Bauer, Keller, and Reiss began by finding when solutions to nonlinear equations will have secondary bifurcation points and where those points will occur. Others had shown previously where primary bifurcation points (the initial branch points of a solution) will occur. If a nonlinear equation is approximated by a linear equation, primary bifurcation points of the solution to the original nonlinear equation occur at specific values, the eigenvalues, of the solution to the linearized equation.

Bauer, Keller, and Reiss used numerical and perturbation methods to show that when eigenvalues of the linearized equation are of a certain type (when they are multiple eigenvalues) secondary bifurcations can occur. Reiss reports that tertiary bifurcations can occur when the secondary bifurcation points are multiple eigenvalues of the problem linearized about the primary bifurcation solutions. He extends this analysis to provide the basis of a general theory, which he calls the theory of cascading bifurcations.

Cascading bifurcations have been observed in the buckling of plates and shells. If a plate is subjected to increasing pressure, it may buckle when the pressure reaches a certain critical value. If the pressure is then increased, it reaches other critical values at which the plate may buckle further. In the mathematical model of plate buckling, each critical pressure corresponds to a bifurcation point of the solution to a nonlinear equation. At each critical pressure, the plate can assume any one of a number of buckled states, corresponding to several solution branches at each bifurcation point.

Bernard Matkowsky and Thomas Mahar of Rensselaer Polytechnic Institute are applying Reiss's method of calculating cascading bifurcations to a mathematical model that some believe may describe the phenomenon of morphogenesis. The model is based on a hypothesis advanced by A. M. Turing, then at the University of Manchester, in 1952. Turing proposed that biological forms can arise from a homogeneous mixture at chemical equilibrium when that mixture is perturbed. The perturbation would set off a sequence of instabilities in the chemical mixture. This concept was formalized by Ilya Prigogine of the University of Brussels. Prigogine proposed a hypothetical "trimolecular reaction" that could be used to model this process. The equations representing this group of reactions are assumed to have solutions that undergo a cascade of bifurcations. At each bifurcation point, the reactions of the chemical mixture would become increasingly complex until, finally, the mixture would take on a new and highly ordered form. Investigators had previously demonstrated only primary bifurcations of solutions of these chemical equations. Now, Matkowsky and Mahar report that secondary, and possibly cascading, bifurcations occur in these solutions.

Bauer, Keller, and Reiss hope to use their theory of cascading bifurcations to describe the way a smoothly flowing liquid becomes turbulent as the strength of the flow is increased. How to describe this process has baffled physicists and engineers for decades.

One hypothesis to explain turbulence was advanced in 1941 by Russian physicist D. Landau. Landau suggested that turbulence may be described by equations whose solutions undergo a sequence of bifurcations. At each bifurcation, the fluid flow would become more complex until finally it would become completely chaotic. The increased emphasis among mathematicians on describing cascading bifurcations and determining when they will occur may lead to a determination of whether Landau's hypothesis can be used to describe the transition to turbulence.

The established field of bifurcation theory is now the subject of renewed interest due to the new theoretical results in cascading bifurcations and chaotic regimes. It is certainly possible that many complex phenomena may be modeled and analyzed in terms of simple nonlinear equations whose solutions appear as cascading bifurcations.—GINA BARI KOLATA

Paleontology: Facing a Choice Between Fossils and Trash

The destruction of historic sites seems more and more to be becoming a commonplace occurrence as man pushes aside the past to build for the future. In many cases, the destruction of artifacts may be unavoidable or perhaps even justified; taken to an extreme, the desire to preserve history would require that nothing ever be destroyed. But occasionally there arises a case where it is patently obvious that preservation is the only acceptable alternative. Such is the case with the Messel Pit, a major paleontological site in West Germany that, if current government plans are carried through, may soon be filled with trash and other wastes.

The Messel Pit, located about 10 kilometers northeast of Darmstadt, is the site of a lake that formed at the junction of two fissures during the Eocene epoch, more than 50 million years ago. The lake, according to Jens Lorenz Franzen and Siegfried Rietschel of the Senckenberg Research Institute and Museum of Natural History in Frankfurt, apparently contained very little oxygen. Consequently, flora and fauna that fell into the lake underwent little degradation before they were eventually covered by the accumulating sediment. The end result was the formation of a large deposit of oil shale—stratified rock containing as much as 22 percent organic matter and large numbers of well-preserved fossils.

Most of the oil shale was mined over a long period ending in 1971; it was used for a variety of purposes, including the production of medicinals, chemicals, and gasoline. The mining left a large crater about 1000 meters long, 700 meters wide, and 60 meters deep. The crater still contains many pockets of oil shale, and it is the fossils found in this shale that are a source of international concern.

German paleontologists have long recognized that the Messel Pit is the source of excellent specimens. For many years, the Darmstadt Landesmuseum had exclusive rights to dig at the site; but the museum had insufficient funds to sponsor the work, so it let amateurs use the site without supervision. Consequently, most of the better specimens from the site have ended up in private collections without having been catalogued. Only very recently has a more concerted effort been carried out at the site by teams of investigators from the Landesmuseum and the Senckenberg Institute.

The specimens they have obtained so far are quite exceptional. Whereas digging at other sites most often produces remnants of fossil skeletons or jumbled collections of fossil bones, Rietschel says, excavation at Messel commonly produces complete skeletons in a remarkable state of preservation. Digging in the past few weeks, for example, has yielded an intact skeleton of an eohippus, a small horse of the Eocene epoch; the skeleton of a large bat with two smaller bats attached at the breasts; and the skeleton of a large snake with, inside it, the intact skeleton of a young crocodile. In many cases, Rietschel says, it is even possible to observe the shadow of the soft

The wooded slopes of the Messel Pit and the lake at its center make it look more like a mountain retreat than a future garbage dump.

parts of the animals' bodies. It is almost, he adds, as if the animals were "still in some way living."

The fossils obtained from the Messel Pit contain higher than normal quantities of both water and organic materials; they thus begin slowly disintegrating almost as soon as they are removed from the protective shale. Many of the earliest specimens obtained from the pit are, in fact, nearly totally worthless. Investigators from the Senckenberg Institute have thus adopted the procedure of encasing newly found skeletons in polyester or other clear plastics to preserve them for future study.

The abundance and quality of the fossils should be enough to ensure the importance of the Messel Pit. But even more important, according to Donald E. Savage of the University of California Museum of Paleontology, Messel is one of only two sites in Europe (the other is in England) that yield complete, articulated fossil skeletons of land vertebrates from the Middle Eocene era, about 48 million years ago. Most other sites on the continent yield only fossils from the more recent Miocene epoch, that is, 13 to 25 million years ago. Furthermore, Savage adds, no more than three other sites in the world produce as diverse and complete a paleontological record of the Eocene, and none of the other sites is so compact, so concentrated, and so manageable as Messel. Similar conclusions are also expressed by Donald E. Russell of the National Museum of Natural History in Paris.

But scientists are not the only ones who think that the Messel Pit is potentially valuable. The German state of Hessen has selected the pit as a potential repository for incombustible trash, ashes, and chemical wastes from the metropolitan area that includes Frankfurt, Offenbach, and Darmstadt. The trash is currently dispersed at a series of sites in a fashion described by Hessen's Minister of the Environment, Willi Gorlach, as "Wild Western." The sites now in use are not only esthetically displeasing, but are also nearing the limits of their capacity. Gorlach estimates that the Messel Pit could hold 25 million cubic meters of trash, a capacity that should serve the region well beyond the year 2000 and that should enable disposal of the wastes in a manner Gorlach terms "civilized."

The federal government of Germany has apparently shown great ambivalence about the project, but has taken little action. The only important action was an assay of the site by the German equivalent of the U.S. Geological Survey. This agency has suggested that the site is ideal for waste disposal, since the oil shale is encased in a much harder rock that should prevent pollutants leached from the wastes from polluting groundwater.

A major factor in the controversy is the owner of the pit, Ytong AG, a Munich-based firm which uses spent shale for the production of building materials. If the pit is used for waste disposal, Ytong will receive about $2.5 million from the government. If it is used for paleontology, the firm may receive no compensation at all. Ytong, furthermore, in conjunction with the German Bureau of Mining, has limited the number of investigators who may work at the site because it fears lawsuits that might result from accidents. The pit is currently licensed for only 24 workers; the museums, unfortunately, have funds to support only about half that number on a regular basis. (Ytong, incidentally, retains title to all the fossils recovered; they are considered to be on indefinite loan to the museums. Many specimens previously recovered by amateurs, furthermore, are thought to be secreted in basements and private collections in the area.)

A final factor is the 3000 residents of Messel, who are mostly opposed to the use of the pit for trash. They fear that the presence of the trash will lower the value of their property, that odors and smoke will emanate from the pit, and—despite the assurances from federal geologists—that pollutants will contaminate the local water supply. These residents, led by Margit Sauerwein, have petitioned the Ministry of the Environment to find another site for the dump and have generally been quite vociferous in their objections.

What, then, are the alternatives? Many groups, including the museum people, suggest that the trash be placed in mounds that, when covered with soil, would create artificial hills for recreational purposes. The Hessen government argues that this approach would be considerably more expensive, but it does not appear to have calculated the actual costs or to have investigated potential sites. No other alternatives seem to have been considered.

For their part, the investigators from the Senckenberg Institute would like to see the Messel Pit turned into an open-air museum of natural history. The site, itself, is actually quite pleasant; the sides of the pit have been reforested naturally and there is a 20-meter-deep lake at the center (which, unfortunately, covers some of the best fossil sites), so that it looks rather like a mountain valley. Franzen suggests that it would make a good recreation area and that sections of it could be set aside for display of fossils from the site and for reconstructions of the animals that once inhabited it. But even if the government should decide not to use the site as a dump, there are no funds available to carry out such a project.

For the present, then, the paleontologists are stalemated and the government seems to have the upper hand. The only hope for saving the Messel Pit now appears to lie in the mobilization of international opinion against the government plans. Several scientific groups have already urged the preservation of the site, including a group from the Paleontology Society of Germany and participants at a recent international symposium on the study of mammals from the European Tertiary period. Franzen and Rietschel hope other groups will also come to their assistance.

—THOMAS H. MAUGH II