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Computers and Future Music

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Technology must always be the tool of the artist and not vice versa, but their relationship is now so intimate that one cannot meaningfully discuss either separately. In a pessimistic moment, in 1963, a few years before he died, Hermann Scherchen said that music was a dead art in the sense that Latin is a dead language (*1*)—that the rich vein of human expression based on modal harmonies and melodies had been exhausted and that no humanly valid alternatives exist, hence no new music could be written. By contrast, Edgar Varèse in 1917 wrote (*2*):

> Music, which should be alive and vibrating, needs new means of expression and science alone can infuse it with youthful sap. . . . I dream of instruments obedient to thought—and which, supported by a flowering of undreamed-of timbres, will lend themselves to any combination I choose to impose and will submit to the exigencies of my inner rhythm.

Perhaps these two apparently contradictory opinions are actually not so different. We believe that current technology—computers, integrated circuits, and loudspeakers—will make possible a new musical art, one that will be immeasurably more potent than existing music in certain ways, but we also believe that most of the new music will be very different from all past music—so different that it may require a new name.

Even though many lovers of existing music might like to have it continue unchanging, music, by its very nature, cannot be preserved on the walls of a museum. Already the concert hall suffers from rigidity in an evolving world. The productivity of most concert halls has not increased, as has that of more modern media; competition from recordings, motion pictures, and television has increased as these media have become cheaper and technically better. The result is bankruptcy for far too many orchestras and poverty for performing musicians.

At the same time, many modern composers, following their personal urges for innovation, have been frustrated in trying to create new expression with traditional instruments, players, listeners, and environments. The result is all too often unplayable or unpalatable "music."

One of the few bright areas is the popular scene. Here, the acceptance of electronic instruments has made good music both easier to play and more powerful than ever before. Twenty years ago, only a handful of the most expert bands could hold the interest of a high school audience. Today, every school in the United States has its own group, who, with the aid of amplifiers, pickups, fuzz tones, and countless decibels, can mesmerize their classmates for hours. In a very real sense, the productivity of the group, measured by listener attention divided by the hours of practice and expertise of the performers, has vastly increased.

A second hopeful area is the recording process and high fidelity reproduction. As H. F. Olson (*3*) has meticulously described, 8- and 16-track tape recorders, multiple microphones, mixing, reverberation, filtering, and other enhancements create phonograph records that many listeners prefer to a concert hall. Indeed, the competition of these magnificent recordings may be live music's greatest economic problem. And yet, live performance generates an excitement that can never be completely reproduced from an unchanging recording.

Can technology provide the tools with which the performing musician can again attract the musical attention and command the economic support of sufficient audiences? Can technology also provide new sounds to awaken the modern composer to new creativity? We believe that the answer to both questions is yes. The power of the computer plus integrated electronic circuits, both as an instrument of unlimited capacity to make new sounds and as a performance instrument that breaks the shackles of mechanical virtuosity, has been clearly demonstrated in the laboratory. We describe, as examples of this potential, the Music V program and the Groove program. Both Music V and Groove have been in operation and in use by musicians for 3 to 5 years. Music V makes it easy to produce a catalog of sounds, which can remove both the theoretical and practical limits on the creation and promulgation of new sounds. Groove has bridged the gap between recordings and live performance.

Although we intend to discuss only these two programs, in order to limit the length of this article and to keep close to our areas of personal interest, they represent only a small portion of the vast new technology that is shaping future music. "Classical" electronic music studios pioneered by Pierre Schaeffer and Karlheinz Stockhausen in Europe and by Vladimir Ussachevsky and Otto Luening in the United States, as well as synthesizers pioneered by Robert Moog and Donald Buchla, have already had great impact. Programs with objectives similar to Music V have been written by Hubert Howe, Arthur Roberts, John Chowning, and Barry Vercoe. A system similar to Groove has been independently created in England by Peter Zinovieff. Computer-controlled sound synthesizers are being developed by Edward Kobrin, Buchla, and Kurt

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Wiggen, among others. Computer-controlled environments have been demonstrated by James Seawright. All of this work is as likely to be relevant to future music as the specific examples discussed below.

Music V and a Catalog of Computer Sounds

A general-purpose computer can directly calculate sound wave forms. We call this process direct digital synthesis of sound; it is described elsewhere (*4*, *5*), hence our explanation here will be brief.

The computer calculates a string of samples of the desired sound waves; these samples are eventually converted into electrical pulses by a digital-to-analog converter. The pulses are smoothed by an appropriate low-pass filter to produce a continuously varying electrical voltage which drives a loudspeaker. The sampling theorem (*6*) states that, provided the sampling rate is high enough (say 40,000 hertz), one can produce *any* band-limited wave form (up to 20,000 hz, which is the upper limit of hearing for most people). We know how to deal with the samples to simulate such operations on continuous waves as filtering or generation of periodic oscillations: digital synthesis potentially allows for a very powerful musical instrument. In essence, the computer directly controls the motion of a loudspeaker, which is the most general source of sound now available. One can envision the synthesis of any sound that could come from a loudspeaker. However, two problems must be solved before music can be usefully produced—one concerns programming; the other, psychoacoustics.

A computer is controlled with programs, and different programs permit different ways of synthesizing sounds. But the process would be very tedious if the user had to write a new program each time he wanted a new sound. What is required is a music interpreter, a program capable of generating musical sounds in a wide variety of ways. Such a program would have to relieve the composer of the near-impossible task of specifying the value of the individual samples, since it takes many thousands of samples to synthesize 1 second of sound. At the same time, it must not restrict the generality of the process, as would be the case if it offered only a few synthesis procedures. The program should afford the composer a choice among many synthesis procedures; the composer should be able to specify to the computer the procedures he has chosen in a convenient and compact way; and the program should compute the many samples of the sound from this compact specification.

Moreover, the sound synthesis program must be efficient in terms of the computer time required for synthesis. Even with fast computers and efficient programs, it requires several seconds of computer time to calculate a second of fairly complex sound: an inefficient program would consume prohibitive computer time and would make the process impractically slow or expensive.

Music V is an example of a solution to the programming problem: it is a music synthesis interpreter that meets the requirements mentioned above. It is available to interested users, and it has been designed to be easily adaptable to various computer facilities: it is written mostly in Fortran, a programming language available on most computers, but the repetitive operations are in separate subroutines in order that they can be rewritten in assembly language for efficiency. It is thoroughly documented (*5*).

The user of Music V specifies the physical structure of the sounds he wants to synthesize. These specifications, usually punched on computer cards, are the necessary input data for the Music V compiler—they form the Music V "score." The score, through the description of the physical structure of the sound, is, at the same time, a recipe for synthesis. Much of the descriptions can be in the form preferred by the user—for example, in physical terms (seconds, decibels, and hertz) or in musical terms (beats, pianissimo-fortissimo, and notes of the scale). Individual notes may be described separately. At a higher compositional level, it is easy to add subroutines that can handle groups of notes in an efficient manner: for instance, designating a melody by name (for example, ALPHA) and having ALPHA + 5 represent the transposition of ALPHA up by five semitones, or 1/ALPHA represent the inversion (in the musical sense) of ALPHA. Such motive-handling subroutines have been developed by Leland Smith (*7*).

At a still higher compositional level, one can use composing programs to produce complete scores, following procedures pioneered by Lejaren Hiller and Leonard Isaacson (*8*). These programs can embody various rules of composition and orchestration. Pieces like "Masquerades" by M. V. Mathews and "Stochastic Quartet" by James Tenney (*9*) are both computer-composed and computer-played.

In addition to specifying the notes to be played, the composer must describe the instrument on which the notes will be played. Building blocks—digital oscillators, multipliers, filters, attack generators, adders, and others—from which the instrument may be assembled are provided, but the composer must know how to describe the sounds he wishes to generate in terms of these subroutines. By contrast, a composer for conventional orchestras knows the sounds of the instruments—violins, horns, and so forth—from long experience and hence has little need to know how they work physically. Providing an adequate physical description of interesting timbres is the psychoacoustic problem raised by direct digital synthesis.

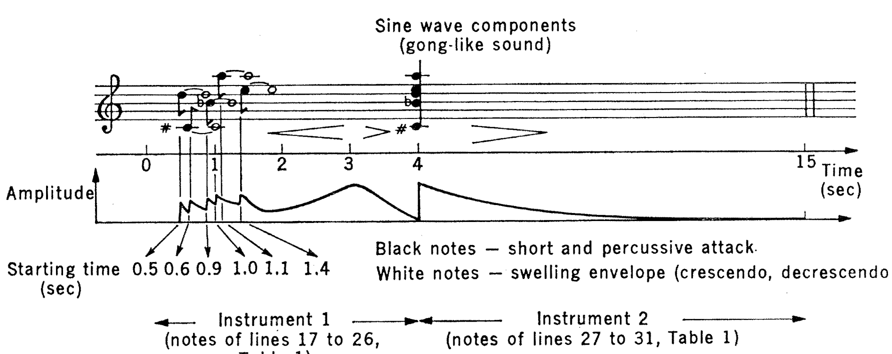


Fig. 1. Musical excerpt from the catalog of computer sounds (*11*), represented in quasi-conventional notation.

The first users of direct digital synthesis were immediately confronted with this fundamental problem. Even familiar sounds, such as those of traditional instruments, are not as easy to imitate as one might think: early attempts to imitate sounds using descriptions from classical acoustics treatises failed, pointing out the inadequacy of these descriptions and the need for more detailed and relevant data (*10*).

In order to make efficient use of direct digital synthesis, a body of psychoacoustical knowledge relating the physical parameters of a sound and its subjective effect is needed. Fortunately, Music V itself provides a language in which to describe such relationships, and a preliminary treatise has been written in the form of a catalog of musical sounds (*11*).

This catalog consists of three parts: the first is a recording of the actual sounds, the second is a simple description of the sounds in traditional musical terms (that is, a score plus a discussion of the structure of the instrument), and the third is a listing of the Music V score that synthesized the sounds. The listing is a complete and precise description of the sounds. By listening to the recording, each user can subjectively evaluate the timbres. The sounds, even those that have been synthesized as the result of long and careful preparatory work, can be immediately resynthesized by any user of the Music V program—or even by users of another sound synthesis process that starts from a specification of the physical structure of the sound, be it another program for direct digital synthesis or a music synthesizer such as the Moog. Thanks to the Music V language, such catalogs constitute unprecedented archives of complex, yet completely known, sounds. Every user's progress can be pooled to build up psychoacoustic know-how and to make available a gamut of musically useful sounds.

To give an idea of the process, we present an example drawn from the catalog. Lacking here is the recording of the sound itself (*12*), but the example will show how the synthesis procedure can be specified in the Music V language.

The sounds are represented in a form close to conventional music notation (Fig. 1): notes of a chord (D, C sharp, B flat, A, E) are played in succession, and the chord is sustained with a crescendo-decrescendo dynamics; then the chord is echoed by a gong-like sound whose frequency components are equal to the fundamental frequencies of the notes of the chord. (Note that computer synthesis permits such fine control over the timbre of the gong-like sound that it is heard as a kind of continuation of the sound of the previous chord. But one hears the chord as a combination of several musical tones and the gong-like sound as a single gestalt. This is one example of the kind of subtle sonic manipulation which can only be achieved by direct digital synthesis of sound.)

The entire computer score is reproduced in Table 1. To completely understand the score, one must know the Music V language (*5, 11*); we will not attempt a detailed explanation here but will point out some major features. Lines 1 to 12 of Table 1 specify the "instruments" used to synthesize the sounds. A block diagram of the instruments is given in Fig. 2, with each block representing a subroutine. Lines 1 to 12 specify which subroutines are used in each of the two instruments and how they are interconnected.

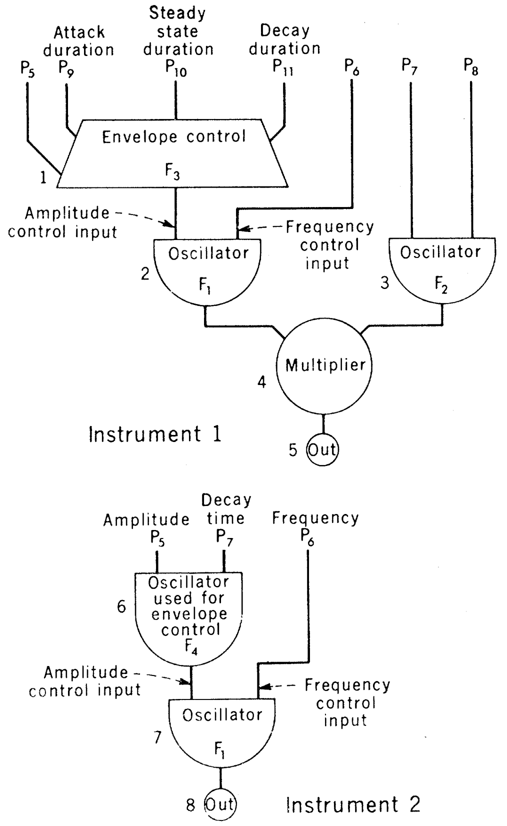
Lines 13 to 16 specify the generation of the four stored functions used by the instruments sketched in Fig. 2. Stored functions are utilized to speed the computation. F_1, a sinusoid, forms each overtone of the gong; F_2, a modified square wave, is used to generate the chord notes; F_3 is the attack and decay of the chord; and F_4 is the decay of the gong components.

Lines 17 to 31 specify the actual notes that make up the sounds. Starting times and durations of the notes are given in columns 2 and 4, and these can be easily related to the Fig. 1 score. For instrument 1 (lines 19 to 26), the pitch of the chord components is produced as a difference tone (which is specified by the difference in the numbers in columns 6 and 8 between oscillators 2 and 3). For instrument 2, the frequencies of the components of the gong-like sound are directly written in column 6. For both instruments, the other columns specify other simple factors such as amplitudes and attack times. Anyone with a small amount of training in Music V can easily and quickly read the score.

Although this example might appear to be closely related to traditional instrumental music, one should not be misled by the terms "instrument" and "note," used here only by analogy. In fact, the example should suggest how direct digital synthesis makes it possible to compose directly with sound, rather than by having to assemble notes—a process that Varèse envisioned very

Table 1. Score for the Music V excerpt shown in Fig 1.

1	INS	0	1;													
2	ENV	P5	F3	B3	P9	P10	P11	P30;								
3	OSC	B3	P6	B3	F1	P29;										
4	OSC	P7	P8	B4	F2	P28;										
5	MLT	B3	B4	B3;												
6	OUT	B3	B1;													
7	END;															
8	INS	0	2;													
9	OSC	P5	P7	B3	F4	P30;										
10	OSC	B3	P6	B3	F1	P29;										
11	OUT	B3	B1;													
12	END;															
13	GEN	0	2	1	1	1;										
14	GEN	0	3	2	0	10	10	10	10	10	0	− 10	− 10	− 10	− 10	− 10
15	GEN	0	6	3	10	1	1	10;								
16	GEN	0	7	4	− 9;											
17	NOT	.5	1	.6	18	424	18	1000	.01	0	.59;					
18	NOT	.6	1	.6	18	727	18	1000	.01	0	.59;					
19	NOT	.9	1	3.6	18	424	18	1000	2.3	.1	1.2;					
20	NOT	.9	1	.6	18	1545	18	2000	.01	0	.59;					
21	NOT	1	1	3.5	18	727	18	1000	2.7	0	.8;					
22	NOT	1.1	1	.6	18	1136	18	2000	.01	0	.59;					
23	NOT	1.3	1	3.2	18	1545	18	2000	1.9	.1	1.2;					
24	NOT	1.4	1	.6	18	1352	18	2000	.01	0	.59;					
25	NOT	1.5	1	3	18	1136	18	2000	1.9	0	1.1;					
26	NOT	1.8	1	2.7	18	1352	18	2000	1.4	.1	1.2;					
27	NOT	4	2	10	400	273	10;									
28	NOT	4	2	7.5	200	455	7.5;									
29	NOT	4	2	4.5	200	576	4.5;									
30	NOT	4	2	6.5	150	648	6.5;									
31	NOT	4	2	4	150	864	4;									
32	TER	15;														



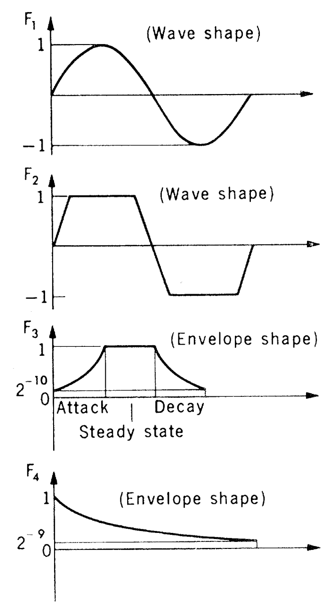


Fig. 2. Diagram of instruments and functions used in the illustration from the sound catalog (*11*).

clearly more than 50 years ago. In our example, each NOT instruction of lines 27 to 31 corresponds not to a note in the usual sense, but only to a tone component. This permits one to freely control the blending of these components into a timbre, whereas, if a composer writes for a gong, he has no control over the harmonic structure of its sound, which depends upon the particular instrument used.

Using this example or others in the catalog, the trained computer musician can identify sounds suitable for his needs by listening to the recording, study a score of the sound in musical notation, obtain a general description of how the instrument functions from its block diagram and textual description, and know exactly and completely how the sound was produced from the Music V score. Never before have complex sounds been described so completely and usefully.

The present catalog includes 28 classes of sounds, ranging from traditional instruments—flutes, brasses, percussion instruments—to novel sounds like the above example, or "endless glissandi." The present catalog in itself is very useful, but the possibilities of continually adding new sounds portends an unlimited future.

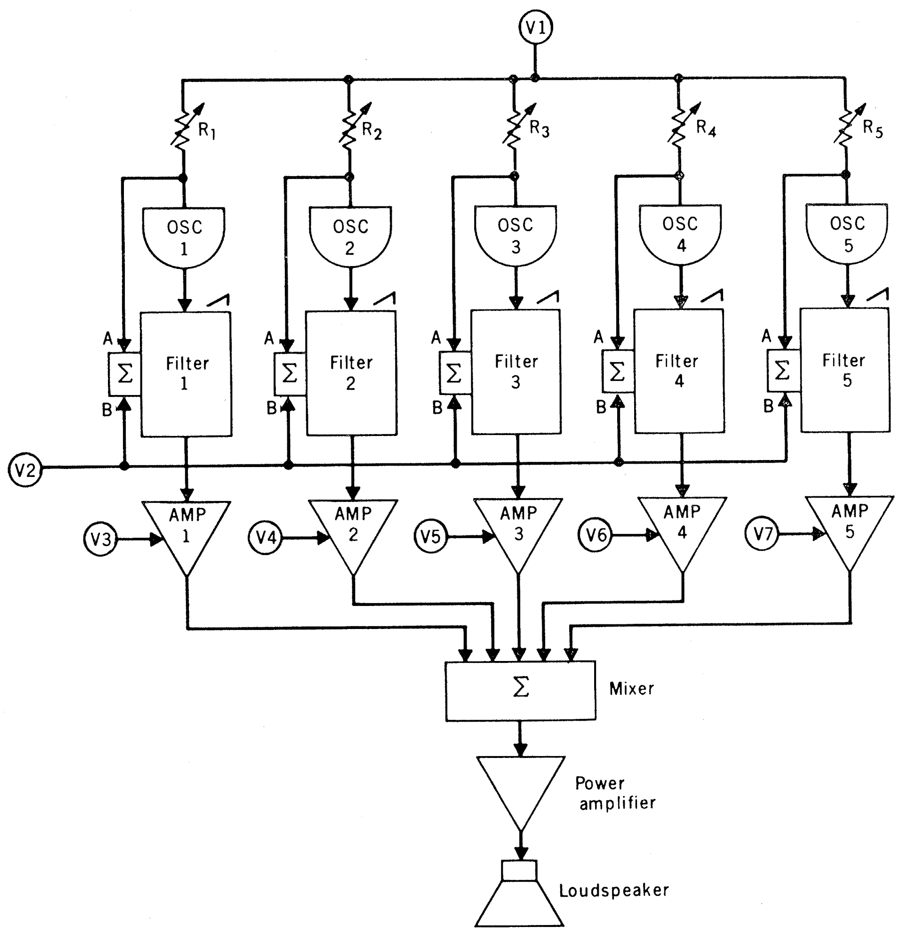


Fig. 3. A Groove instrument for synthesizing a wide variety of sounds. Control-voltage one (V1) controls the pitch of the five oscillators (*OSC*), V2 controls the number of harmonics that will be admitted through the filters, and V3 through V7 control the amplitude (*AMP*) envelopes of each of the five components of the sound. The relations among the five pitches are set by variable resistors (potentiometers) R_1 through R_5.

Groove, A Real-Time Music Program

Use of the sound catalog has produced recordings with rich and varied timbres that compare favorably with other recorded art forms. A criticism often made of all records is the lack of a "spontaneity" that is inherent in a live concert. Spontaneity is especially hard to introduce into computer music because the composer must specify all aspects of the synthesized sound in advance; no subtle adjustments in the music, called "performer nuance," can be made by the computer. In order to add interpretations during a performance, it is necessary to provide a means by which a human being can work on his own time scale—that is, in real-time. Our first realization of a real-time system is called Groove (generated real-time operations on voltage-controlled equipment) (*13*). Groove is a hybrid system that interposes a digital computer between a human composer-performer and an electronic sound synthesizer. All of the manual actions of the human being (playing a keyboard, twisting knobs, and so forth) are monitored by the computer and stored in its disk memory as digitally sampled functions of time. These functions of time are selectively output through digital-to-analog converters to become time-varying voltages that control the various acoustic parameters of the sound synthesizer, such as pitch, loudness, and timbre.

To avoid limiting the capabilities of the system to what could be accomplished by a person controlling a sound synthesizer directly, a new concept in control is necessary—what we call the "conductor concept." Simply stated, it is that the relation between the performer and the computer is not that between a player and his instrument, but rather that between a conductor and an orchestra. The conductor does not play every note in the score; instead, he influences (hopefully controls) the way in which the instrumentalists play the notes. In Groove, this concept led to the creation of a memory for functions of time that embody the score of the composition. In the playback mode, the performer-conductor determines how these functions are reproduced from the computer memory: he can vary the tempo or emphasize a particular musical voice, at will, by twisting appropriate knobs. In the editing mode, new time functions may be added, existing ones altered, and time itself may be made to move forward or backward, or even be stopped momentarily, under the explicit control of the performer.

Currently, Groove utilizes analog electronic devices such as voltage-controlled oscillators, filters, and amplifiers, whose frequency of oscillation, bandwidth, and gain are responsive to the control voltages generated at the computer. A typical Groove "instrument" is shown in Fig. 3. It is "constructed" by patching together the several general-purpose analog elements shown and is capable of synthesizing a wide variety of interesting sounds, such as the gong-like sound described in the Music V example above. Control voltage V1 from the computer is applied simultaneously to five voltage-controlled oscillators, generating sawtooth wave forms. The sawtooth wave forms are filtered by five voltage-controlled filters. The cutoff frequencies of the filters are determined by the sum of two control inputs. Control input A causes the filter to track the frequency of its associated oscillator. Control voltage B determines how many of the harmonics of the sawtooth wave form will be admitted through the filter. Thus, by varying control voltage V2 from the computer, variable harmonic spectrums may be obtained. Each of the five basic component signals are routed through individual voltage-controlled amplifiers, allowing the computer to generate separate amplitude envelopes for each of these components by way of voltages V3 through V7. Given the "instrument" shown, the Groove user might instruct the computer to assign the time functions he will generate, as shown in Table 2. A simple algebraic interpreter is incorporated in the program, and this enables the user to define functions in terms of the normal arithmetic operators, plus special subroutines that read keys depressed on the keyboard, read the position of knobs, and compute such relations as exponential envelopes. Details of the algebra have been published (*13*)—Table 2 is intended only to give an idea of the possibilities. As shown in Table 2, the user has instructed the computer that he wishes to control the pitch of his instrument with the keyboard, and the timbre (number of harmonics) with knob 1. He wishes to use an exponential envelope with knob 2 controlling the amplitude, knob 3 the attack time, and knob 4 the decay time. A new envelope will be triggered each time V1 changes value (that is, each time a key is depressed). The expressions for V4 through V7 cause the envelopes of the higher frequency components to be the square, cube, and so forth of V3, thus making the higher frequency components attack and decay more abruptly. The user then may improvise a composition or play from a prepared score, while the computer memorizes all of his actions. After playing once through his composition, he may backtrack through the stored functions and alter or augment these until satisfactory results are obtained. When the final performance has been stored in the computer's memory, he may tape record the result.

Table 2. Slightly simplified Groove instructions that assign humanly controlled input devices to the various control functions. V1 will be determined by which note is depressed on an organ-like keyboard; V2 will be set by a knob that the composer can turn in real-time; V2 through V7 are computed by an envelope generator from information taken from the keyboard and from other knobs.

```
V1 = KEY*55
V2 = KNOB1
V3 = EXPENEVELOPE (AMPLITUDE
     (KNOB2), ATTACK (KNOB3),
     DECAY (KNOB4), TRIGGER (V1))
V4 = V3*V3
V5 = V4*V3
V6 = V5*V3
V7 = V6*V3
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Although Groove has been in existence for only a little more than 3 years, many hours of music have been composed and, even more encouraging, the musicians who have tried it are highly enthusiastic. The facilities for playing —with powerful timbres, with the same immediacy as a conventional instrument, and with the possibilities of easily correcting mistakes and adding later interpretations—are much appreciated.

Current experiments with the Groove system include the construction of special-purpose analog circuits, which extend the sound-generating palette, and the use of special-purpose digital circuits as sound synthesizers. Such digital circuits will provide a generality and accuracy akin to that of Music V and will allow one to use the sound catalog. Within the next few years, a Groove-like system consisting of composer, computer, and digital sound synthesizer will allow the interactive generation of virtually any sound that can come from a loudspeaker in real-time.

Musical Futures

In the preceding sections we have described innovations that we hope are convincing examples of the technical possibility of making musical sounds in very new and different ways. However, the mere existence of technology in no way guarantees its musical exploitation or, even less, that it will be effective in leading music toward a new era. Here we will speculate on possible new directions that we feel are technically possible and that music may take, although we fully recognize the fallibility of predictions.

It seems clear to us that, even with no additional developments, computers are powerful instruments for creating recorded music. Therefore, we anticipate that computer-synthesized sound tracks for films, television background music, and phonograph records will become important. Music V makes available a range of sounds, an ease of description of sound quality, and a precision that have never before been possible. Groove is an instrument that makes real-time performance in the studio easy and that makes possible editing, correcting, and augmenting existing Groove pieces in ways that are far more flexible than any past ones.

Although technology may augment and change performance, we do not minimize the importance of recorded sound. Indeed, we believe recorded sights and sounds are the outstanding medium of the present era. One is led to ask whether technology can rescue live performance from the stiff competition of recordings.

There is every reason to expect that Groove-type synthesizers will soon be small enough and reliable enough to be used in concerts, either alone or in combination with other instruments. Using large-scale integrated circuits, which created the pocket calculator, one can easily construct a computer synthesizer that will fit in the volume of existing instruments. The latest Allen organ already attests to the musical use of these circuits.

Both prerecorded tapes and noncomputerized synthesizers have already been used in concert, and both have serious limitations that are overcome by Groove. The prerecorded tape imposes its own tempo on the composition; synthesizers are one of the most difficult instruments to play rapidly. Groove can reproduce a prerecorded score at any tempo and allows the player to "interpret" the score in any way that he physically can with computer assistance.

One of the most interesting possible developments is the electrical connection of existing instruments to Groove. At present, the only Groove control that resembles an instrument is the organ keyboard. The pitch and intensity of most instruments can be measured, and these signals have much potential as Groove functions.

In addition to the important but obvious augmentation of sound quality and power, what musical potentialities are created by the instrument-computer-synthesizer combination that we have envisioned? There exist possibilities for relations among composer, performer, and listener which are very different from those we now know. Almost all existing music falls into one of two categories—completely prespecified pieces, in which each note is written in the score, or improvisational pieces, in which the performer must select the notes with little guidance from the composer. With the computer, there are many possibilities between these extremes. The composer may create a framework of relations that limit or direct the performer's choices. He may compose in terms of harmonic laws, rhythmic or metric groups, and gradual changes in overall dynamics or tempo. Some aspects of improvisational music such as jazz can be incorporated into the performance. But at the same time, overall structures and precise interrelations between voices, which are all too often missing from improvisational music, can be retained.

The new freedom in performance may mean that many pieces will never be repeated exactly. In a sense, this music will become a consumable rather than an archival expression; there is much to be said for a phoenix-like art —which is consumed in the fires of performance but which contains the genes of reincarnation in its program.

What of the audience? It may continue unchanged for music that we call a "virtuoso-spectator experience." However, we are attracted to a form in which the audience and performers coalesce. We see the possibility of a broadly popular "participator art" whose purpose is self-expression. Technology will provide an instrument that can be played as simply as lowering a needle on a phonograph record, but that has no visible limits on the powers of expression and variation.

If there comes into being a medium in which a composer writes a program, in which one or perhaps many performer-listeners execute the program on a computer synthesizer, in which the performer-listeners can interact with the program in complex ways to influence the course of the sound, in which there may be no audience, either by choice of the performers or because the potential audience prefers to do its "own thing," should the medium be called music?

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