to Isolde at CERN, and a powerful velocity separator. At least one group will use the mass separator, with a molten metal target, to try to find superheavies; two other groups will use thick solid targets and chemical separation techniques, to try to find superheavies on the basis of their predicted chemical properties.

The nuclear physics studies at GSI will be very broad in scope, covering not only the high-spin states and the high-energy excitations mentioned by the Nobel laureates, but also testing the interaction mechanisms of very heavy-ions when they hit each other. Studies at Berkeley have already shown that a type of prompt interaction that was little noted with lighter ions tends to dominate the reactions of heavy-ions. Called deep inelastic scattering, the process appears to be the passage of the lighter ion straight through the heavier one, exchanging perhaps 5 to 25 nucleons as it goes by, but not fusing to form a composite system. When uranium was bombarded with argon-40, a composite system was formed half the time. With a projectile twice as heavy, krypton-84, fusion occurred only about 4 percent of the time.

It is considered that only a fusion reaction, and a very special type at that, could produce a superheavy stable nucleus. So in the face of unsuccessful searches so far, and a trend in the reaction mechanism that suppresses fusion, most nuclear scientists are now somewhat pessimistic about the possibility of making them with accelerators. As Ben Mottelson said in Washington, "People are convinced they are there, but all the reactions that have been studied so far produce them in such small numbers that it will be very hard to detect them."

Many scientists have also searched for evidence of the superheavy elements in nature, and Edward Anders and his colleagues at the University of Chicago have reported evidence of fission products from a superheavy element (Z near 114) in the Allende meteorite [*Science* **190**, 1262 (1975)].

Technical Excellence

In a world where low-energy nuclear physics was long ago overshadowed by the size and power of high-energy physics, the scale of GSI is startling. Most nuclear physics laboratories are fairly small facilities, designed to serve a small group of scientists. But GSI is being planned as a national facility. The best comparison in the United States would be with a high-energy physics laboratory. There are 400 people on the staff of GSI, including 200 technicians and support personnel who make up the "accelerator infrastructure." The scientific staff is now 40, including 8 accelerator physicists. With a yearly operating budget that is now $20 million and is not expected to fall below $16 million even after construction is completed, the funding for GSI is at least four times that of the Berkeley SuperHILAC laboratory ($4 to $5 million per year), and only slightly less than that of the Stanford high-energy accelerator.

To a visitor to the GSI laboratory, its technical sophistication is obvious. Almost every bit of equipment incorporates the latest development, whether in electronics, computers, or custom-made experimental instruments. The laboratory has two computers, an IBM 370-168 and a Sigma 6, for batch processing, on-line data analysis, and eventually computer control of the accelerator. Presently ten programmers are working on the accelerator control system, which will continually take diagnostic data from five satellite minicomputers to control the operation of the Unilac. This is particularly important for a heavy-ion machine, because the sources are continually changing as the exit orifice erodes away, and tuning and focusing the accelerator is different for every isotope, ion and energy. The accelerator is being run under manual control now, and computer control will begin in late 1977.

A nice example of the technical adroitness of the Unilac is the method used to control the ion sources, which cannot be wired to the outside because the 250-kilovolt potential for preacceleration would cause shorts. Older accelerators have used fishline, glass rods, and various other homemade inventions to stretch safely across the large voltage drop, but the Unilac uses a glass fiber optic cable. The glass is, of course, an insulator, and the data transmission cable will allow direct control of the source by computer.

Technical competence is only one of many ingredients that go into the successful operation of a research enterprise, but it is an indispensable ingredient. Until the early 1980's, when the new French project is scheduled to be working, GSI should have the premier facility for heavy-ion reserach. No one knows what new things will be found, but with as fine a facility as the Unilac, the West Germans have a good chance of finding them.—WILLIAM D. METZ

Mathematical Proofs: The Genesis of Reasonable Doubt

Mathematicians have known for 40 years that infinitely many statements in mathematics are undecidable—that is, their truth or falsity can be neither proved nor disproved. This disquieting result had a profound philosophical impact on mathematicians because it imposed a barrier within mathematics itself to the formerly invincible methods of proof. Yet gradually mathematicians came to accept and live with this result and to believe, as an act of faith, that showing a statement is decidable is tantamount to showing it can be proved. Now, however, a new twist to this undecidability question has come up. Investigators are finding that even theoretically decidable questions may have proofs so long that they can never be written down, either by humans or by computers.

To circumvent the problem of impossibly long proofs, Michael Rabin of the Hebrew University in Jerusalem proposes that mathematicians relax their definition of a proof. In many cases it may be possible to "prove" statements with the aid of a computer if the computer is allowed to err with a predetermined low probability. Rabin demonstrated the feasibility of this idea with a new way to quickly determine, with one chance in a billion of being wrong, whether or not an arbitrarily chosen large number is a prime.* Because Rabin's method of proof goes against deeply ingrained notions of truth and beauty in mathematics, it is setting off a sometimes heated controversy among investigators.

Rabin became convinced of the utility of a new definition of proof when he considered the history of attempts to prove theorems with computers. About 5 years ago, there was a great deal of interest in this way of proving theorems. This interest arose in connection with research in artificial intelligence and, specifically, in connection with such problems as designing automatic de-bugging procedures to find errors in computer programs. Researchers soon found, however, that

*Rabin presented this result at the symposium on New Directions and Recent Results in Algorithms and Complexity, held at Carnegie-Mellon University in Pittsburgh on 7 to 9 April 1976.

proofs of even the simplest statements tend to require unacceptable amounts of computer time. Rabin believes that this failure at automatic theorem proving may be due to the inevitably great lengths of proofs of many decidable statements rather than to a lack of ingenuity in the design of the computer algorithms.

About 4 years ago, Albert Meyer of the Massachusetts Institute of Technology demonstrated that computer proofs of some arbitrarily chosen statements in a very simple logical system will necessarily be unfeasibly long. The system consists of sets of integers and one arithmetic operation—the addition of the number 1 to integers. It had long been known that any statement in this logical system can be proved true or false with a finite number of steps, but Meyer showed that this number of steps can be an iterated exponential, that is, an exponential of an exponential of an exponential, and so on. A statement of length n can require

$$2^{2^{2^{2^{\cdot^{\cdot^{\cdot}}}}}}$$

steps, in which the number of powers is proportional to n.

Meyer's method of demonstrating how extremely long proofs of some statements must be has now been applied to other logical systems. Meyer and others have obtained similar statements about the unfeasibility of proofs for almost all the familiar decidable results in logic.

About 2 years ago, Meyer and Larry Stockmeyer, now at IBM's Thomas J. Watson Research Center in Yorktown Heights, New York, obtained a more concrete result about lengths of computer proofs. They focused on the original logical system for which Meyer obtained unfeasibility results and asked what length statements would be completely impossible to prove. They defined "completely impossible" as requiring a computer network of 10^{123} components which, according to Meyer, is an estimate of the number of proton-sized objects that would densely fill the known universe. Then, they showed that in order to prove an arbitrary statement consisting of 617 or fewer symbols, a computer would require 10^{123} components.

The problem with proofs, Rabin decided, is the demand that they be correct, with no probability of error. Yet humans constantly make errors in mathematics and all other endeavors. Perhaps because of this, humans who solve problems tend to finish their tasks, whereas computers often stop for lack of time. Rabin then set out to find examples of instances in which a computer cannot finish a task if it is not allowed to err but can complete it if it is permitted the possibility of being wrong. This led him to the result involving prime numbers.

Rabin's test for prime numbers is based on a recent result of Gary Miller of the University of Waterloo in Canada. Miller discovered that if a number n is a prime, every integer between 1 and n will pass a certain mathematical test. If any such integer fails the test, n is not a prime. Miller showed, moreover, that not all integers between 1 and n need be tested. It is sufficient to test integers from 1 to a certain number m that depends on n. If n is not a prime, some integer will fail the test by the time m is reached. Miller's test has the advantage of being relatively quick but has the disadvantage that the number of integers that must be tested increases as n increases.

Rabin found that if n is not a prime, at least half the integers between 1 and n will fail Miller's test. Thus if some number between 1 and n is chosen at random and tested, there is at least a ½ chance it will fail the test if n is not a prime. If two numbers are chosen at random and tested, there is at least a ¾ chance that one of them will fail if n is not a prime. If 30 numbers are chosen at random, there is at least a $1 - (½)^{30}$ chance that one will fail the test if n is not a prime. The chance that 30 randomly chosen numbers between 1 and n all pass the test and that n is not a prime, then, is only $(½)^{30}$ or 1 in 1 billion. This probabilistic method involves the testing of relatively few integers. The number of integers tested is independent of the size of n, but does depend upon what chance of being wrong is risked.

Rabin's probabilistic test is far more rapid than exact tests. Exact tests take so long that the only numbers larger than about 10^{60} that have been tested are of special forms. Rabin can test numbers of that size in about 1 second of computer time. As an example of the potential of his method, he and Vaughn Perapp of the Massachusetts Institute of Technology showed that $2^{400} - 593$ passes his test and thus is a prime "for all practical purposes."

Peter Weinberger, now at Bell Laboratories in Murray Hill, New Jersey, asks what it means to be a prime "for all practical purposes." The uses of large prime numbers include the generation of random numbers and the calculation of fast Fourier transforms. Weinberger contends that existing exact methods to find large primes are sufficient for such purposes. Of Rabin's contention that probabilistic methods of proof are necessary, Weinberger answers, "I'm willing to be convinced. Just show me one substantial example."

Donald Knuth of Stanford University speculates that although Rabin's example has no immediate new applications, once a way to calculate something is found, someone always finds a use for it. He suggests that other difficult problems, such as a class of problems in computer science, called NP complete problems, are good candidates for probabilistic approaches. He believes, however, that the primary impact of Rabin's result is esthetic rather than practical. And on the grounds of esthetics, the contention begins.

Typical of the reactions of many mathematicians is that of one who said he does not accept a probabilistic method of proof because the "glory of mathematics is that existing methods of proof are essentially error-free." Ronald Graham of Bell Laboratories in Murray Hill and others reply that they have more confidence in results that could be obtained by probabilistic methods such as Rabin's prime test than in many 400-page mathematical proofs. Such proofs can often be nearly impossible to check, as is evidenced by a debate over a particular result in homotopy theory, which is a subject in topology. One investigator came up with a proof of a statement and another came up with a proof of its negation. Both proofs were long and very complicated, hence the two investigators exchanged proofs to check each other's work. Neither could find a mistake in his colleague's proof. Now a third investigator has come up with still another complicated proof that supports one of the two original proofs. The verdict, then, is 2 to 1 in favor of one proof, but the problem is still not resolved.

Graham is concerned that long and involved proofs are becoming the norm rather than the exception in mathematics, at least in certain fields such as group theory. He speculates that this situation may arise because there are relatively few interesting statements with short proofs compared to the total number of possible interesting mathematical statements. Fewer and fewer statements with short proofs remain to be worked on. He and Paul Erdös believe that already some of the long proofs being published are at the limit of the amount of information the human mind can handle. Thus Graham and others stress that verification of theorems by computers may necessarily be part of the future of mathematics. And mathematicians may have to revise their notions of what constitutes strong enough evidence to believe a statement is true.—GINA BARI KOLATA