motion of the water in such a way as to exaggerate the perceived upward angle by a factor of many times its actual magnitude, causing the waters to appear to be poised as if to engulf the observer. Careful attempts made by us to determine the angle of rise in the presence of this phenomenon resulted in an estimate of 2°. Precise measurement from a shore position under equivalent conditions yielded a reading of 2′. The perceived angle, due to the effects and influences described, was 60 times as great as the actual angle. The presence of another vessel within the range of view would only strengthen the impression of being within the funnel of a vortex. Even at distances of only a few kilometers (see scale diagram, Fig. 4) the waters beyond would loom much higher than a vessel such as a Norse longship. The same effect can occur on ice surfaces. Thompson wrote: "On one occasion, going to an Isle where I had two traps for Foxes, when about one mile distant, the ice between me and the Isle appeared of a concave form, which, if I entered, I should slide into its hollow, sensible of the illusion, it had the power to perplex me. I found my snow shoes, on a level, and advanced slowly, as afraid to slide into it . . ." (*9*, p. 120).

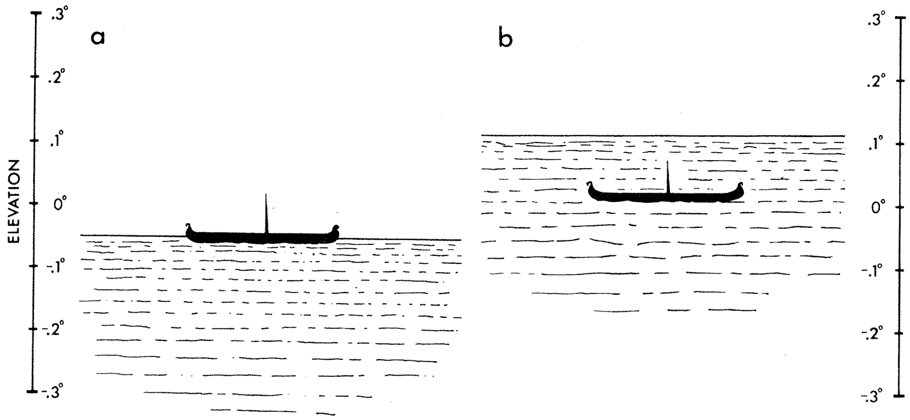

Fig. 4. Boat of the dimensions of a Viking longship as seen from eye level (3 meters) at a distance of 5 kilometers. The scales give the angle that the image subtends at the eye of the observer. The vertical temperature gradient in (a) is normal; in (b) it is 0.5°C per meter. The horizon distance is about 7 kilometers in (a) and 51 kilometers in (b).

Celtic Exploration

Turning now to early exploration, consider that the maximum overwater distance across the North Atlantic, from landfall to nearest landfall, is the 385 kilometers from the Faroe Islands to southeastern Iceland (Fig. 3). Consider further that it was Celtic people from the Faroes—not Norsemen—who, before A.D. 800, first reached and returned from Iceland (*21*), presumably in their relatively fragile skin-covered currachs, and the supposition becomes realistic that they ventured forth in response to some body of advance information concerning the possible existence of land in the ocean to the northwest. In so doing, they and the Norse seafarers who came after them would encounter phenomena which would "substantiate" legendary concepts of the nature of the world on the approaches to its limits.

Information as to the existence of land beyond the limits of the known world could have been acquired in ways often supposed. It could, indeed, have been discovered as a result of involuntary storm-driven voyages. However, the persuasiveness of this supposition is undermined by a consideration of the small size and relative fragility of the Celtic vessels, and by the obvious necessity for at least some of the participants in such a voyage or voyages to have survived and returned to tell of their discovery. More persuasive is the suggestion that observation of the annual migrations of birds could have led people on the farthest fringes of Europe, who for their very existence needed to be keen observers of natural phenomena, to reason that there must be land to the northwest.

To this very probable factor, then, add the fact that the mighty Vatna glacier rears to a height of 2000 meters almost from the water's edge in extreme southeast Iceland and would reflect a diffused cast of light, or "iceblink," visible for a great distance, against the sky. This phenomenon might, by the practiced eye, have been rendered of the information it bore in the same way that the Eskimo today reads the arctic sky for suggestions on the nature of the terrain beyond the range of direct vision. The possibility, however, considering the distance involved, would present itself only in the latter part of a voyage in the waters near Iceland, or in the event of a deep inversion spanning the entire region between the Faroes and Iceland and bringing with it, inevitably, the arctic mirage. Such inversions, as demonstrated by satellite photographs, occur with some frequency today. Since the available evidence suggests that during the latter part of the first millennium A.D. the climate of the North Atlantic was somewhat milder than now, they may have occurred with greater frequency then (*22*, *23*). NATO air patrols operating out of Keflavik describe the range of visibility under such conditions as "unlimited"—that is, to or beyond the maximum range of resolution of the human eye. Under such meteorological conditions the arctic mirage *must* occur, thus setting the stage for the derivation of information from beyond the normal limits set by the horizon or by diminished transparency of the air.

Consider also that (i) the highest levels of atmospheric stability, under the conditions envisaged, occur from dawn until sometime after sunrise (*24*), and (ii) the direction of potential discovery was to the northwest, and the observer on or near the Faroe Islands would have the light of the morning sun over his shoulder and in consequence would enjoy the best viewing conditions possible. Since it is quite probable that the inversion would be of somewhat lesser depth in the vicinity of the Faroes than farther northwest, nearer the heart of the polar high-pressure cell, it is reasonable to suppose that the surface inversion could on occasion have modest but pervasive cloud cover extending from the fringes of Europe toward Iceland. In that event the object (Iceland) would be brightly illuminated while much of the intervening space to the Faroes would be relatively dark. Under those conditions the potential for the transmission of visual information over large distances would be superb. Indeed, such conditions appear to have prevailed at the time of Captain Bartlett's observation of 17 July 1939 (*25*). Moreover, the arctic mirage, at these latitudes, considering the relatively low angle of the sun and the character of the cold-water surface, is hardly affected by surface heating as the day advances, and good viewing conditions may be experienced throughout the daylight period. [We made excellent observations on frozen Lake Winnipeg (51° to 52°N) in the afternoon in late April and on God's Lake (54°N) in the afternoon in mid-June. The best observations during a brief reconnaissance in Iceland in May 1973 occurred in midmorning in one case and early evening in another.]

Assume the extreme case, of light passing from southeast Iceland to the farthest outpost of Europe. That light would, under the conditions described, retain some 4 percent of its initial intensity upon arriving in the Faroes—a level well within the information-resolving capability of the human eye (*26*). With this information, alone or in conjunction with others of the factors cited, the practiced observer might logically have become persuaded that something more than water lay in the regions to the northwest. Indeed, if light reflected from the upper elevations of the Vatna glacier encountered the upper strata of a deep inversion at a shallow angle, it could, instead of heading out into space, be refracted downward continuously so as to intersect the surface of the earth again. That the observational skills involved in interpreting information from such sources are hardly a topic of discussion today need not amaze us. The stepping-stones across the North Atlantic were discovered more than a thousand years ago. Skills such as those here imputed to certain folk in those times have survived into the present. However, only for the few for whom they have not been crowded out of memory by the technology which has replaced them do they still hold practical value.

The Norse

From 874, when the Norse arrived in Iceland (having, during the 80 or so years since the first Celtic voyages, taken possession of the Faroes), the Celtic role recedes into the dimmest recesses of legend. It is from the Norse sagas that we have inherited provocative accounts of important voyages—provocative in terms of the thesis propounded here. When Gardar Svafarsson, the Swede who first circumnavigated Iceland, set out in search of it some time after its "discovery" by the Norse, the pilot on the voyage was his mother, who was "possessed of second sight" (*18*, p. 17) and guided the ship on a direct course from Norway. More provocative, however, is the legend of the discovery of Greenland by Erik the Red.

Erik, so the sagas have it, was aware of a persistent legend in northwest Iceland to the effect that, off on the northwest horizon, there had been repeatedly discerned something which was neither sea ice nor water (*27, 28*). Indeed, when he set out to find a place in which to live out his period of exile, he did so from northwest Iceland on a northwest heading that describes the shortest distance, 300 kilometers, between Iceland and Greenland. Now, it is not possible that this legend arose simply because Greenland is directly visible from Iceland under normal atmospheric conditions. It is not. The curvature of the earth interrupts, by some 63 kilometers, a line of sight through a "normal" atmosphere joining even the highest points near the respective coasts. From a 100-meter elevation—the limit of pasture and therefore of relatively frequent human activity—the minimum visual gap is 120 kilometers and, from a height of 3 meters at the shore, 155 kilometers. There are no significant fishing grounds near northwest Iceland. It therefore seems improbable that people would have set out the minimum of 155 kilometers into the Denmark Strait necessary to allow for direct sighting of Greenland. Yet, in the absence of advance information there is much that argues against Erik's projected route. The winds, pouring off the Greenland icecap, would be almost inevitably contrary, as are the ocean currents. Erik and his people, considering the lack of ability of the Norse ships to tack or even to sail very close to the wind, would almost certainly have been forced to row most or all of the way, only to encounter a permanent and continuous band of sea ice many kilometers wide interdicting any landing on that hostile coast. Only after setting south for hundreds of kilometers did they reach the less inhospitable fjord country of southern Greenland, where the Eastern and Western Settlements were established. Had they taken an immediate heading for southern Greenland, they could have ridden the southward-setting Greenland Current. At the latitude of southern Iceland they would have intercepted the polar easterlies and the westward-setting Irminger Current, and arrived, with luck, almost effortlessly at their destination. The logical basis of Erik's line of action is that he possessed information, which well may have been transmitted by the arctic mirage.

It is not necessary to assume that voyages undertaken in response to information gleaned, at least in part, from the arctic mirage were dependent for their success on the phenomenon occurring throughout the voyage. The Norse, lacking any knowledge of the computation of longitude, possessing only dim notions of latitude, and unable to see the polestar in the summer sky, are known to have used ravens as pilots between northwestern Europe and Iceland (*29*). Iceland itself would have been relatively easy to locate, since, relative to the Faroes, it subtends an angle of approximately 30°. Once well out to sea, however, ravens were released at intervals, on the assumption that these sharp-eyed birds would take the shortest route "as the crow flies," thereby providing a basis for course corrections. At the same time such action might be construed as an act of piety, placing the fate of the voyage in the hands of Odin, sacred to whom was the raven.

Summary

The arctic mirage is a phenomenon that is common in higher latitudes. It occurs under conditions of pronounced temperature inversion, which impart to the air a refractive capability that may equal or exceed the curvature of the earth. Manifestations of the arctic mirage, though largely forgotten in modern times, are described in the earliest accounts of North Atlantic discovery.

This interdisciplinary investigation, combining historical induction with scientific observation and analysis, has suggested a new interpretation of historical events. We believe that information gleaned from these mirages was vital to Norse navigation and exploration in the North Atlantic. We further contend that the mirage may furnish a logical basis for the pervasive ancient and medieval concept of the flat or saucer-shaped world.

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14. The authority on which Nansen based his discussion was research carried out by Alf Torp.
15. There is also an ancient lay, the "*Hafgerdingadrápa*," about this phenomenon.
16. Ivar Bárdsson was sent to Greenland in 1341 by Bishop Hákon of Bergen and served at Gardar in the Eastern Settlement (*12*, vol. 2, pp. 107 and 241).
17. This attempt at explaining the legend in terms of the Moskenström appears to have been advanced first by the historian Peter Frederik Suhm (1728–1798) in his *Historie af Danmark* (1790). See also Nansen (*12*, vol. 2, p. 154).
18. J. Jóhannesson, *A History of the Old Icelandic Commonwealth*, H. Bessasson, Transl. (Univ. of Manitoba Press, Winnipeg, 1974). The reason for the small number of voyages to Iceland appears to lie, in part at least, in the fact that the Icelanders themselves (possibly from lack of timber) owned few vessels capable of trading between Iceland and Europe. See also Nansen (*12*, vol. 2, pp. 98–99).
19. ———, "The date of the composition of the saga of the Greenlanders," reprinted from *Saga-Book*, vol. 16, part 1 (University College, London, 1962).
20. On the other hand, there appear to have been occasional protracted periods in winter when the sea between the Faroes and Iceland was calm and permitted of voyages, even in small craft. Calm conditions would indicate the presence of an extensive high-pressure system which would, almost inevitably, be accompanied by the *hillingar* effect.
21. C. O. Sauer, *Northern Mists* (Univ. of California Press, Berkeley, 1968), p. 175.
22. E. Le R. Ladurie, *Times of Feast, Times of Famine* (Allen & Unwin, London, 1966), pp. 259–261. It has been established that the content of the isotope oxygen-18 in rain and snow decreases with decreasing temperature. Analysis of borings taken from the Greenland icecap corroborates the evidence obtained from plant remains in European bogs, namely that there occurred "an intense warming up which lasted from the seventh to the eleventh century" (p. 261).
23. H. H. Lamb, *The Changing Climate* (Methuen, London, 1966).
24. This is because radiation and conduction cooling reach their maximum cumulative level just at sunrise, after which incoming radiation tends to diminish atmospheric stability.
25. Captain Bartlett's observation of the Snaefells Jökull on 17 July 1939 was made with the sun in the southwest, and he consequently had the light over his shoulder also. At noon of that day, and again at 4 p.m., when the sighting of the Snaefells Jökull was made, the ship was in sunshine. By 6 p.m., however, it was rainy, the air having been warm and calm and the sea smooth throughout (*11*).
26. F. Baur, *Meteorologisches Taschenbuch* (Akademische Verlagsgesellschaft, Leipzig, 1970), vol. 2, p. 525.
27. P. Gayet-Tancréde, *Golden Iceland*, M. Magnusson, Transl. (Almenna Bókafélagid, Reykjavik, 1967), p. 63; A. E. Jensen, *Iceland: Old-New Republic* (Exposition Press, New York, 1954), p. 328.
28. V. Stefansson [*Greenland* (Doubleday, New York, 1947), p. 43] matter-of-factly substantiates this legend: ". . . you cannot go far offshore from northwestern Iceland without being able to see both countries. . . . You cannot climb the mountains of northwestern Iceland on many different days without catching a glimpse of the [Greenland] coast. The mountains . . . are of such height that the tops are inter-visible whenever the skies are clear." For this to be so, however, the *hillingar* effect must be present. Under clear skies, indicative of anticyclonic conditions, it almost inevitably occurs.
29. See the account of the voyage of Hrafn-Floki in H. Pálsson and P. Edwards, Transls., *The Book of Settlements* (Univ. of Manitoba Press, Winnipeg, 1972), p. 17.

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Energy Conservation in New Housing Design

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The total annual energy use, including space heating and cooling, in a typical detached house built before 1970 in Baltimore, Maryland, with a floor area of 140 m^2, amounts to an average 187 gigajoules (10^9 joules) (*1*) of heating fuels and 44 gigajoules of electrical energy (*2*). At 1975 average prices for natural gas and electricity (*3*), the corresponding annual cost is about \$840. If the house has electric resistance heating, its 1975 annual energy costs could be as much as \$1400. These household expenditures are nearly double the expenditures for the same energy requirements in 1970. Thus a primary incentive for increasing the thermal efficiency design of new housing is the increasing cost of energy. However, this is not the only factor stimulating energy consciousness in housing design. The more or less regular recurrence of brownouts in some large cities since 1970, the subsequent unavailability of natural gas to new residential customers in many parts of the country, and finally, of course, the Arab oil embargo of November 1973 all served to place in focus both the vulnerability of the United States to an energy shortage and changing and uncertain patterns of future fuels availability (*4*). Hence the more efficient use of energy has become an urgent national goal which is providing a further impetus to the consideration of energy conservation design features in housing.

The potential impact of these strong economic and national policy thrusts on both existing and new housing is significant. Existing dwellings that have low thermal efficiency and cannot be improved economically will become increasingly obsolete. This will result in decreased property values (relative to

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