Reports

Past Orientation of the Lunar Spin Axis

Abstract. *The orientation of the lunar spin axis is traced from the early history of the earth-moon system to the present day. Tides raised on the earth by the moon have caused an expansion of the lunar orbit. Tides raised on the moon by the earth have de-spun the moon to synchronous rotation and driven its spin axis to a Cassini state—that is, in a coprecessing configuration, coplanar with the lunar orbit normal and the normal to the Laplacian plane (which is at present coincident with the normal to the ecliptic). This combination of events has resulted in a complex history for the lunar spin axis. For much of the period during which its orbital semimajor axis expanded between 30 and 40 earth radii, the obliquity of the moon was of order 25° to 50°. In fact, for a brief period the obliquity periodically attained a value as high as 77°; that is, the spin axis of the moon was only 13° from lying in its orbit plane.*

It is well known that the present dynamical configuration of the earth-moon system bears little resemblance to its original state at the time of its formation, having been subjected throughout geological time to the influence of mutual tidal torques of the earth, moon, and sun. Although the principal consequences of the tides have been an increase in the lunar semimajor axis and a lengthening of the day, there have also been important changes in the obliquity of the earth and in the inclination of the lunar orbit to the ecliptic (*1, 2*). It is the purpose of this report to point out that the tides are additionally responsible for a rather remarkable history of the lunar spin axis.

Although the current lunar obliquity (the angle between the orbit normal and the spin axis) is small, a major reorientation of the lunar spin axis appears to have taken place early in the life of the earth-moon system, when the lunar semimajor axis was between 30 and 40 earth radii. Theoretical calculations of the past lunar spin state indicate that the obliquity was of order 25° to 50° for a significant fraction of this period. Indeed, for a brief interval of a few hundred thousand years, an obliquity as high as 77° was probably attained. This unusual behavior was generated by the tidal evolution of the lunar orbit, which in turn produced a change in the nature of the moon's rotation state.

At present, the lunar orbit precesses in 18.6 years while its pole (orbit normal) maintains a constant inclination of 5.1° to the normal to the ecliptic, **k**. The lunar spin axis, **s**, is inclined to **k** by 1.6° and is in a coplanar configuration with this vector and the orbit normal, **n**. This situation is illustrated in Fig. 1a. The spin axis is believed to have been driven to this state by tides raised on the moon by the earth—the same mechanism thought to be responsible for the synchronous rotation of the moon (*3, 4*). This specialized state of rotation is known as Cassini motion after G. D. Cassini, who described it in 1643 (*5*). This coplanar configuration of the spin axis, orbit normal, and normal to the ecliptic is maintained by a coprecession of **s** and **n** about **k**. The precession of the lunar orbit is produced by gravitational torques exerted by both the sun and the equatorial bulge of the earth, while it is the attraction of the earth for the figure of the moon that gives rise to the precession of the lunar spin axis. It has been shown (*6*) that coprecession is possible for only certain values of the lunar obliquity, θ, those which satisfy the expression

$$3/2n[(C-A)/C]\sin\theta\cos\theta + 3/8n[(B-A)/C]\sin\theta(1-\cos\theta) + \dot{\Omega}\sin(\theta - I) = 0 \quad (1)$$

where n is the orbital mean motion, $A < B < C$ are the principal moments of inertia of the moon, I is the orbital inclination, and $\dot{\Omega} < 0$ is the precession frequency of **n** about **k**. For other values of θ, the spin axis precesses either too quickly or too slowly to keep pace with the orbit normal. Adopting values $(C-A)/C = 6.2 \times 10^{-4}$, $(B-A)/C = 2.1 \times 10^{-4}$, $2\pi/|\dot{\Omega}| = 18.6$ years, and $I = 5.1°$ for the moon yields the present obliquity, $\theta = 6.7°$, as one solution of Eq. 1. A solution to Eq. 1 is called a generalized Cassini state.

The Hamiltonian describing the spin axis motion

$$H = 1/2Cn^2 - Cn\dot{\Omega}(\mathbf{s}\cdot\mathbf{k}) - n^2(C-A)[3/4(\mathbf{n}\cdot\mathbf{s})^2 - 1/4] - n^2(B-A)\{3/16(1+\mathbf{n}\cdot\mathbf{s}) + 3/8[1-(\mathbf{n}\cdot\mathbf{s})^2]\} \quad (2)$$

referred to a coordinate frame attached to the uniformly precessing orbit plane (*7*), is a constant of motion in the absence of dissipation, such as tidal friction. Equation 2 can be rearranged into a form more easily recognized as a parabolic cylinder. The spin axis trajectory is given by the intersection of this with the unit sphere (*8, 9*)

$$s_x^2 + s_y^2 + s_z^2 = 1 \quad (3)$$

The Cassini states of Eq. 1 are simply those points where the parabola is tangent to the sphere. Since varying H is equivalent to sliding the parabolic cylinder along its axis, Cassini states are also extrema of the Hamiltonian (*6*). In general, there exist either two or four such points, depending on the value of the ratio $n/\dot{\Omega}$ (on which both the axis and the latus rectum of the parabola depend). Peale (*6*) has numbered these states, and Fig. 1b illustrates their nature schematically.

Today, the status of the moon is such that only states 2 and 3 are possible for coprecession (Fig. 1a). The moon occupies state 2. State 3 (with the obliquity near 180°; that is, retrograde rotation) turns out to be unstable in the presence of tides raised on the moon by the earth (*3, 10*) and we shall ignore this state from now on.

It is clear from Eq. 1 that a change in any three of the parameters n, $\dot{\Omega}$, or I will cause a drift in the positions of the Cassini states. Furthermore, from theories of the lunar orbit it appears that all three parameters have been altered by tidal friction (*1, 2*). For a large enough algebraic value of $n/\dot{\Omega}$, such as existed early in the history of the earth-moon system, states 1 and 4 are also possible solutions to Eq. 1.

We cannot in the brief space available here describe all the complexities of the past lunar orbit necessary for an accurate calculation of the past Cassini states. The essential features of the motion can, nevertheless, be illustrated by the use of a simplified model first suggested by Öpik (*11*): that of treating the orbital evolution of a satellite of negligible mass. Under this assumption

$$\dot{\Omega}^2 = [\dot{\Omega}_\oplus\cos^2(\epsilon - \Phi) + \dot{\Omega}_\odot\cos^2\Phi] \times [\dot{\Omega}_\oplus\cos 2(\epsilon - \Phi) + \dot{\Omega}_\odot\cos 2\Phi] \quad (4)$$

with

$$\dot{\Omega}_\odot = 3/4n(M_\odot/M_\oplus)(a/a_\odot)^3 \quad (5)$$

$$\dot{\Omega}_\oplus = nJ(R_\oplus/a)^2 \quad (6)$$

where $M_{\odot}$ is the solar mass, $M_{\oplus}$ is the mass of the earth, a is the lunar semimajor axis, $a_{\odot}$ is the earth-sun distance, $R_{\oplus}$ is the radius of the earth, and $J = 3/2(C_{\oplus} - A_{\oplus})/M_{\oplus}R_{\oplus}^2$. The obliquity of the earth is denoted by ϵ, and Φ measures the angular separation of **k** from the true normal to the ecliptic (*12*). Equation 5 arises from the solar torque and is at present the dominant term. Equation 6 is due to the torque from the oblate earth. From Eq. 4, the dominance of $\dot{\Omega}_{\odot}$ at large a implies $\dot{\Omega} \propto a^{3/2}$, so that the orbital precession was slower in the past. However, for very small a, the torque from the earth dominates and $\dot{\Omega} \propto a^{-7/2}$; that is, the precession becomes more rapid for smaller a. Goldreich has determined that $\dot{\Omega}_{\oplus} \sim \dot{\Omega}_{\odot}$ at $a/R_{\oplus} \sim 17$. This calculation takes into account the change in the oblateness of the earth with the shortened day (*1*).

Past positions of Cassini states 1, 2, and 4 are presented in Fig. 2 as a function of $a/R_{\oplus}$, taking into account the changes in n, Ω, and I brought about by tidal evolution. For large $|n/\dot{\Omega}|$ (small $a/R_{\oplus}$), state 1 lies near the orbit normal while states 2 and 4 are located at large obliquities ($\pm$ 86°). (Negative obliquities indicate that states 1 and 4 lie on the opposite side of **n** from **k**.) As $|n/\dot{\Omega}|$ decreases, states 1 and 4 approach each other, eventually merging at a critical obliquity, $\theta_{crit} = -26°$. This situation develops at $(a/R_{\oplus})_{crit} = 34.2$. For $a/R_{\oplus} > (a/R_{\oplus})_{crit}$, states 1 and 4 cease to exist. Throughout the orbital evolution, state 2 drifts from large obliquities toward **k**.

The motion of a spin axis near a Cassini state can be easily approximated by linearizing the equations of motion obtained from the Hamiltonian. The trajectories are ellipses about Cassini states 1 and 2. On the other hand, state 4 is always dynamically unstable (*3*).

It can be shown that tides not only despin the moon, but drive its spin axis into either Cassini state 1 or 2 (*3, 4*). Hence, elliptical motion near a state is generally short-lived with a tidal decay time of order

$$\tau_{spin} = (C\Delta\dot{\psi}a^6/3GM^2_{\oplus}R^5_{\text{☽}})(Q/k_T)_{\text{☽}} \quad (7)$$

where $\Delta\dot{\psi}$ is the change in the lunar spin frequency, Q is the specific dissipation function, k_T is the tidal Love number, G is the gravitational constant, and $R_{\text{☽}}$ is the radius of the moon. There is an important characteristic of this system which should be stressed: the time scale for orbital evolution

$$\tau_{orbit} = (2M_{\oplus}a^5/3nmR^5_{\oplus})(Q/k_T)_{\oplus} \quad (8)$$

is much longer than the time scale for decay of the spin axis to a Cassini state given by Eq. 7 (*13*). It is this behavior which allows us to determine the history of the spin axis with relative confidence. Regardless of its original orientation at the time of lunar formation, the spin axis drifts into a Cassini state very early. We simply take up the lunar history from that point on—whatever that appropriate semimajor axis might be.

We are now ready to trace the history of the lunar spin axis. Figure 2 shows its trajectory superimposed on the plot of past Cassini states. Provided the moon initially

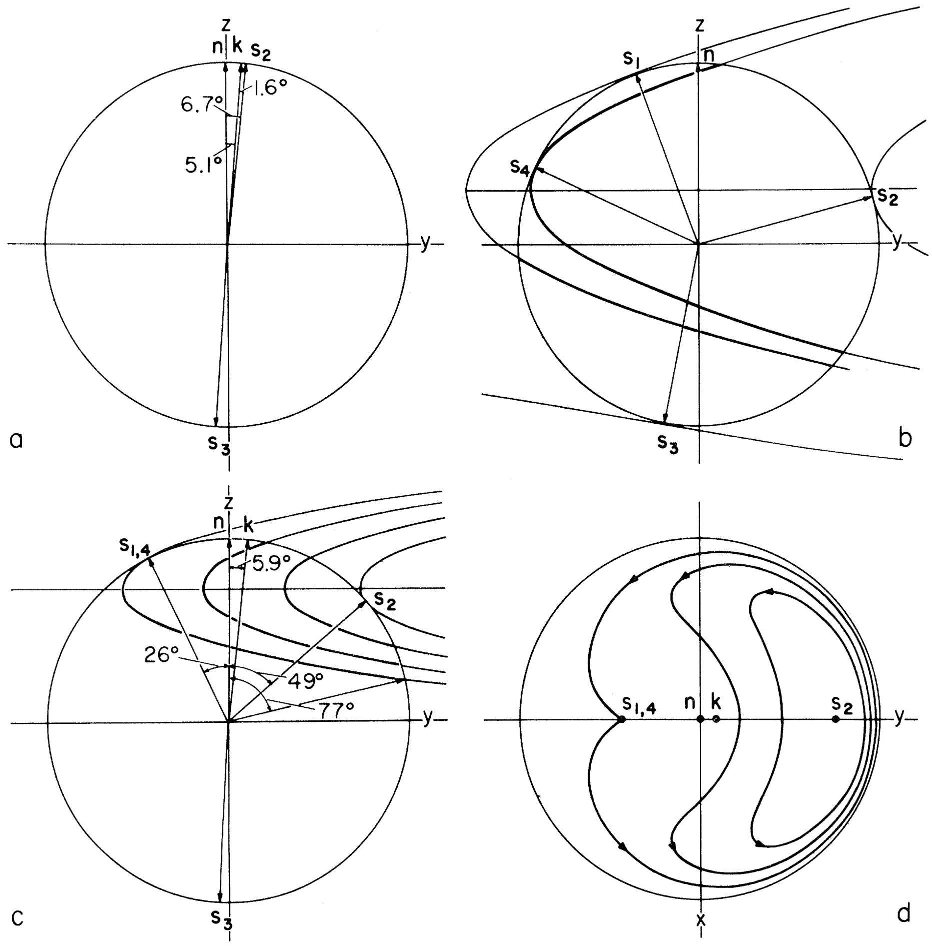

Fig. 1. Relative orientation of the lunar orbit normal, **n**; the normal to the Laplacian plane, **k**; and the lunar spin axis, **s**; viewed from a coordinate system precessing with the orbit plane with the z-axis in the direction of the orbit normal and the x-axis along the line of the ascending node of the orbit on the Laplacian plane. In this coordinate system, **n** and **k** are fixed in the z-y plane. (a) Present coplanar configuration of **n**, **k**, and **s** for the moon. (b) Schematic illustration of the nature of Cassini states 1 through 4. The points where the parabolic cylinder obtained from the Hamiltonian is tangent to the unit sphere locate the state positions. (c) Possible spin axis trajectories following merger of states 1 and 4 with resulting loss of state 1 stability. Tides drive the spin axis through a succession of trajectories to state 2. (d) View of spin axis motion (following merger) projected onto orbit plane.

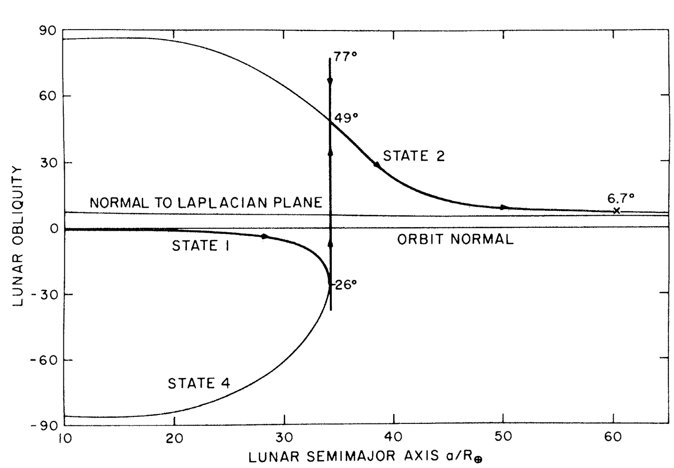

Fig. 2. History of the lunar spin axis and past positions of the Cassini states plotted as a function of lunar orbital semimajor axis measured in earth radii. The spin axis of the moon is driven to state 1 by tidal friction soon after formation. As the orbit expands, the obliquity of the moon increases to 26° at $a/R_{\oplus} = 34.2$, at which time state 1 becomes unstable and the spin axis starts moving on a trajectory surrounding state 2. (A projection of this motion is also shown.) The maximum obliquity during this reorientation is 77°. As the tides drive the spin axis to state 2, the obliquity decays to 49°. Continued expansion of orbit then causes the spin axis to drift to its present position (marked by $\times$).

formed in an orbit interior to $a/R_{\oplus} = 34.2$ its primordial spin decayed rapidly and its spin axis drifted into state 1 (*14*) on a time scale of order (a few) $\times 10^6$ years at $(a/R_{\oplus}) = 30$ or (a few) $\times 10^5$ years at $(a/R_{\oplus}) = 20$ (assuming an initial rotation period of $2\pi/\dot{\psi} = 10$ hours). Following this event, state 1 begins to slowly drift away from the orbit normal, carrying the spin axis with it as the orbit expands. The solid angle traced out by the precession of the spin axis about the Cassini state behaves as an adiabatic invariant: that is, although not exactly constant, it remains nearly so provided the drift rate, $\dot{\theta}$, is slow enough (*3, 15*). In particular, if the spin axis is initially in the direction of a Cassini state, it remains very nearly so.

As the semimajor axis approaches 34.2 earth radii states 1 and 4 merge, and the spin axis, which now has an obliquity of $-26°$, begins to move on a trajectory surrounding state 2. Figure 1, c and d, shows a succession of trajectories converging on state 2 through which the tides drive the spin axis. (These are obtained from the Hamiltonian, Eq. 2.) A maximum obliquity of 77° is reached while traversing the initial trajectory following loss of state 1 stability. Then, as the tides change the value of H, the parabolic cylinder slides along its axis until it is tangent to the unit sphere at state 2. Equation 7 with $\Delta\dot{\psi} \sim n$ and $a/R_{\oplus} = 34.2$ yields $\tau_{\text{tidal}} \sim$ (a few) $\times 10^5$ years before the spin axis settles into state 2 with an obliquity of 49° (*16*). Thereafter, the obliquity continues to decrease as the orbit expands and state 2 drifts toward the normal to the invariable plane. At $a/R_{\oplus} = 60.3$, the present value of 6.7° is achieved.

Although a simplified model of the lunar orbit has been discussed here, a more accurate treatment of earth-moon dynamics has actually been employed in the analysis leading to Fig. 2 (*17*). It appears that in addition to drastically altering the lunar orbit, the tides have resulted in a complex history for the lunar spin axis characterized by very large excursions of this axis from the lunar orbit normal during the early history of the earth-moon system.

WILLIAM R. WARD
Center for Astrophysics, Harvard College Observatory and Smithsonian Astrophysical Observatory, Cambridge, Massachusetts 02138

References and Notes

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5. G. D. Cassini, *Traite de L'Origini ede Progres de L'Astronomie* (Paris, 1693).
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7. The orbit normal is chosen as the z-axis and the x-axis is in the direction of the ascending node of the orbit on the Laplacian plane.
→ G. Colombo, *Astron. J.* **71**, 891 (1966).
9. In writing Eqs. 1 and 2, the eccentricity of the lunar orbit has been ignored and synchronous rotation has been assumed.
→ P. Goldreich and S. J. Peale, *Astron. J.* **75**, 273 (1970).
11. E. J. Öpik, *Irish Astron. J.* **10**, 190 (1972). The advantage of this procedure is that the spin axis of the earth can be regarded as fixed (aside from a very slow precession due to the solar torque on the figure of the earth).
12. The lunar spin axis precesses about the normal to the Laplacian plane, **k**. Although **k** is nearly coincident with the normal to the ecliptic today, it was not always so. A decrease in the semimajor axis causes a rotation of the Laplacian plane toward the equator plane of the earth. However, the inclination, I, of the lunar orbit to the Laplacian plane behaves as an adiabatic invariant (*15*) during the process of orbit expansion and is not significantly changed by this rotation. The inclination has, however, been somewhat altered by direct tidal decay (*1*).
13. Even for an initial spin period of 10 hours and a ratio $[(Q/k_T)_{☽}/(Q/k_T)_{\oplus}]$ of order 10^3, one finds τ_{spin} to be at least an order of magnitude less than τ_{orbit} for $a/R_{\oplus} \geq 5$.
14. Peale has shown that when state 1 is near the orbit normal, it is the most probable end point of spin down (*3*). During this initial event, the Cassini states will undergo some drift, since a more general form of Eq. 1 shows that they are a function of $\dot{\psi}$ also.
15. P. Goldreich and A. Toomre, *J. Geophys. Res.* **74**, 2555 (1969). For the problem at hand, sufficiently slow can be taken to mean $\omega_i \gg \dot{\theta}$, where ω_i is the frequency for spin axis precession about Cassini state i. The constancy of the solid angle about the Cassini state can be appreciated if one imagines the spin axis alternately leading and then trailing the drifting state position. Decreases in the angular separation between the spin axis and Cassini state that occur when the spin axis is leading are very nearly canceled by increases when **s** is trailing if the precession is rapid.
16. Since the spin is already synchronous, changing the obliquity involves changing components of **s** by amounts of order $\Delta\dot{\psi} \sim n$.
17. In constructing Fig. 2 we have included the finite mass of the moon in the analysis of the past orbit, the changes in J for both the earth and the moon, and the tidal decay of the inclination, I, of the lunar orbit to the Laplacian plane. Past positions of the Laplacian plane and the obliquity of the earth have also been found. A more detailed account of this analysis will be presented elsewhere (*18*).
18. W. R. Ward, in preparation.
19. Several valuable suggestions from F. Franklin, H. Mitler, M. Lecar, and A. G. W. Cameron have greatly improved the clarity of the manuscript. Supported by NASA NGR 22-007-269 and NSF GP-40776 at the Harvard College Observatory.

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Catagonus, an "Extinct" Peccary, Alive in Paraguay

Abstract. *A third species of peccary, discovered in the Chaco of Paraguay, is added to the living members of family Tayassuidae. It is assigned to the genus* Catagonus *Ameghino, heretofore considered confined to the Pleistocene. The new peccary is conspecific with* Catagonus wagneri *(Rusconi), a species placed in the related extinct genus* Platygonus *LeConte when it was described from pre-Hispanic archeological deposits of Argentina.*

Organic remains of extinct mammals, such as hides and dung of ground sloths and frozen carcasses of woolly rhinoceroses and mammoths, have been noteworthy findings in the history of mammalogy. There have also been several genera of mammals discovered as fossils and later found to be still living (*1*). We are pleased to announce an addition to this group. A living peccary, related to the genus *Platygonus* LeConte of the upper Pliocene to early postglacial times and to the genus *Catagonus* Ameghino of the Pleistocene, has been found in South America. Twenty-nine specimens were obtained in recent fieldwork (*2*) in the Chaco of western Paraguay.

Peccaries are members of the family Tayassuidae, New World relatives of the pigs or Suidae. Until now, only two species were known to have survived to the present (*3*): the collared peccary or javelina *Dicotyles tajacu* (L.), distributed from the southwestern United States to Argentina, and the white-lipped peccary *Tayassu pecari* (Link) which occurs from southern Mexico to Argentina. But large portions of South America, particularly the thorn forests of the Gran Chaco, extending from southeastern Bolivia through western Paraguay to northern Argentina, remain relatively unstudied. Earlier investigations in Paraguay (*4*) were biased toward the more accessible eastern third of that nation. It is possible that specimens of the third peccary are concealed in some of the

Table 1. Comparison of specimens from Paraguay with *C. wagneri* (holotype and No. 6, pre-Hispanic) and *C. carlesi* (holotype, Pleistocene) (*9*). For specimens from Paraguay, the mean ± standard deviation of the sample and (in parentheses) the observed range and number in sample are given. Abbreviations: C[1], upper canine; PM[2], first upper premolar; M[1-3], first to third upper molars; and M[1], first upper molar.

Specimen	Measurement (mm)					
	Condylobasal length	Width across canines	C[1]–PM[2] diastema	M[1-3] length	M[1] length	M[1] width
Paraguay	266.7 ± 7.7 (255–278) (N = 16)	61.2 ± 1.8 (58.4–63.6) (N = 13)	23.4 ± 3.5 (18.8–28.7) (N = 16)	58.5 ± 2.6 (54.1–64.9) (N = 13)	16.8 ± 0.9 (15.0–18.7) (N = 26)	15.4 ± 0.6 (14.3–16.4) (N = 26)
C. wagneri	—, 265	61, 60	20, 20	57, 54	17, 15	15.5, 16
C. carlesi	287	70	32	51	12.3	14