25 April 1975, Volume 188, Number 4186 SCIENCE

Terrestrial Timekeeping and General Relativity—A Discovery

W. H. Cannon and O. G. Jensen

Terrestrial Timekeeping

Time is a primitive element in the logical structure of physics. Consequently physics does not explicitly define time but rather specifies operational procedures for its measurement in units of seconds. Any such specified procedure to measure time constitutes the definition of a "clock" and its corresponding "time scale." Clocks may be material objects, the observation of which indicates time in seconds directly, or they may be abstract devices by which time is inferred from a set of physical observations through the intermediary of a theory. The former are known as "real clocks," of which the wristwatch is an example, and the latter are known as "paper clocks," of which the orbital motions of the planets are an example.

The quality of a clock is judged by two criteria applied to its time scale: resolution and uniformity. Resolution pertains to the ability of a clock to resolve events closely spaced in time and assign them to different points on its time scale. Today there are clocks in use in the field of particle physics which provide temporal resolution of events spaced only 10^{-16} second apart. Uniformity of a time scale is a subtler notion. A time scale is said to be uniform if the observed dynamical phenomena of the universe, when measured against that time scale, are in accordance with those predicted by theory. If, according to accepted theory, free particles travel in straight lines at a constant speed, then the observed position of a free particle as a function of any given time scale will provide a test of the uniformity of that time scale. If the paths of free particles are observed not to be straight lines, then either there are hitherto unsuspected forces accelerating the particles and they are not free at all or the clock providing the time scale against which the particles' motions are observed does not run uniformly. Faced with such a situation, the physicist must determine which of these two alternatives prevails and either expand his theoretical framework to include the new forces or define a new clock and its corresponding time scale which "straightens out" the paths of the free particles.

Historically the effort of defining clocks and time scales possessing ever greater resolution and uniformity has led man to the use of a variety of ingenious paraphernalia such as candles, water clocks, and sundials. Following the invention of the telescope by Galileo in 1610 the most uniform time scale was provided by the rotation of the earth. The rotation of the earth was regarded as a frequency standard which, through observations of the transit of the stars or the sun, provided one with sidereal and solar time scales, respectively. The secular decrease of the earth's rotation rate due to tidal interaction with the sun and moon was known to 19th-century physicists, who theorized about the existence of short-term fluctuations in the rotation rate of the solid earth due to daily, seasonal, decadal, and irregular changes in its moment of inertia as well as exchanges of angular momentum between the solid earth and the oceans, atmosphere, and liquid core. By the early 1930's the steady improvement in terrestrial timekeeping devices had revealed such fluctuations (*1*), and the earth's rotation rate was abandoned as a primary frequency standard and replaced by the orbital motions of the planets. The sidereal and solar time scales were superseded by the ephemeris time scale. Ephemeris time is the independent variable in the gravitational theory of planetary orbits and is by definition uniform. Ephemeris time is provided by a paper clock which uses photographic observations of the position of the moon relative to a background of fixed stars and possesses a resolution of about ± 5 seconds.

Atomic time (*2*) originated in the late 1950's with the development of stable atomic oscillators whose frequencies are derived from the quanta radiated during transitions between excited states of various atoms or molecules. The Bureau International de l'Heure (BIH) in Paris has maintained a scale of atomic time since 1958. This time scale, designated A1, was provided by a real clock in Paris and was designed as the continuation without time step of UT2 past the epoch January 1958 00^{h} UT2. (UT2 is the most uniform solar time scale, solar time having been corrected first for the motion of the earth's pole of rotation to give UT1, and then for seasonal changes in rotation rate to give UT2.)

In 1967, at the request of the International Astronomical Union (IAU), A1 was replaced by A3, which became both an official international scale of atomic time and a paper clock. The frequency standard providing A3 was defined in 1967 as the weighted mean of an initial set of three atomic frequency standards maintained at the Physikalisch-Technische Bundesanstalt (PTB), Braunschweig, Germany; the

W. H. Cannon is assistant professor of earth science, Department of Physics, York University, Toronto, Canada. O. G. Jensen is assistant professor of geophysics, Department of Mining and Metallurgical Engineering, McGill University, Montreal, Canada. Both authors are members of the Aurora Institute for Advanced Studies.

Observatoire de Paris (OP), Paris, France; and the U.S. Naval Observatory (USNO), Washington, D.C. The frequencies of these standard oscillators were intercompared by very low frequency radio transmissions, and the difference between the computed mean normalized frequency, a_3, for the paper clock providing A3 and the normalized frequency, e_{OP}, of the standard in Paris was continuously monitored. The atomic time, A3, corresponding to any epoch t could be materialized in Paris by numerically computing the integral

$$\mathrm{A3} - \mathrm{AT_{OP}} = \mathrm{A3} - \mathrm{AT_{OP}}|_{t_0} + \int_{t_0}^{t} (a_3 - e_{OP})\,dt$$

where $\mathrm{AT_{OP}}$ is atomic time reckoned by the OP frequency standard and where

$$a_3 = \frac{\sum_{i=1}^{3} p_i e_i}{\sum_{i=1}^{3} p_i}$$

and where e_i are the normalized frequencies of the standards of the three contributing observatories and p_i are their weights. Atomic time could be materialized at any other of the three contributing observatories, providing a frequency standard had remained operational there, by calculating

$$\mathrm{A3} - \mathrm{AT}_i = (\mathrm{A3} - \mathrm{AT_{OP}}) - (\mathrm{AT}_i - \mathrm{AT_{OP}})$$

Toward the end of 1968 the Loran-C (Long Range Aid to Navigation) chains became operational. The Loran-C network consists of about 30 stations grouped into eight chains known as the Hawaiian, Aleutian, U.S. East Coast, Mediterranean, North Atlantic, Northwest Pacific, Southeast Asian, and Norwegian chains. Each chain consists of one master and several slave stations. The Loran-C stations broadcast precisely spaced pulsed time signals on a 100-kilohertz carrier which is phase locked throughout the chain to a cesium frequency standard at the master station. In the case of the U.S. East Coast chain the master station at Cape Fear, North Carolina, is synchronized with the frequency standard at the USNO. The emissions from the slaves are delayed by a precise amount relative to the master, and lines of constant delay form a hyperbolic network of coordinates useful for navigation over a range of about 3000 kilometers.

The operation of the Loran-C network and the high quality of its frequency standards facilitated the synchronization of remote clocks by low frequency ground wave phase delay measurements to ± 0.2 microsecond once the necessary propagation delays had been measured by clock transportation. Largely as a result of this new capability, International Atomic Time was redefined in 1969 to be AT, the weighted mean of the individual atomic time scales AT_i maintained at the contributing observatories. The number of the contributing observatories was expanded to include, in addition to the initial set of three, the National Research Council (NRC), Ottawa, Canada; Royal Greenwich Observatory (RGO), Herstmonceux, England; National Bureau of Standards (NBS), Boulder, Colorado; and Observatoire de Neuchâtel (ON), Neuchâtel, Switzerland. Thus in 1969 A3 was superseded by AT

$$\mathrm{AT} = \frac{1}{\sum_i p_i} \sum_i p_i[\mathrm{AT}_i + B_i(t - t_0) + A_i]$$

where the constants A_i were chosen so that no time step appeared on 1 January 1969 between A3 and AT. The constants B_i should have been chosen to account for (i) systematic errors in the clocks which would appear if the clocks were running side by side, manifesting themselves in a correction β_i, and (ii) relativistic effects between the various clock sites, manifesting themselves in a correction γ_i

$$B_i = \beta_i + \gamma_i$$

However no international agreement on the appropriate definition of the B_i has been reached, and the BIH has been forced to unilaterally establish a set of B_i, which are chosen to remove long-term trends between the individual atomic time scales AT_i subject to the constraining condition that

$$\sum_i p_i B_i = 0$$

to ensure that their choice did not affect the scale of International Atomic Time, AT.

Also beginning in 1969, as a service to surveyors and navigators who require solar time, the national time services were charged with the responsibility of maintaining Coordinated Universal Time (UTC) which was to be derived from AT subject to the requirement that it remain within 0.1 second of UT2. To this end, each observatory maintained a time scale UTC_i, which was derived from AT_i by internationally agreed on frequency offsets and time steps announced in advance by the BIH, as well as small frequency offsets and time steps locally chosen by observatories for coordination with one another. Frequency offsets in UTC_i of plus or minus an integral multiple of 50 parts in 10^{10} occurred when necessary at the beginning of each year, and time steps in UTC_i of plus or minus an integral multiple of 100 msec occurred when necessary at the beginning of each month. Locally chosen perturbations to UTC_i for purposes of coordination could occur at any time.

This rather unsatisfactory situation prevailed from 1969 until 1972 when the BIH and its contributors adopted recommendation 460 of the International Radio Consultative Committee (CCIR) which had convened in New Delhi in 1970. Recommendation 460 of the CCIR provided that from a specified date (i) the carrier frequencies of the atomic oscillators contributing to UTC would be kept constant, (ii) the intervals on the time scales UTC_i of the contributing observatories would correspond to the adopted definition of the second, and (iii) UTC would be stepped to maintain approximate agreement with UT2 only by the internationally agreed on insertion or deletion of intervals of exactly 1 second, otherwise known as positive or negative leap seconds.

Recommendation 460 was implemented by the BIH and its contributors on 1 January 1972 at $00^h 00^m 00^s$ AT, at which time UTC was set to $00^h 00^m 00^s$ UTC. Since that date the BIH has determined, at intervals of 10 days, the values of the interval residuals $\mathrm{UTC} - \mathrm{UTC}_i$ between the clocks of the contributing observatories and the paper clock UTC. These interval residuals are generally determined by Loran-C transmissions with occasional control checks by actual clock transportation. Clock transportations, when they occur, corroborate the interval residuals determined by Loran-C to within a few tenths of a microsecond and frequently better. Insofar as an observatory abides by the ruling of recommendation 460 the time scale UTC_i should correspond, apart from systematic errors and leap seconds, to a measure of proper time at that location. Unlike the published Atomic Time interval residuals $\mathrm{AT} - \mathrm{AT}_i$, which have had linear trends between clocks empirically removed by the B_i's, the Coordinated Universal Time interval residuals $\mathrm{UTC} - \mathrm{UTC}_i$ published since January 1972 have been left unaltered.

Relativity

The special theory of relativity is founded on two hypotheses:

1) The laws of physics do not distinguish between coordinate frames moving at constant relative velocity.

2) The speed of light in free space is the same for all observers.

These postulates lead directly to the Lorentz transformation and the invariance under that transformation of the interval ds

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \quad (1)$$

where c is the velocity of light, and x, y, z are the Cartesian spatial coordinates and t the time coordinate assigned to events by an observer in an inertial frame S. Inertial frames are defined in relativity theory as the class of coordinate frames in which Newton's first law is valid. More precisely, an inertial coordinate frame is one with respect to which a free particle either remains at rest or persists in uniform rectilinear motion. Inertial frames of reference S and S′, characterized respectively by coordinates x, y, z, t and x', y', z', t' related one to the other by linear transformations of the Lorentz type, are known as Galilean or Lorentz frames.

It follows from Eq. 1 that, for an observer in an inertial frame, proper time intervals (that is, time intervals measured by an ideal clock at rest with respect to the observer) are proportional to the length of the observer's world line, $\int ds$. The converse is also true: the length of the world line of an observer in uniform motion in an inertial frame can be directly measured by the timekeeping of a comoving ideal clock.

At this point in the development of the theory there is a question concerning accelerated observers. Since accelerated observers are not considered by hypotheses 1 and 2 one must ask, Does the same relationship hold between the length of an accelerated observer's world line, $\int ds$, and the timekeeping of an ideal comoving clock? This question is usually answered in the affirmative by the introduction of a third hypothesis (*3*, p. 49):

3) The inertial (nongravitational) acceleration of a clock relative to an inertial frame has no influence on the rate of the clock.

That this constitutes a third, often unstated, hypothesis of relativity theory has been clearly pointed out by Fock (*4*, p. 234). Furthermore, like hypothesis 1 and 2, hypothesis 3 is a physical hypothesis and amenable to empirical test.

Equation 1 can be written in tensor notation as a quadratic form

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu \quad \mu, \nu = 0, 1, 2, 3 \quad (2)$$

where $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, and $\eta_{\mu\nu}$ is the metric tensor of special relativity or "flat" space-time.

$$\eta_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (3)$$

A theory of gravitation, often referred to as the general theory of relativity, can be constructed from hypotheses 1, 2, and 3 with two additional hypotheses:

4) Inertial mass and gravitational mass are directly proportional.

5) There exist, even in the presence of gravitational fields, space-time coordinate frames which are locally Galilean.

The modern theory of gravitation leads to a generalization of the quadratic form for the interval

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad \mu, \nu = 0, 1, 2, 3 \quad (4)$$

where $g_{\mu\nu}$ is the metric tensor of a curved space-time manifold whose form is determined through a set of field equations (*5*, p. 406)

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}R = 8\pi T_{\mu\nu} \quad (5)$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R its trace, and $T_{\mu\nu}$ the stress-energy tensor. In this theory of gravitation the concept of the world line of an observer in uniform motion in an inertial frame which obtains in special relativity generalizes to the concept of a timelike geodesic of the space-time manifold. Timelike geodesics of the metric are defined by the standard geodesic equation (*5*, p. 211)

$$\frac{d^2x^\mu}{ds^2} + \Gamma^{\mu}_{\sigma\lambda}\frac{dx^\sigma}{ds}\frac{dx^\lambda}{ds} = 0$$
$$ds^2 > 0 \quad (6)$$

and are in reality the world lines of observers in free fall in the presence of gravitational fields. It follows from these hypotheses that the length of the world line of an observer in free fall in a gravitational field—or equivalently, the length of a timelike geodesic of the curved space-time manifold—is measured directly by the timekeeping of a comoving ideal clock.

At this point in the development of the modern theory of gravitation there is a question concerning observers not traveling timelike geodesics of the space-time manifold. Such observers are not falling freely in gravitational fields and are hence being accelerated. This question is a generalization of that previously stated concerning accelerated observers in special relativity: Does the same relationship hold between the length of the world line of an observer not traveling a geodesic of the space-time manifold and the timekeeping of an ideal comoving clock? Once again this question is usually answered in the affirmative by an appeal to hypothesis 3.

Many of the five hypotheses and their consequences have been tested and confirmed by a host of experiments. Hypotheses 1 and 2 have been precisely tested and confirmed by observations of the predicted relativistic changes in mass, energy, and momentum of atomic and subatomic particles moving with large relative velocities. Hypothesis 4 was tested and confirmed by Galileo in the 16th century, but more recently by Braginsky and Panov (*6*) to a precision of 1 part in 10^{12}. Hypothesis 5 was tested by and confirmed to an accuracy of 1 percent by the gravitational redshift experiment of Pound and Snider (*7*). Hypothesis 3 is the exception, having never been tested or confirmed with macroscopic timekeepers.

In this article, we present the results of an empirical test of hypothesis 3 involving almost ideal macroscopic timekeepers: modern atomic clocks. We do this by comparing the predicted behavior of atomic clocks under the assumption of the validity of hypothesis 3 to the predicted behavior of the same set of atomic clocks when the assumption of hypothesis 3 is relaxed. If one is to relax the assumption of hypothesis 3, one must necessarily replace it with another. Of the infinity of possible choices, we shall consider an extension of the principle of equivalence whereby we assume that accelerations are capable of direct influence on clock rates. Before doing so, it is instructive to examine the content of the principle of equivalence.

The principle of equivalence proposed by Einstein (*8*) in 1911, asserts the complete physical equivalence of the effects of a homogeneous gravitational field and a field of uniform acceleration relative to some inertial frame. Since any real gravitational field (the gravitational field of a finite mass) is nonhomogeneous, it is known that the principle of equivalence can only be true over an infinitesimal or local region of space. The principle asserts that if measurements are restricted to an infinitesimal region of space it is not possible by any means to distinguish the effects of gravitational fields from the effects of inertial accelerations. The principle of equivalence rests heavily on hypotheses 4 and 5 while containing something of the sentiment of hypothesis 1.

Appealing to the principle of equivalence, Einstein (*8*) was able to deduce, among other things, the gravitational redshift effect. He then used this result to argue that the rate of timekeeping of an ideal clock should depend on the value of the gravitational potential at the point in question. Although he expressed this in a somewhat confusing fashion, Einstein asserted that the relationship between the time intervals $d\tau_1$ measured by an ideal clock in a region of gravitational potential Φ_1 and the time intervals $d\tau_2$ measured by an ideal clock in a region of gravitational potential Φ_2 would be

$$\frac{d\tau_1}{d\tau_2} = 1 + \frac{\Phi_1 - \Phi_2}{c^2} \qquad (7)$$

where $\Phi \leqq 0$ is given by the volume integral

$$\Phi(\mathbf{r}) = -G \int \frac{\rho(\mathbf{R})}{|\mathbf{R} - \mathbf{r}|} dv \qquad (8)$$

Clocks located at positions of relatively higher gravitational potential would run faster according to the principle of equivalence.

These sorts of arguments culminated finally in the publication of Einstein's general theory of relativity wherein the gravitational field is manifested by an appropriate scaling of the units of measure of the space-time manifold and incorporated into the geometry of space-time through the metric tensor $g_{\mu\nu}$. The effects of gravitational fields on relative rates of clocks transported along geodesics of space-time can be viewed, even without the assumption of hypothesis 3, as relative differences between the magnitudes of the units of measure along their respective paths. The statement of the principle of equivalence can be extended to suggest that inertial accelerators are equally capable of directly altering the units of measure of clocks. Nothing compels us to assume that clocks with identical velocities but different accelerations will be going at the same rate. Although such an assumption is built into relativity theory and constitutes the essence of hypothesis 3, it is equivalent to the assertion that two identical clocks side by side and at relative rest, one in the gravitational field and one not, will also run synchronously. However, the physical meaning of the tensor $g_{\mu\nu}$, describing gravitational fields, would deny the synchronism of such clocks. While the physical realization of the above condition is difficult to conceive of, it can be duplicated instantaneously by inertial accelerations. Clocks can be subjected to different accelerations while having identical velocities. In such instances, the equivalence principle suggests that synchronism between such pairs of clocks must also be denied. Accordingly, we shall construct an ad hoc theory, an extension beyond current relativity theory, which at least for the case of uniform rotation will attempt to incorporate the possible effects of inertial accelerations on clock rates.

Given a field of acceleration $\mathbf{a}(\mathbf{r})$ it is possible to define acceleration potentails $U(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ such that

$$\mathbf{a}(\mathbf{r}) = -\nabla U(\mathbf{r}) - \nabla \times \mathbf{A}(\mathbf{r})$$

where, for an appropriate choice of gauge, $U(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ are both given by a Poisson equation and hence are uniquely determined to within an arbitrary constant.

Newtonian potentials $\Phi(\mathbf{r})$ are single valued in $\mathbf{r}$ for all observers. This is a consequence of the equality of inertial and gravitational mass. All observers at the same point in a gravitational field experience the same gravitational acceleration. Unlike Newtonian potentials, acceleration potentials $U(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ need not be single valued in $\mathbf{r}$ for all observers. It is possible for two observers to occupy the same point in space, $\mathbf{r}_0$, and experience different inertial accelerations $\mathbf{a}_1(\mathbf{r}_0)$ and $\mathbf{a}_2(\mathbf{r}_0)$. These observers would generally regard their accelerating forces as being derived from potentials $U_1(\mathbf{r}_0)$, $\mathbf{A}_1(\mathbf{r}_0)$, and $U_2(\mathbf{r}_0)$, $\mathbf{A}_2(\mathbf{r}_0)$.

The principle of equivalence serves as a useful guide in examining the possible effect that inertial accelerations might have on the timekeeping of ideal clocks. The principle of equivalence applies only to observers who are restricted to making local space-time measurements. To such an observer, the field of acceleration $\mathbf{a}(\mathbf{r})$ would be derivable from a potential $U(\mathbf{r})$ by

$$\mathbf{a}(\mathbf{r}) = -\nabla U(\mathbf{r}) \qquad (9)$$

for only by nonlocal measurements could the observer detect terms in his acceleration field involving $\nabla \times \mathbf{A}(\mathbf{r})$. By an argument similar to that presented by Einstein (*8*), it is possible to argue that the relative rate of timekeeping between an inertially accelerated clock and a clock at rest in an inertial frame is

$$\frac{d\tau}{dt} = 1 + \frac{U}{c^2} \qquad (10)$$

The local comparison of two inertially accelerated clocks would yield a relative proper time rate of

$$\frac{d\tau_1}{d\tau_2} \cong 1 + \frac{U_1 - U_2}{c^2} \qquad (11)$$

Equation 11 expresses the fact that the arbitrary constant in the potential is the same for all clocks, which follows from the necessity that two identical ideal clocks, side by side in identical states of acceleration, must reckon proper time identically. Furthermore, the requirement that this theory reduce in the absence of inertial accelerations to the standard theory of gravitation compels the arbitrary constant in the potential to be identically zero.

The two lines of argument, one based on hypothesis 3 and the one given above based on an extension of the principle of equivalence, lead to a contradiction. It is possible with modern atomic clocks to resolve this contradiction with an empirical test. Such a test is the subject of the remainder of this article.

Terrestrial Timekeeping and General Relativity

All theories of gravitation are constrained by the conditions that in the limit of low mass and energy densities, implying weak gravitational fields and low velocities, they must degenerate to the theory of special relativity. This can be expressed analytically by insisting that in such cases

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad (12)$$

where $|h_{\mu\nu}| \ll 1$. This suggests the possibility of a perturbation expansion for the elements of $g_{\mu\nu}$ in terms of the small quantities of order ε, namely mass and energy densities, responsible for the departure of the metric from the form $\eta_{\mu\nu}$.

Such a perturbation expansion has been carried out (*5*, p. 1080) and to first order, $O(\varepsilon)$

$$h_{\mu\nu} = \begin{bmatrix} 2\Phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (13)$$

where Φ is now the dimensionless Newtonian gravitational potential

$$\Phi(\mathbf{P}_i) = -\frac{G}{c^2} \int \frac{\rho_o(\mathbf{P})}{|\mathbf{p} - \mathbf{p}_i|} dv \qquad (14)$$

Setting $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and considering a static distribution of mass in which mechanical stresses are negligible compared to mass-energy density, namely

$$T_{\mu\nu} = \begin{pmatrix} c^2\rho_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (15)$$

we find that Einstein's field equations, Eq. 5, yield Laplace's equation for the gravitational potential

$$\nabla^2\Phi = \frac{1}{c^2} 4\pi G\rho_0 \quad (16)$$

and the equation for the geodesics, Eq. 6, yields

$$\frac{1}{c^2}\frac{d^2x^i}{dt^2} = -(\nabla\Phi)_i, \; i = 1, 2, 3 \quad (17)$$

Equation 17 expresses the fact that in Newtonian dynamics the gravitational acceleration on a freely falling mass is everywhere in the direction of local **g**.

The second-order terms in this perturbation expansion yield "post-Newtonian" corrections to the space-time metric (*5*, p. 1080),

$$h_{\mu\nu} = \begin{bmatrix} 2\Phi & O(\epsilon^3) & O(\epsilon^3) & O(\epsilon^3) \\ O(\epsilon^3) & 2\gamma\Phi & O(\epsilon^4) & O(\epsilon^4) \\ O(\epsilon^3) & O(\epsilon^4) & 2\gamma\Phi & O(\epsilon^4) \\ O(\epsilon^3) & O(\epsilon^4) & O(\epsilon^4) & 2\gamma\Phi \end{bmatrix} \quad (18)$$

where γ is a parameter taken to be unity in Einstein's theory but assuming other values in other theories of gravity.

To examine the world lines of terrestrial timekeepers it is necessary to specify coordinates of these world lines relative to some inertial frame. It is convenient to use an inertial frame comoving with the center of mass of the earth as it travels in free fall about the sun. Such a geocentric inertial frame must initially be confined to an infinitesimally small region surrounding the geocenter, for any attempt to extend the frame globally to provide space-time coordinates for the world lines of surface terrestrial timekeepers is immediately met with gravitational forces. However by hypothesis 5 the construction of such a global coordinate frame is possible, and the solution to the problem is provided by the theory of gravitation and the post-Newtonian metric tensor. The post-Newtonian metric tensor provides us with a global coordinate frame which is nearly inertial everywhere and relative to which one is able to compute the lengths of world lines, $\int ds$, for terrestrial timekeeping observatories.

From Eqs. 4, 12, and 18 it follows that the line element for the terrestrial post-Newtonian metric, valid for weak gravitational fields ($|\Phi| \ll 1$) and low velocities ($|v/c| \ll 1$), is

$$ds^2 = (1 + 2\Phi)c^2dt^2 - (1 - 2\gamma\Phi)(dx^2 + dy^2 + dx^2) \quad (19)$$

If Φ is interpreted as the Newtonian gravitational potential in the vicinity of the earth, then Eq. 19 describes the interval between space-time coordinate points x, y, z, t and $x + dx, y + dy, z + dz, t + dt$ in the neighborhood of the earth. The interval between space-time coordinate points $P_1 = (x_1y_1z_1t_1)$ and $P_2 = (x_2y_2z_2t_2)$ separated by a timelike geodesic, G, is given by

$$\Delta s_G = \int_G ds \quad (20)$$

The theory of gravitation asserts that Δs_G is directly proportional to the interval of proper time $\Delta\tau_G$ reckoned by an ideal clock comoving with an observer who is on a free-fall trajectory (geodesic) passing through points P_1 and P_2

$$\Delta\tau_G = \frac{1}{c}\Delta s_G \quad (21)$$

It is possible for an observer who is inertially accelerated to pass through points P_1 and P_2 along a general timelike curve C which is not a geodesic of the space-time manifold. Let us assume that the interval of space-time Δs_C traversed by such an observer is given as before by

$$\Delta s_C = \int_C ds \quad (22)$$

for the current theory of gravitation holds that an observer's path through the space-time manifold cannot affect its geometry.

However, as we pointed out above, one can legitimately ask, Does the same relationship hold between the interval Δs_C and the interval of proper time $\Delta\tau_C$ measured by a comoving ideal clock when the clock is carried by an inertially accelerated observer?

An appeal to hypothesis 3 answers the question in the affirmative. According to hypothesis 3, Eq. 19 applies to both gravitationally and nongravitationally accelerated observers; as before

$$\Delta\tau_C = \frac{1}{c}\Delta s_C \quad (23)$$

An extension of the principle of equivalence answers the question in the negative. This use of principle of equivalence asserts that accelerations, like gravitational fields, affects the rates of timekeeping of ideal clocks and that the interval of proper time $\Delta\tau_C$ is not related to the traversed interval of space-time Δs_C by the simple proportionality of Eq. 23

$$\Delta\tau_C \neq \frac{1}{c}\Delta s_C \quad (24)$$

Using the principle of equivalence in this way as a guide in a search for an expression to replace Eq. 24 we suggest that, in the case of an inertially accelerated clock, it may be possible to identify the increment of proper time $d\tau_C$ with the quantity $d\Sigma$ where

$$d\Sigma^2 = ds^2 + d\sigma^2 \quad (25)$$

where ds^2 is given by Eq. 19 and where $d\sigma^2$ is a "correction" term allowing for the observer's departure from free fall. Examination of Eqs. 7, 11, and 19 further suggests that $d\sigma^2$ be given to first order by

$$d\sigma^2 = 2U\, c^2\, dt^2 \quad (26)$$

where U is the dimensionless acceleration potential obtained from Eq. 9 by dividing by c^2. Accordingly $d\Sigma$ can be written to first order

$$d\Sigma^2 = [1 + 2(\Phi + U)]c^2dt^2 - (dx^2 + dy^2 + dz^2) \quad (27)$$

The quantity $d\Sigma$ can be integrated along the world line of the inertially accelerated clock

$$\Delta\Sigma_C \int_C d\Sigma \quad (28)$$

and the relationship between the quantity $\Delta\Sigma_C$ and the proper time interval $\Delta\tau_C$ reckoned by the inertially accelerated clock is

$$\Delta\tau_C = \frac{1}{c}\Delta\Sigma_C \quad (29)$$

It is possible to use the terrestrial atomic clocks of the national time standards of the world to compare empirically the accuracy with which the two expressions for ds (Eq. 19) and $d\Sigma$ (Eq. 27) describe the intervals of proper time $d\tau$ reckoned by these accelerated clocks. To examine terrestrial timekeepers it is convenient to cast the problem in cylindrical coordinates. Since Eq. 27 contains Eq. 19 as a special case where $U = 0$ we shall consider only

$$d\Sigma^2 = [1 + 2(\Phi + U)]c^2dt^2 - (dr^2 + r^2d\lambda^2 + dz^2) \quad (30)$$

For a clock fixed on the surface of the earth $dr/dt = 0$, $dz/dt = 0$, and $d\lambda/dt = \Omega$, where Ω is the earth's sidereal rotation rate. Since the elements $|h_{\mu\nu}| \ll 1$ Eq. 30 yields

$$d\Sigma = [1 + \Phi + U - \frac{r^2\Omega^2}{2c^2}]cdt \quad (31)$$

Table 1. Locations and equipment of time laboratories.

Code	Laboratory and location	Equipment	Latitude (°N)	Elevation (m)
PTB	Physikalisch-Technische Bundesanstalt, Braunschweig, Germany	Six HP Cs* One primary Cs	52.30	80
USNO	U.S. Naval Observatory, Washington, D.C.	Sixteen HP Cs Two hydrogen maser	39.92	80
OP	Observatoire de Paris, Paris, France	Three HP Cs	48.83	60
NBS	National Bureau of Standards, Boulder, Colorado	Eight HP Cs One primary Cs	40.05	1650
RGO	Royal Greenwich Observatory, Herstmonceux, England	Five HP Cs	52.86	25
NRC	National Research Council, Ottawa, Canada	Three HP Cs One primary Cs	45.45	100
ON	Observatoire de Neuchâtel, Neuchâtel, Switzerland	One E Cs† Two HP Cs	47.00	490

*HP Cs, Hewlett-Packard cesium standard. † E Cs, Ebauches cesium standard.

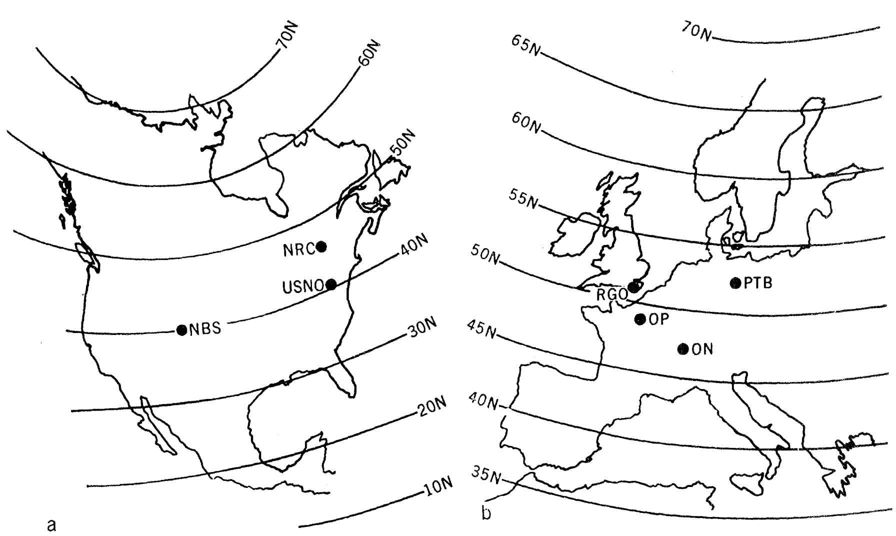

Fig. 1. Geographical locations of the atomic timekeeping laboratories contributing to International Atomic Time during 1972 and 1973 in (a) North America and (b) Europe.

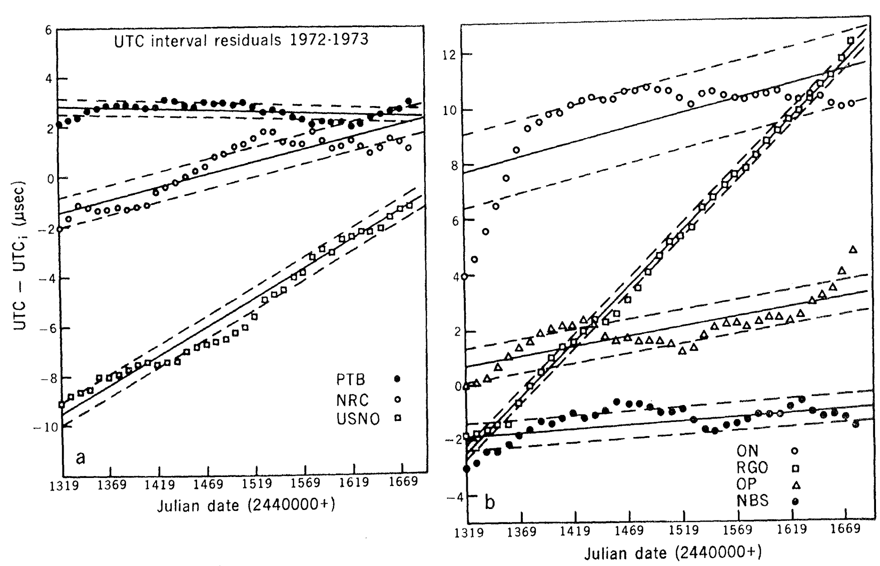

Fig. 2. (a and b) Linear regression analysis of 1 year of UTC interval residuals for the seven observatories contributing to International Atomic Time. The dashed lines indicate the standard deviation on the regression line.

It follows that for an ideal clock at position $\mathbf{p}_i$ (with spatial coordinates r_i, λ_i, z_i) on the earth's surface the rate relative to coordinate time is

$$R_i = \frac{d\tau_i}{dt} = \frac{1}{c}\frac{d\Sigma_i}{dt} \quad (32)$$

$$R_i = 1 + \Phi_i + U_i - \frac{r_i^2\Omega^2}{2c^2} \quad (33)$$

This result yields $\beta_{ij} = R_i/R_j$, the relative rate of two ideal clocks on the earth's surface at positions $\mathbf{p}_i$ and $\mathbf{p}_j$, respectively,

$$\beta_{ij} = \frac{d\tau_i}{d\tau_j} \quad (34)$$

which to order v^2/c^2 becomes

$$\beta_{ij} = 1 + \Phi_i + U_i - \Phi_j - U_j - \frac{\Omega^2}{2c^2}(r_i^2 - r_j^2) \quad (35)$$

Equation 35 expresses the relative rate of two ideal clocks on the earth's surface as predicted by the principle of equivalence. To obtain the appropriate expression for β_{ij} as predicted by hypothesis 3 one must set $U_i = U_j = 0$ in Eq. 35 to obtain

$$\beta_{ij} = 1 + \Phi_i - \Phi_j - \frac{\Omega^2}{2c^2}(r_i^2 - r_j^2) \quad (36)$$

At position $\mathbf{p}_i$ the Newtonian potential $\Phi(\mathbf{p}_i)$ is

$$\Phi(\mathbf{p}_i) = -\frac{G}{c^2}\int \frac{\rho_0(\mathbf{p})}{|\mathbf{p} - \mathbf{p}_i|}\,dv \quad (37)$$

where $\mathbf{p}$ is the position of an elemental volume of material of density $\rho_0(\mathbf{p})$ and the integral is taken over the volume of the earth.

The field of acceleration $\mathbf{a}(\mathbf{p})$ resulting from a uniform rotation is lamellar (that is, $\nabla \times \mathbf{A} \equiv 0$) and can be represented exactly by the gradient of a scalar potential U which, like the Newtonian potential Φ, is given by a Poisson equation

$$\nabla^2 U(\mathbf{p}) = -\nabla \cdot \mathbf{a}(\mathbf{p})$$

Uniform axial rotation yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial U(\mathbf{p})}{\partial r}\right] = -\frac{2\Omega^2}{c^2}$$

which, upon integration, gives for the acceleration potential at $\mathbf{p}_i$

$$U(\mathbf{p}_i) = -\frac{\Omega^2 r_i^2}{2c^2} \quad (38)$$

where the arbitrary constant in the integral has been set equal to zero. It should be pointed out that the contribution to the clock rate derived from the acceleration potential is exactly the same as the contribution to the clock rate derived from the spatial terms of the line element.

Table 2. Regression line fitting. The intercept is for 3 January 1972; σ_L^2 and σ_S^2 are the line and slope variances, respectively.

Station	Intercept (μsec)	σ_L^2 (μsec)^2	Slope (μsec/ 10 day)	σ_S^2 (μsec/ 10 day)^2
PTB	2.82	0.094	−0.011	0.000022
USNO	−9.51	0.190	0.226	0.000045
OP	0.69	0.356	0.071	0.000084
NBS	−1.86	0.243	0.029	0.000058
RGO	−2.50	0.069	0.402	0.000016
NRC	−1.42	0.335	0.098	0.000079
ON	17.7	1.73	0.104	0.00041

For the earth, the sum of the Newtonian potential Φ and the acceleration potential U is known as the geopotential, Φ_g. Surfaces of constant geopotential are known as geops and the geop coinciding with the mean undisturbed surface of the ocean is called the geoid. For any two points $\mathbf{p}_{gi}$ and $\mathbf{p}_{gj}$ on the geoid

$$\Phi(\mathbf{p}_{gi}) + U(\mathbf{p}_{gi}) = \Phi(\mathbf{p}_{gj}) + U(\mathbf{p}_{gj})$$

or

$$\Phi_{gi} = \Phi_{gj} \qquad (39)$$

In general, the Newtonian potential for a clock at position $\mathbf{p}_i$, displaced in elevation from the geoid by an amount $\mathbf{h}_i$, can be expressed in terms of the Newtonian potential $\Phi(\mathbf{p}_{gi})$ at the point $\mathbf{p}_{gi}$ on the geoid (directly above or below $\mathbf{p}_i$) by the addition of two terms:

1) A free-air potential Φ_{fi} allowing for the displacement from the earth's center of mass

$$\Phi_{fi} = \nabla\Phi(\mathbf{p}_{gi}) \cdot \mathbf{h}_i \qquad (40)$$

2) A Bouguer potential (9, p. 132) Φ_{bi} allowing for the gravitational effects of the mass interposed between the clock and the surface of the geoid

$$\Phi_{bi} = \frac{\pi G \rho_{bi}}{c^2} |\mathbf{h}_i|^2 \qquad (41)$$

where ρ_{bi} is the Bouguer density appropriate to position $\mathbf{p}_i$.

Thus for the general case of a clock at $\mathbf{p}_i$ anywhere on the earth

$$\Phi(\mathbf{p}_i) = \Phi(\mathbf{p}_{gi}) + \nabla\Phi(\mathbf{p}_{gi}) \cdot \mathbf{h}_i + \frac{\pi G \rho_{bi}}{c^2} |\mathbf{h}_i|^2 \qquad (42)$$

For displacements $\mathbf{h}_i$ from the geoid the ratio of the change in acceleration potential δU to the change in Newtonian potential $\delta\Phi$ is $\delta U/\delta\Phi \lesssim 1/300$, and so for terrestrial clock sites $\mathbf{p}_i$ we can take

$$U(\mathbf{p}_i) \cong U(\mathbf{p}_{gi}) \qquad (43)$$

Substituting the general expression for the Newtonian potenital $\Phi(\mathbf{p}_i)$ into Eqs. 35 and 36 and using Eq. 43 yields expressions for the relative rate of ideal clocks at arbitrary points on the earth's surface.

1) According to hypothesis 3 (standard relativity theory)

$$\beta_{ij} = 1 + (\nabla\Phi_{gi} \cdot \mathbf{h}_i - \nabla\Phi_{gj} \cdot \mathbf{h}_j) + \frac{\pi G}{c^2}(\rho_{bi}|\mathbf{h}_i|^2 - \rho_{bj}|\mathbf{h}_j|^2) \qquad (44)$$

2) According to the extended principle of equivalence (ad hoc theory)

$$\beta_{ij} = 1 + (\nabla\Phi_{gi} \cdot \mathbf{h}_i - \nabla\Phi_{gj} \cdot \mathbf{h}_j) + \frac{\pi G}{c^2}(\rho_{bi}|\mathbf{h}_i|^2 - \rho_{bj}|\mathbf{h}_j|^2) - \frac{\Omega^2}{2c^2}(r_i^2 - r_j^2) \qquad (45)$$

The principal difference between Eqs. 44 and 45 lies in the fact that, for clocks at the same geopotential, Eq. 44 predicts equal proper time rates anywhere on the earth and Eq. 45 predicts a latitude dependence of proper time rate because of the earth's rotation. Equation 44 follows from the assumption that accelerations do not affect clock rates and Eq. 45 from the principle of equivalence.

Method of Analysis

The annual report of the BIH (2) provides tabulated differences of the UTC − UTC$_i$ interval residuals which constitute a high-quality data set established by large numbers of clocks (about 60) operating in carefully controlled environments. We have examined these data in an attempt to resolve the contradiction between Eqs. 44 and 45.

Among the 17 laboratories contributing to the data set only the 7 (see Fig. 1, a and b) listed in Table 1 contribute to the definition of International Atomic Time, AT, and Coordinated Universal Time, UTC).

Standard regression analysis (*10*) was applied to the seven data sets of UTC − UTC$_i$ interval residuals to obtain in each case (i) an estimate, $\hat{R}_i$, of the drift rate of the ith clock relative to UTC, which is given by the slope of the regression line, (ii) the variance on the drift rate, and (iii) the variance on the regression line. The results are presented in Fig. 2, a and b, and summarized in Table 2.

Without loss of generality one can regard UTC as the coordinate time t since the relevant observable quantities, namely the relative drift rates between clocks, are independent of the choice of coordinate time (see Eqs. 44 and 45). Thus the experimentally determined drift rate $\hat{R}_i$

$$\hat{R}_i = \left(\frac{dt_{\mathrm{UTC}\,i}}{dt_{\mathrm{UTC}}}\right)_{\mathrm{estimate}}$$

corresponds to an estimate of the theoretical quantity R_i (see Eq. 33). As a consequence of Eq. 34, the quantity

$$\hat{\beta}_{ij} = \hat{R}_i/\hat{R}_j$$

becomes an experimental estimate of the relative drift rate β_{ij} of the ijth clock pair. The variance on the regression line is an experimental estimate of the stability of each time standard relative to UTC. The 1σ level of stability, that is the standard deviation on the regression line divided by the experiment duration (Table 3), has been found to range from roughly 4 parts in 10^{14} (ON) to less than 1 part in 10^{14} (RGO) for a duration of 1 year. An average of or a choice among a group of N clocks could be expected to improve stability by approximately a factor of $N^{1/2}$. However, since no stations possess more than 16 clocks (USNO) and few stations possess more than 5, all stations appear to demonstrate greater stability than would be expected on the basis of the quoted stability (*11*) of 1 part in 10^{12} over 1 year for a single cesium beam oscillator. Although hydrogen masers (*11*) could provide the stability observed in these oscillators—roughly 2 parts in 10^{13} over 1 year—such instruments were contributing to UTC$_i$ only at USNO during 1972. On the basis of this evidence it seems reasonable to suggest that individual cesium clocks currently possess stabilities better than 3 parts in 10^{13} over 1 year when operated in the ideal environments provided by the international time laboratories. Analysis of the terms appearing in Eqs. 44 and 45 will show that with stabilities of this order it should be possible to test the predicted relativistic effects on proper time on the earth's

Table 3. Drift rates ($\hat{R}_i$), standard deviations (σ_S), and stability measures (σ_L/year).

Station	$R_i \times 10^{-14}$	$\sigma_S \times 10^{-14}$	σ_L/year $\times 10^{-14}$
PTB	−1.3	0.5	1.0
USNO	26.2	0.8	1.4
OP	8.2	1.1	1.9
NBS	3.4	0.9	1.6
RGO	46.5	0.5	0.8
NRC	11.3	1.0	1.9
ON	12.0	2.3	4.2

surface and resolve the contradiction in these equations.

An approximate comparison of the magnitude of the terms of Eqs. 44 and 45 yields estimates of

1) The free-air potential effect. This is the so-called gravitational redshift and is proportional to

$$|\nabla\Phi_g| \sim g/c^2 = 1.1 \times 10^{-16}\ \mathrm{m}^{-1}$$

where g is the magnitude of gravitational acceleration on the geoid.

2) The Bouguer potential effect. For a reasonable density of the earth's crustal materials, $\rho_b \sim 3\ \mathrm{g/cm}^3$

$$\frac{2\pi G}{c^2}\rho_b|\mathbf{h}_i| \sim 1.4 \times 10^{-20}|\mathbf{h}_i|\mathrm{m}^{-1}$$

Since $|\mathbf{h}_i| \sim 10^4$ m or less for any terrestrial location the Bouguer term is negligible compared to the free-air term for any terrestrial clock site and will not be included in further analysis.

3) The latitude effect. This results from the velocity of the clock relative to a geocentric global inertial frame, and for a spherical earth of radius a = 6400 km it is proportional to

$$\frac{\Omega^2 r}{c^2} = \frac{\Omega^2 a\cos\phi'}{c^2} \leqslant 3.8 \times 10^{-16}\ \mathrm{km}^{-1}$$

where ϕ' is the geocentric latitude of the site.

According to Eq. 45 ideal clocks separated in latitude and elevation on the earth's surface could be expected to exhibit rate differences ranging from several parts in 10^{13} to more than 1 part in 10^{12} in special cases. On the basis of the observed stability of the seven time laboratories, any observed relative drift rates $\hat{\beta}_{ij}$ between clocks should be due in part to relativistic effects. If either theory (Eq. 44 or 45) is correct then appropriately correcting for such effects should reduce the relative drift rates of the residual level of systematic differences between clocks.

In order to definitively test these theories relativistic corrections to the clock rates in each case must be made with a precision of 1 part in 10^{15}. From the partial derivatives of clock rate with respect to elevation h (gravitational redshift) and axial distance r (latitude effect) computed above it is clear that clock elevations h (height above the gravitational equipotential surface of the geoid) and axial distances r (distances from the rotation axis of the earth measured parallel to the plane of the equator) must be known to accuracies of ± 10 m and ± 3 km, respectively. The reference surface for the coordinates of the European stations is the International Ellipsoid with datum at Potsdam, Germany, equatorial radius $a_e = 6.378388 \times 10^6$ m, and flattening $f = 1/297.0$. The reference surface for the coordinates of the North American stations is the Clarke Ellipsoid with datum at Meades Ranch, Kansas, $a_e = 6.378206 \times 10^6$ m, and $f = 1/294.98$. The centroids of these two reference surfaces are removed from the geocenter by Cartesian displacements of a few hundred meters in each case. In computing the axial distances r_i, $i = 1, 2, \ldots 7$, Gaposchkin's (*12*) recommended values for the required geophysical parameters have been applied. The adopted reference ellipsoid has $a_e = 6.378140 \times 10^6$ m and $f = 1/298.258$. This results in errors in the axial distances of the timekeeping stations of about 0.5 km and thus allows timekeeping corrections for the latitude effect well within the required limits of precision.

Table 4. Relativity corrections.

Station	Correction $\times 10^{-15}$	
	Elevation	Latitude
PTB	8	−451
USNO	8	−730
OP	6	−523
NBS	178	−707
RGO	3	−441
NRC	11	−594
ON	53	−561

The differences in gravitational potential $\Phi_i - \Phi_j$ between two sites $\mathbf{p}_i$ and $\mathbf{p}_j$ is expressed in Eqs. 44 and 45 in terms of the dot product of the gradient normal to the geoid $\nabla\Phi_g$ and the elevation from the geoid $\mathbf{h}$. In computing this correction the gradient normal to the reference ellipsoid was taken to be (*13*)

$$|\mathbf{g}| = g_e(1 + B_2\sin^2\phi' + B_4\sin^2 2\phi') \tag{46}$$

where

$$B_2 = \frac{5}{2}m - f - \frac{17}{14}mf$$

$$B_4 = \frac{1}{8}(f^2 - 5mf)$$

and

$$m = \frac{a_e{}^3\Omega^2}{MG}$$

is the ratio of the centrifugal to gravitational acceleration at the equator. Gaposchkin's (*12*) values for the required geophysical constants adopted in these equations are (i) the magnitude of the equatorial gravitational acceleration $g_e = 9.780327\ \mathrm{m/sec}^2$, (ii) the angular rotation rate of the earth relative to the fixed stars $\Omega = 7.292115 \times 10^{-5}$ rad/sec, (iii) the product of the gravitational constant and the mass of the earth $MG = 3.986013 \times 10^{14}\ \mathrm{m}^3/\mathrm{sec}^2$, and (iv) the speed of light $c = 2.997925 \times 10^8$ m/sec.

However, since Eq. 46 provides the gradient of the gravitational potential normal to the reference ellipsoid rather than the geoid, it is in principle necessary, when computing the relativistic corrections to clock rates, to allow for (i) the difference in direction of the normals to the two surfaces and (ii) the difference in the radial distance of the two surfaces from the center of mass of the earth.

In practice these considerations are not necessary. The difference in direction of the two normals is at most a few seconds of arc and hence is negligible. An examination of the fits of the International and Clarke ellipsoids to the geoid in the regions of continental Europe and North America, respectively, indicates that geoid-ellipsoid departures do not exceed 10 m at the locations of the timekeeping laboratories and are in most cases less than 5 m. Thus gravitational redshift corrections to the clock rates can be made with the requisite accuracy by ignoring both these factors.

Apart from NBS (elevation 1650 m) and ON (elevation 490 m), for which the gravitational redshift corrections are of the order of 1 part in 10^{13}, the relatively low elevations of the remaining five sites contribute less than 1 part in 10^{14} to the relative rates of ideal clocks. However, Eq. 45 predicts that the effect of the earth's rotation should contribute several parts in 10^{13} to the relative rates of ideal clocks located at the laboratory sites. The axial distance (r coordinate) of a position $\mathbf{p}_i$ was computed from the standard formula (*14*)

$$r_i = (a_e C_i + h_i)\cos\phi_i$$

where h_i is the magnitude of the elevation of $\mathbf{p}_i$ above the reference ellipsoid, ϕ_i is the geodetic latitude, and C_i depends on the flattening as

$$C_i = [\cos^2\phi_i + (1-f)^2\sin^2\phi_i]^{-\frac{1}{2}}$$

The magnitudes of both the free-air and latitude corrections for each of the seven laboratory sites are listed in Table 4. Equation 44 implies that only the elevation correction should be applied to the clock rate, and Eq. 45 implies that both the elevation correction and the latitude correction should be applied to the clock rate.

Theoretical values for the relative rates β_{ij} of pairs of ideal clocks at the locations of the seven contributing time laboratories were computed according to both Eq. 44 and Eq. 45. The relative rate remainders $\beta_{ij} - 1$ are tabulated in each case in matrix form in Table 5. The observed relative rate remainders $\hat{\beta}_{ij} - 1$ derived from the regression analysis are shown in Table 6 for comparison. The rate residuals $\hat{\beta}_{ij} - \beta_{ij}$, the relative rate remainders after corrections for the effects of relativity, are presented according to both Eq. 44 and Eq. 45 in Table 7.

Discussion of Results

Clock pairs have been grouped into four classes according to the magnitudes of their relative rate remainders:

1) Clock pairs whose relative rate remainder is less than $\pm$ 50 parts in 10^{15},

2) Clock pairs whose relative rate remainder is between $\pm$ 50 and $\pm$ 100 parts in 10^{15},

3) Clock pairs whose relative rate remainder is between $\pm$ 100 and $\pm$ 200 parts in 10^{15}, and

4) Clock pairs whose relative rate remainder is greater than $\pm$ 200 parts in 10^{15}.

A grouping of the 21 independent clock pairs of the worldwide contributors to atomic time into the above four categories (i) before relativistic corrections, (ii) after relativistic corrections for the gravitational redshift (Eq. 44), and (iii) after relativistic corrections for the gravitational redshift and accelerations (Eq. 45) is shown in Table 8.

Column 2, which includes corrections to the clocks for the gravitational redshift, does not indicate any significant improvement in worldwide timekeeping. The number of "good" clock pairs (classes 1 and 2) is 8 both before and after correction, and the number of "bad" clock pairs (classes 3 and 4) is 13 both before and after correction. Column 3, which includes corrections to the clock rates for both gravitational redshift and acceleration, indicates a significant improvement in worldwide timekeeping. Good clock pairs now number 14, while bad clock pairs now number 7. Furthermore six of the seven bad clock pairs involve comparisons with RGO, which can be seen even from the raw data (see Fig. 3a) to be running at an anomalous rate. These results confirm the validity of Eq. 45 and suggest that, as predicted by the equivalence principle, accelerations do affect the rates of ideal clocks and that hypothesis 3 is incorrect.

Of the seven contributing time standard laboratories, three (NRC, NBS, and USNO) are intercompared by using the U.S. East Coast Loran-C chain and four (PTB, OP, ON, and RGO) are intercompared by using the Norwegian Loran-C chain. The data thus fall naturally into two subsets: a North American data set and a European data set. The classification of the independent clock pairs in the same manner as in Table 8 is presented for these two data sets in Table 9.

Taking the European data alone, a comparison of columns 1 and 2 of Table 9 indicates no significant improvement in European timekeeping by correcting only for the gravitational

Table 5. Theoretical relative rate remainders derived from Eqs. 44 and 45 and expressed in parts in 10^{15}.

	PTB	USNO	OP	NBS	RGO	NRC	ON
			Equation 44				
PTB	0	1	−1	173	−7	3	45
USNO		0	−2	172	−7	2	44
OP			0	174	−5	4	46
NBS				0	−179	−170	−128
RGO					0	9	52
NRC						0	51
ON							0
			Equation 45				
PTB	0	−278	−73	−83	5	−139	−65
USNO		0	205	195	282	138	213
OP			0	−10	78	−67	8
NBS				0	88	−57	18
RGO					0	−144	−70
NRC						0	75
ON							0

Table 6. Observed relative rate remainders derived from the regression analysis and expressed in parts in 10^{15}.

	PTB	USNO	OP	NBS	RGO	NRC	ON
PTB	0	−274	−95	−46	−477	−125	−132
USNO		0	179	228	−204	148	141
OP			0	49	−383	−31	−38
NBS				0	−432	−80	−87
RGO					0	352	345
NRC						0	−7
ON							0

Table 7. Rate residuals or relative rate remainders after correction for the effects of relativity, according to Eqs. 44 and 45, and expressed in parts in 10^{15}.

	PTB	USNO	OP	NBS	RGO	NRC	ON
			Equation 44				
PTB	0	−275	−94	−219	−470	−128	−177
USNO		0	181	56	−197	146	97
OP			0	−125	−378	−35	−84
NBS				0	−253	90	41
RGO					0	343	293
NRC						0	−58
ON							0
			Equation 45				
PTB	0	4	−22	37	−482	14	−67
USNO		0	−26	33	−486	10	−72
OP			0	59	−461	36	−46
NBS				0	−520	−23	−105
RGO					0	496	415
NRC						0	−82
ON							0

redshift. The numbers of good and bad clock pairs remain the same before and after such corrections. A comparison of columns 1 and 3 of Table 9 indicates an improvement in European timekeeping as a result of correcting for both gravitational redshifts and accelerations. The number of good clock pairs is three after corrections, increased from two, while the number of bad clock pairs is three after corrections, decreased from four.

Taking the North American data alone, the results tabulated in columns 1 and 2 of Table 9 indicate that correcting for the gravitational redshift among the North American clock pairs results in an improvement in North American timekeeping, with the number of good clock pairs increased from one to two. This is not surprising as NBS, which is at an altitude of 1650 m, is most strongly affected by the gravitational redshift, and this correction improves its performance markedly. A comparison of columns 1 and 3 of Table 9 indicates a large improvement in North American timekeeping when corrections are made for both gravitational redshift and acceleration, with the number of good clock pairs increased from one to three and the number of bad clock pairs decreased from two to zero.

The residual rates between North American clock pairs after correction for both gravitational redshift and acceleration do not exceed a few parts in 10^{14}—an average reduction of residuals or an average improvement in timekeeping of almost an order of magnitude. Furthermore it is evident from Table 1 that the corrections between USNO and NBS should result almost entirely from elevation differences, as the two observatories lie at nearly the same latitude, while the corrections between USNO and NRC should result almost entirely from latitude differences, as the two observatories have similar elevations.

Within the European subset RGO, the most stable clock of all, tends to disagree consistently by large amounts with theoretical predictions. This clock alone is responsible for six of the seven bad clock pairs in the worldwide data set (see Table 8, column 3) and for all three bad clock pairs in the European data set (see Table 9, column 3). The reason for RGO's anomalous behavior is unknown and merits investigation. Apart from RGO, ON, the least stable clock, exhibits the largest residuals after corrections for both gravitational redshift and acceleration. This is probably due to an obvious change in the performance of ON in early 1972 (see Fig. 2a). By eliminating the first six ON data points, for the time when the laboratory apparently ran anomalously slowly relative to UTC, the residuals involving ON can be reduced significantly.

Table 8. Classification of the 21 independent clock pairs of the seven contributing observatories before relativistic corrections, after relativistic corrections by Eq. 44, and after relativistic corrections by Eq. 45.

Class	Number of clock pairs		
	Before corrections	After correction by Eq. 44	After correction by Eq. 45
1	5	2	10
2	3	6	4
3	5	5	1
4	8	8	6

Table 10 shows the normalized residuals. Normalization has been effected by dividing the residuals by the square root of the sum of the variances on the drift rates obtained from the regression analysis. Clearly, most stations agree to within a few standard deviations. Systematic differences between the clocks, which produce drift rates large compared to relativistic drift rates, probably account for lack of even closer agreement. The largest normalized residuals, which indicate the largest disagreement with theory, are associated with RGO, as would be expected from the raw data (see Fig. 2a).

The relative drift rate of any pair of clocks is due to both the relativistic effects described by the theory and the systematic differences (errors) between the clocks. The data presented here cannot distinguish systematic errors and long-term (duration greater than 1 year) instabilities or fluctuations in the clock rates. Both these effects are contained in the rate residuals of Table 7.

Table 9. Classification of the six independent European clock pairs and the three independent North American clock pairs before relativistic corrections, after relativistic corrections by Eq. 44, and after relativistic corrections by Eq. 45.

Class	Number of clock pairs		
	Before corrections	After correction by Eq. 44	After correction by Eq. 45
	European clock pairs		
1	1	0	2
2	1	2	1
3	1	1	0
4	3	3	3
	North American clock pairs		
1	0	0	3
2	1	2	0
3	1	1	0
4	1	0	0

The results of this analysis are displayed in Fig. 3, which shows, for each of the seven clocks contributing to International Atomic Time, regression lines fitted to the raw data (Fig. 3a), regression lines fitted to the data and corrected for the gravitational redshift (Fig. 3b), and regression lines fitted to the data and corrected for both the gravitational redshift and the acceleration (Fig. 3c). The site of USNO was arbitrarily adopted as a standard location, and relativistic "corrections," theoretically mainfesting themselves between this location and the sites of the remaining six observations, were appropriately applied. Since parallel lines in Fig. 3 indicate identical clock rates, it is clear from a comparison of Fig. 3a and Fig. 3c that corrections for both gravitational redshift and acceleration result in a great improvement in terrestrial timekeeping. With the exception of RGO, the corrected intervals measured by each clock over a year agree within roughly 1 μsec in most cases.

The discovery that inertial accelerations offset ideal clock rates has immediate implications for international timekeeping. If the geoid can no longer be regarded as a surface of constant proper time it would be desirable to define a scale of atomic time independent of any particular location on the surface. A convenient and obvious choice of such a standard location would be the geocenter, for there $U \equiv 0$ and Φ is unique. Such a scale of atomic time, which might be called geocentric atomic time (GAT), could be generated from the seven contributors to International Atomic Time AT_i, $i = 1, 2, \ldots 7$) by the corrections proposed in this article in much the same ways as UT1 and UT2 are generated, by appropriate corrections, from observations of $UT0_i$.

As a result of the effects predicted here, the equator has apparently experienced about 1 day less proper time than the poles when integrated over the age of the earth, 4.5×10^9 years. Observable geophysical manifestations of this effect are difficult to imagine, and it is a tribute to the precision of modern atomic clocks that the effect is revealed in their short record of timekeeping. In principle, radioactive ore bodies which have resided at equatorial latitudes for geologic time should appear younger by about 1 day when compared with their high-latitude counterparts. However current techniques cannot establish geochronologic dates with a precision even approaching parts in 10^{13}, and in any event the effect would be obscured by polar wander and continental drift.

Finally, the implications for the theory of relativity of the discovery of the effect of accelerations on proper time measure are nontrivial.

Einstein was guided by the principle of equivalence to deduce the effect of gravitational potentials Φ on ideal clock rates. He predicted a scaling by the amount $1 + \Phi$ of an observer's time scale when in the presence of a gravitational field. In order to preserve the constancy of the velocity of light for observers in gravitational fields it was necessary to appropriately scale the observer's distance measure. This lead to the concept of a space-time metric $g_{\mu,\nu}$ which would carry out the necessary scaling of space and time and describe gravitational fields geometrically as space-time curvature. The world geometry described by the tensor $g_{\mu\nu}$ is an absolute feature of the space-time manifold. Although usually referred to today as the theory of gravitation, Einstein's theory has been called the general theory of relativity—a name which is ironical because there is nothing relative about absolute space-time geometry.

Table 10. Normalized differences between theory and experiment, in units of standard deviation on the ratio of the drift rates. The formula used is $(\beta_{ij} - \hat{\beta}_{ij})/(\sigma_{si}^2 + \sigma_{sj}^2)^{1/2}$.

	PTB	USNO	OP	NBS	RGO	NRC	ON
PTB	0	−0.4	1.8	−3.6	67	−1.2	2.8
USNO		0	2.0	−2.8	54	−0.8	2.9
OP			0	−4.2	40	−2.4	1.8
NBS				0	52	1.7	4.2
RGO					0	−44	−17
NRC						0	3.2
ON							0

It was realized that the world lines of observers in uniform motion relative to the inertial frames of special relativity (whose lengths could be measured by proper time intervals reckoned by comoving ideal clocks) generalized in the theory of gravitation to the set of timelike geodesics traveled by observers in free fall in the gravitational field. The lengths of such timelike geodesics could also be measured by proper time intervals reckoned by comoving ideal clocks. Thus it was concluded that observers in inertial frames can measure the space-time metric by using ideal clocks and rods. The theory to this point does not say how accelerated observers (not in inertial frames) are to measure the metric.

By making the assumptions (*3*, p. 49; *4*, pp. 228 and 234) that (i) observers are equipped with rigid rods whose lengths are unaffected by accelerations and (ii) observers are equipped with ideal clocks whose timekeeping is unaffected by accelerations, it was possible to argue that all observers, accelerated or not, will measure the invariant space-time metric in the same way. We have presented a test of the

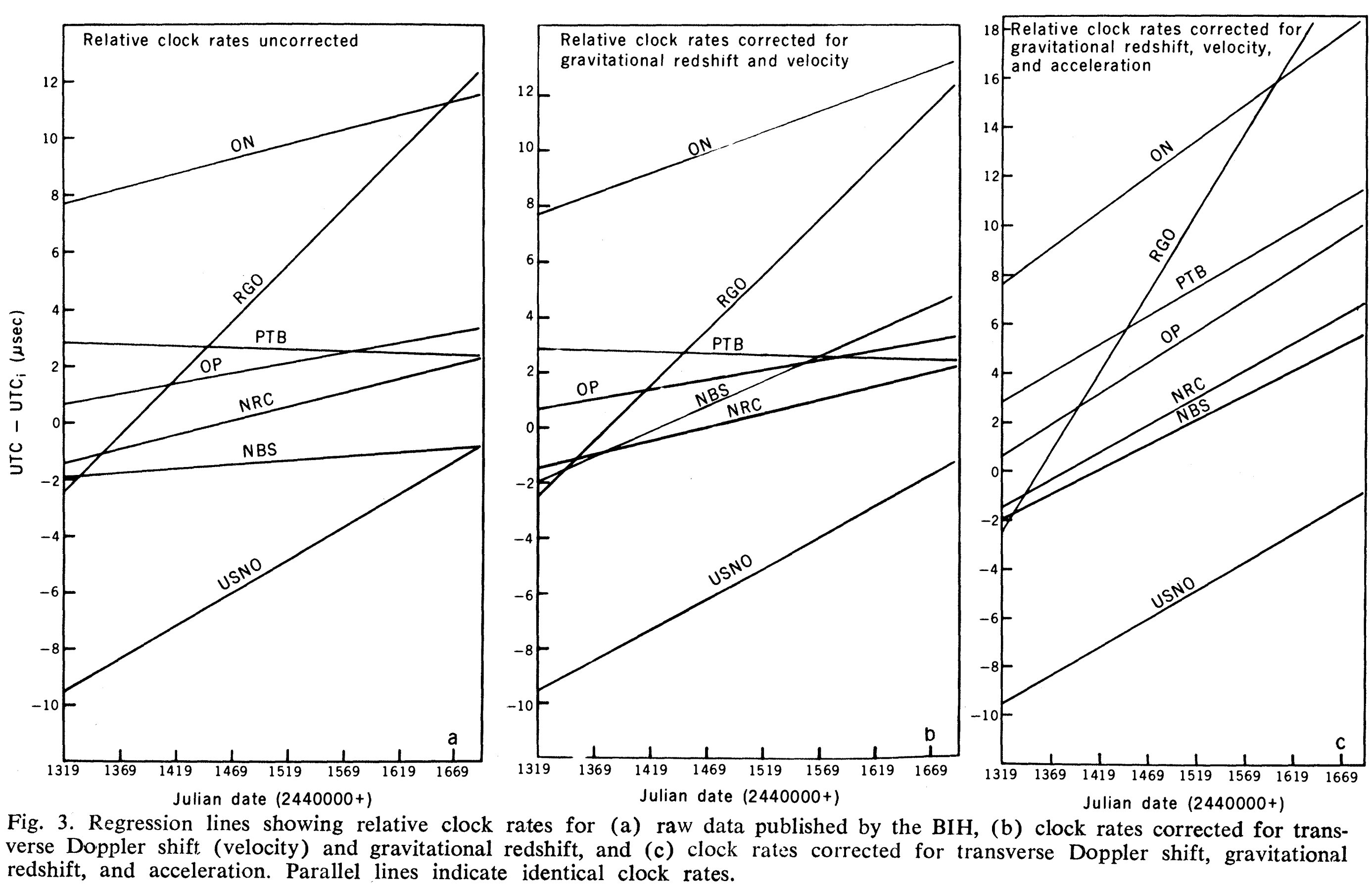

Fig. 3. Regression lines showing relative clock rates for (a) raw data published by the BIH, (b) clock rates corrected for transverse Doppler shift (velocity) and gravitational redshift, and (c) clock rates corrected for transverse Doppler shift, gravitational redshift, and acceleration. Parallel lines indicate identical clock rates.

second assumption and find it to be false. Accelerations do affect the rates of ideal clocks. This brings us face to face with a problem similar to that which confronted Einstein in 1911 (*8*).

If it is now known that, as a result of his accelerations, an observer's time scale is scaled by an amount $1 + U$, where U is the acceleration potential, and if we are to retain the spirit of relativity and insist that even accelerated observers must measure c for the velocity of light, we must be prepared, as Einstein was, to appropriately scale the distance measure for inertially accelerated observers. This implies that inertial accelerations will affect an accelerated observer's perception of the world geometry with

$$g_{\mu\nu} \rightarrow g_{\mu\nu}'$$

where $g_{\mu\nu}'$ differs from $g_{\mu\nu}$ due to the effects of acceleration.

Such a viewpoint is consistent with the theory of gravitation. The world geometry described by the tensor $g_{\mu\nu}$ is fundamental and absolute and would be deduced from measurements with ideal clocks and rods by any observer in an inertial frame, that is, in free fall. Since the geometry of the space-time manifold is absolute, any departure of an observer's motion from the path of a timelike geodesic is absolute. Accelerated observers do not travel timelike geodesics of the space-time manifold. The departure of an observer from free fall (a state of zero acceleration) is an absolute condition which can be measured locally with accelerometers. The results presented in this article indicate that such inertial accelerations, uniquely and absolutely determined for each observer, will affect the observer's measurement of space and time. Such effects on an observer's measurements are not described by the present theory of gravitation since they have been assumed away. The present theory of gravitation deals correctly with observers in free fall and cannot be generalized to inertially accelerated observers without the introduction of an additional hypothesis (*4*, p. 234).

In searching for a first-order theory which would account for the effects of acceleration on an observer's perception of the world geometry we have been guided by the principle of equivalence and the necessary correspondence between the limiting case of zero acceleration (motion along a timelike geodesic of the space-time manifold) and the standard theory of gravitation. According to our view, the observer's perception of the world geometry depends on his state of acceleration, and we believe that a theory which describes these effects correctly would constitute true "general theory of relativity," which would include the theory of gravitation as a special case.

References and Notes

1. W. H. Munk and G. J. F. MacDonald, *The Rotation of the Earth—A Geophysical Discussion* (Cambridge Univ. Press, Cambridge, England, 1960), p. 77.
2. *Annual Reports for 1968–1972* (Bureau International de l'Heure, Paris, 1969–1973).
3. C. Møller, *The Theory of Relativity* (Oxford Univ. Press, New York, 1952).
4. V. Fock, *The Theory of Space, Time and Gravitation* (Pergamon, New York, 1964).
5. C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
6. V. B. Braginsky and V. I. Panov, *Zh. Eksp. Theor. Fiz.* **61**, 873 (1971) [English translation in *Sov. Phys.-JETP* **34**, 464 (1971)].
7. R. V. Pound and J. L. Snider, *Phys. Rev. B* **140**, 788 (1965).
8. A. Einstein, *Ann. Phys.* **35**, 898 (1911); *The Principle of Relativity* (English translation) (Dover, New York, 1923).
9. C. D. Garland, *The Earth's Shape and Gravity* (Pergamon, New York, 1965), pp. 33–37.
10. H. J. Halstead, *Introduction to Statistical Methods* (Macmillan, Toronto, 1960), p. 132.
11. D. Halford, in *Measurement of Frequency Stability*, Proceedings of the Frequency and Time Seminar, National Bureau of Standards, Boulder, Colo., 28 February to 1 March 1968.
12. E. M. Gaposchkin, *EOS Trans. Am. Geophys. Union* **52**, 30 (1971).
13. I. I. Mueller, *Spherical and Practical Astronomy as Applied to Geodesy* (Ungar, New York, 1961).
14. G. W. Wilkins and A. W. Springett, Eds., *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac* (Her Majesty's Stationery Office, London, 1961), p. 57.

15. We are indebted to D. E. Smylie for his encouragement and critical reading of the manuscript and for the provision of the BIH reports. This research was supported by National Research Council of Canada operating grants to W.H.C. and D. E. Smylie.

On the Role of Themata in Scientific Thought

Gerald Holton

When the historian of science studies a product of scientific work—a published paper, a laboratory record, a transcript of an interview—he is dealing first of all with an event. A number of different facets of the event can engage his attention. One can distinguish at least eight such facets, corresponding to different types of interesting questions:

First is of course the understanding of the scientific content of the event (E) at a given time, both in contemporaneous terms and, separately, in terms of what we now believe to be the case. What did the scientist claim was at issue? What was he in fact confronted with? For this we need to establish the awareness, within the area of public scientific knowledge at the time of the event, of the so-called scientific facts, data, laws, theories, techniques, lore. I would include under this heading the larger part of historical research on what are called scientific world views, paradigms, and research programs; chiefly, however, historians and scientists are still concerned with digging out the concepts and propositions embodied in the event studied and with rendering them in empirical and analytical language.

Second is the time trajectory of the state of shared (that is, "public") scientific knowledge (let us call it S_2) that led up to and perhaps goes beyond the time chosen above. Establishing this means, so to speak, the tracing of the world line of an idea or a subject of research, a line on which E is a point. Whether we are studying the problem of falling bodies from Kepler to Newton, or the flowering of quantum electrodynamics from Feynman to the last issue of *Physical Review Letters*, under this heading we are dealing with antecedents, parallel developments, continuities and discontinuities, and the like.