institute serving Congress has already seen done (see box, page 37).

Most of the report's recommendations are, in fact, not new. What lends them considerable cogency is the authors' knowledgeability about weapon systems. They emphasize, for example, that the useful life of a weapon system does not begin until nearly a decade after Congress has authorized production. This fact and the longevity of major weapon systems [the B-52 bomber and the Poseidon (née Polaris) submarine will figure importantly in the U.S. arsenal for a quarter century or more] are key factors in strategic planning.

The report also cautions Congress about using weapon systems as bargaining chips in arms control negotiations, a ploy which seems to be much in fashion. CED recommends the following approach.

> We believe that Congress has a positive role to play in the process of strategic arms limitation bargaining. But in view of the risks involved, we urge Congress to be doubly cautious about authorizing any system that is justified principally in terms of its bargaining value. If the five-year authorization process can be augmented by contingent or conditional authorizations, Congress can help the executive branch to clarify its intention with respect to weapons under negotiation, can clarify its own intentions, and can communicate to the Soviet Union the conditional status of systems under negotiation, due for negotiation, or not available for negotiation.

The new report is one of a CED series concerned with improving the decision-making process in government. CED takes no position on whether the military budget is too large. The view expressed is that "Most Americans are willing to pay a high price for a peaceful nation and a peaceful world, but the Committee questions whether tax dollars are always spent in the most effective ways to pursue these goals." The committee also eschews judgments on the "choice, timing, or validity of individual weapon systems or programs" and the lack of such judgments deprives the report of some force and substance.

Committee Attitudes

A serious question for anyone proposing reforms for Congress is whether the legislators are disposed to accept the advice. Observers of Congress tend to feel that the committees authorizing and appropriating funds for the military are among the least suggestible in this respect. The habits of these committees have been shaped by a conception of the congressional role which dates at least three decades. It has been regarded as not only proper but patriotic to give the Pentagon most of what it asks for. The habit of congressional deference to the Pentagon professionals was set during World War II and the Cold War. Throughout this whole period this deference was reinforced by the interest of some committee members in the military installations and defense industries in their districts and states.

Vietnam and the increasing expense of weapons systems has had a dampening effect on the old attitude, and the arrival of some younger, more skeptical members has introduced a sharpened note of dissent into both House and Senate discussions.

Congress as a whole is developing more analytical horsepower. The General Accounting Office has moved increasingly from acting simply as an auditing agency to carrying out critical evaluations of programs. The Congressional Research Service in the Library of Congress is now bigger and better financed, although still overburdened with trivial assignments from individual legislators. The recent creation of an Office of Technology Assessment to serve Congress is a key experiment in strengthening congressional resources in policy analysis. And the establishment of a joint congressional committee on the budget is an important step endorsed by the report. Congress has a long way to go to match the resources of the Executive branch, but more is being done now in this cause than at any time since the advent of the so-called Imperial Presidency.

—JOHN WALSH

RESEARCH NEWS

The 1974 Fields Medals (I): An Algebraic Geometer

The highest award to which a mathematician can aspire is the Fields Medal, an award comparable in many respects to a Nobel Prize in the prestige it confers. J. C. Fields, who set up a trust for the gold medals that constitute the award, said only that they should be made "in recognition of work already done and as an encouragement for further achievements on the part of the recipient." This has been interpreted to mean that the medals should be given to young mathematicians (generally those under the age of 40), a tradition that has been closely followed since the first two medals were awarded in 1936. The Fields Medals are given out only every 4 years, at the quadrennial convening of the International Congress of Mathematicians. This year Fields Medals were presented to David B. Mumford of Harvard University for his work in algebraic geometry and to Enrico Bombieri of the University of Pisa, Pisa, Italy, for his work in number theory and minimal surfaces.

David B. Mumford was awarded the Fields Medal for his many fundamental contributions to algebraic geometry. Mumford was born on 11 June 1937 in Three Bridges, Sussex, England. His father was a British subject with original and forward-looking ideas about education in the colonies, who taught in Tanzania and London and later worked at the United Nations. Mumford was educated at Phillips Exeter Academy and Harvard. An early recognition of his promise was a Westinghouse Talent Search prize given him for his construction of a model computing machine which was logically quite intricate and powerful, though mechanically unreliable. His enthusiasm for algebraic geometry first became evident when he wrote a term paper on infinitely near points of plane curves in a course I gave. While he learned much from A. Grothendieck, his principal teacher is Oscar Zariski, who now has the unique distinction of having been the main teacher of two Fields Medalists, Mumford and H. Hironaka.

Mumford was a Junior Fellow at Harvard from 1958 to 1961 and has been on the Harvard faculty since then. He spent the year 1962–1963 at the Institute for Advanced Study in Princeton and at Tokyo University. In 1967–1968 he spent 7 months at the Tata Institute in Bombay and 2 months at the Institut des Hautes Etudes Scientifiques in Paris. He was the Nuffield Professor at the University of Warwick in England during 1970–1971. He has been full professor at Harvard since 1967.

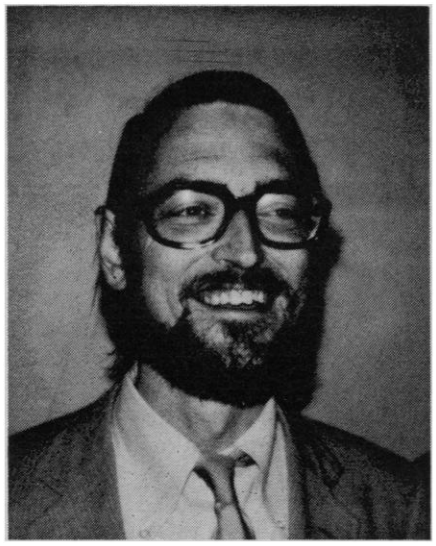
The central topic in Mumford's research has been the theory of moduli. Although this is hard to explain in nontechnical language, one may get an idea of what it involves as follows. It was discovered early in the 19th century that there was no elementary expression for the elliptic integrals

$$I_\lambda(y)=\int_{y_0}^{y}\frac{dx}{[x(x-1)\ (x-\lambda)]^{1/2}}$$

and even that knowledge of I_λ for one λ does not enable one to compute in an elementary way the other I_λ's: that is, λ is an essential parameter that cannot be eliminated by a subtle substitution. This can be seen either algebraically or geometrically. The algebraic way is by showing that the field

$$K_\lambda=\mathbf{C}\ (x,[x(x-1)\ (x-\lambda)]^{1/2})$$

that is, the set of rational expressions in x and $[x(x-1)\ (x-\lambda)]^{1/2}$ with complex coefficients, varies essentially with λ (in technical terms, these fields are not isomorphic). The geometric way is by considering the surface X_λ which is the 2-sheeted covering of the complex x-plane defined by the 2 branches of the analytic function $[x(x-1)(x-\lambda)]^{1/2}$, and showing that except for a number of special cases, X_λ and X_μ cannot be mapped one-to-one and conformally onto each other. The parameter λ is called a modulus or invariant to distinguish the different fields K_λ or surfaces X_λ. (Actually, one must take into account the exceptions where $X_\lambda \approx X_\mu$ and the correct modulus is a slightly complicated function j of λ.) Much of Mumford's work concerns extending this taxonomy to many other families of so-called algebraic varieties and finding suitable moduli for classifying them; more precisely, finding a variety of moduli whose points are in one-to-one correspondence with the varieties to be



David B. Mumford [Courtesy Paul R. Halmos, Indiana University]

classified. For instance, one of the most powerful methods for getting such moduli is by infinite series known as theta functions. The simplest of these is

$$\sum_{k_1=-\infty}^{\infty}\cdots\sum_{k_n=-\infty}^{\infty}\exp\left(\pi i\sum_{p,q=1}^{n}k_pk_q\,\Omega_{p,q}\right)$$

where the $\Omega_{p,q}$ are particular definite integrals (periods) of the type I_λ but depend on more parameters. Mumford analyzed the internal symmetry of these theta functions and found it involved a discrete analog of the symmetry of the position and momentum observables in quantum mechanics (Heisenberg's commutation relations), which he then exploited in various ways.

Another method for finding moduli is by the classical theory of invariants. This was a very active field in the late 19th century, but had been widely considered to have been "killed" by Hilbert, when he proved such very strong and general results that all earlier work seemed pointless. As in many other cases, this "death" was only superficial and resulted from always posing the same questions about invariants. Mumford asked what the geometric significance was in this algebraic theory and, pursuing one of Hilbert's own ideas, was led to his concept of "stable" objects in a modulus problem, which has proved quite fruitful.

Once one has constructed a variety of moduli it is important to study how the objects it parameterizes can degenerate, or in other words, how the variety can be "compactified" by the addition of points on the boundary. A new tool for doing this which yields a much more detailed picture than had been obtained before is Mumford's theory of toroidal embeddings. This theory, which unifies ideas that had appeared earlier in the works of several investigators, reduces the study of certain types of varieties and singularities to combinatorial problems of polyhedral decompositions in the space of exponents.

These three theories, of theta functions, geometric invariants, and toroidal embeddings are based on computational methods which are, in principle, totally elementary. Mumford has the genius to see the theory behind the calculation. In general he points out that modulus problems lead eventually to more and more concrete and explicit questions about particular classes of varieties rather than to the impulse to generalize and build broad theories.

Two interesting results of Mumford which do not have to do with the theory of moduli and are easy to state are the following. Let V be an algebraic surface in complex n-space, that is, the set of complex solutions $z=(z_1,\dots z_n)$ of a system of polynomial equations $P_i\ (z_1,\dots z_n)=0$ such that in the neighborhood of most of its points V is a complex 2-manifold. Mumford proved that if a point P on V has a neighborhood which is a one-to-one continuous image of a neighborhood in real 4-space, then it has a neighborhood which is a one-to-one *analytic* image of a neighborhood in complex 2-space. This result was well known for algebraic curves, but E. Brieskorn showed it to be false for varieties of complex dimension greater than 2. Consideration of the corresponding problem in higher dimension led to the discovery of interesting new relations between alegbraic geometry and differential topology.

Let n be an integer >2. Fermat claimed to have proved that there is no solution of the equation $x^n+y^n=z^n$ in positive integers x,y,z, but he did not publish the proof and the problem ("Fermat's Last Theorem") is still open today. Mumford showed at least that such solutions are rare, in the sense that if (x_m,y_m,z_m) is an infinite sequence of them, arranged according to increasing z_m, then there are constants $a>0$ and b such that

$$z_m>10^{(10^{am+b})}.$$

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