Book Reviews

Much-Esteemed Theory

The Large Scale Structure of Space-Time. S. W. HAWKING and G. F. R. ELLIS. Cambridge University Press, New York, 1973. xii, 392 pp., illus. $28.50. Cambridge Monographs on Mathematical Physics.

Ask a prospective graduate student in theoretical physics what area he hopes to work in, and the chances today are better than 50-50 that he will reply "gravitation theory." This has caused problems for some physics departments, but it shows where the action is—or at least where many students think the action is. This prejudiced reviewer happens to think the students are right—in that the theory of gravity poses some of the most challenging conceptual problems in physics, problems that touch the foundations of nearly all physical theories. The students are simply expressing a gut awareness of the fact.

The theoretical framework for gravitational research has remained unaltered for nearly 60 years. Although proper attention, both experimental and theoretical, has been given to alternative theories, *the* theory is still Einstein's general relativity, as it was in 1916. The remarkable stability of this theory may fairly be attributed to the extreme practical difficulties one encounters in attempting to devise crucial experiments to challenge it. The challenges it faces today have mostly been devised from within, slowly and with many false starts, by men and women who were willing to gamble (and in some cases throw away) their professional careers by studying the internal consistency of general relativity and elucidating, with pencil, paper, and computer, its many fantastic predictions, most of which Einstein himself never knew.

The book by Hawking and Ellis contains a canonical sampling of these fantastic predictions, as well as a full and rigorous account of the chinks in the armor of the theory, which have been uncovered in the past decade. As a fundamental physical theory general relativity is a failure. It is a failure because it predicts that, under very general conditions, singularities must occur in space-time, beyond which the theory is incapable of saying anything. That is, the theory predicts that it cannot predict. It is not fundamental enough. It must eventually be superseded by something more universal.

This is an old story in physics: Classical electrodynamics, hydrodynamics, statistical mechanics, and quantum field theory are all examples of theories whose incompleteness can be shown on internal grounds alone. (Think of point charges, shock waves, phase transitions, and quantum field theoretical divergences.) But these theories by virtue of their overwhelming utility and beauty are still part of our standard curriculum, and we think none the less of them for their failings. Neither do we demote general relativity in our esteem. On the contrary, we see in the problems it presents to us only wonderful adventures for the future. The student who wishes to share in these adventures must master the material that Hawking and Ellis cover.

Beginning with a 50-page résumé of differential geometry in modern notation and a 20-page statement of general relativity theory as a set of postulates about a mathematical model for space-time, the book proceeds briskly to the construction of the tools needed for proving the main theorems. These tools include the theory of geodesics and conjugate points, Raychauduri's equation, energy conditions, the conformal definition of "infinity," topological constructs necessary for analyzing the causal structure of space-time, precise statements of various more or less physically reasonable causality conditions, and the definitions of Cauchy surfaces, Cauchy horizons, global hyperbolicity, and asymptotic simplicity. These tools are then applied in successive chapters to the Cauchy problem in general relativity, to the proofs of the great singularity theorems, to the prediction and analysis of black holes, and to the implications of general relativity for the cosmos as a whole (a singularity in our past).

Except for the black-hole theorems of Israel and Carter, which are stated without proof, all of the major theorems of the book (the majority of which were given in original form by Choquet-Bruhat, Geroch, Hawking, and Penrose) are proved in full in their most general forms. Utmost attention is given throughout to precision of statement, including statements of the minimum differentiability or continuity conditions that must be imposed on the metric. Chapter 5 contains an excellent collection of exact solutions of Einstein's equation together with descriptions of their causal structures. These structures range the gamut of known behaviors and motivate many of the later definitions: Cauchy horizons, event horizons, trapped surfaces, imprisoned curves, incomplete geodesics, and the Schmidt *b*-boundary.

The book is a masterpiece, written by sure hands. But it is a flawed masterpiece. The student who conquers it will be richly rewarded. But he will have to work unnecessarily hard. This is because the authors are unable to decide who their audience is. The weight of internal evidence suggests that they are writing for mathematicians and not for physicists. Certainly the authors make few concessions to the nonmathematician. Too often lemma follows lemma and proposition follows proposition in a way that only those who believe a theorem is its own reward will find stimulating. Too often, especially in the crucial sixth and eighth chapters (causal structure and singularities), the running commentary is held to an absolute minimum, with the significance of a given proposition being discussed after its proof rather than before. A happy exception to this rule is the third section of chapter 9 (gravitational collapse), where all proofs are postponed to the end. Why this felicitous procedure could not have been adopted throughout is a mystery.

That the authors must occasionally have believed they were writing for physicists is clear from the 50-page introduction to differential geometry and from the material on stellar collapse and the expansion of the universe that prefaces chapters 9 and 10. The latter material surely could not have been intended for the average mathematician. It assumes far too much knowledge of graduate-level physics and is too condensed. The authors even lapse on one occasion into that awful sin of astrophysicists: giving formulas containing observational numbers but without stating the units.

The book should be about 100 pages

longer. The discussions preceding the difficult theorems should be greatly expanded. At least a dozen more examples and counterexamples should be given, motivating the introduction of technical machinery. And many more diagrams could profitably be included. What a great book for students it would then be!

The book contains a few eccentricities: use of "data" as a singular noun; references to "measurements" in general relativity (for example, physical determination of a C^{r+1} atlas!) that would mystify an experimental physicist; and calling the founder of black-hole theory (in an appended translation of his original paper on the subject) by the name Peter Simon Laplace. One is reminded of those famous ghosts of Westminster Abbey: Michel Faraday and Jacques Maxwell.

The book also contains one failure to distinguish between mathematics and physics that is actually serious. This is in the proof of the main theorem of chapter 7, that given a set of Cauchy data on a smooth spacelike hypersurface there exists a unique maximal development therefrom of Einstein's empty-space equations. The proof, essentially due to Choquet-Bruhat and Geroch, makes use of the axiom of choice, in the guise of Zorn's lemma. Now mathematicians may use this axiom if they wish, but it has no place in physics. Physicists are already stretching things, from an operational standpoint, in using the axiom of infinity. It is not a question here of resurrecting an old and out-of-date mathematical controversy. The simple fact is that the axiom of choice never is really needed except when dealing with sets and relations in nonconstructible ways. Many remarkable and beautiful theorems can be proved only with its aid. But its irrelevance to physics should be evident from the fact that its denial, as Paul Cohen has shown us, is equally consistent with the other axioms of set theory. And these other axioms suffice for the construction of the real numbers, Hilbert spaces, C^* algebras, and pseudo-Riemannian manifolds—that is, of all the paraphernalia of theoretical physics.

In "proving" the global Cauchy development theorem with the aid of Zorn's lemma what one is actually doing is assuming that a "choice function" exists for every set of developments extending a given Cauchy development. This, of course, is begging the question. The physicist's job is not done until he can show, by an explicit algorithm or construction, how one could in principle always select a member from every such set of developments. Failing this he has proved nothing.

Happily, every other theorem in the book is as sound as a rock, and students could not ask for better navigators through space-time than Hawking and Ellis.

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Solar Physics

The Quiet Sun. EDWARD G. GIBSON. National Aeronautics and Space Administration, Washington, D.C., 1973 (available from the Superintendent of Documents, Washington, D.C.). xviii, 330 pp., illus. $6.20. NASA SP-303.

This book resulted from the discovery by the author, while training for his role as a Skylab astronaut, that no book or compendium dealing with the physics of the whole sun had been written for over two decades. The title *The Quiet Sun* is used as a device for avoiding detailed discussions of active phenomena, not as an attempt to suggest a sharp distinction between a quiet and an active sun.

The chapter topics follow in an orthodox order. After two chapters dealing with the general characteristics of the sun, they are: "The interior," "The photosphere," "The chromosphere," and "The corona." The subject matter contained under the last three of these headings, however, departs quite strongly from that in earlier books. This difference reflects, in part, new ideas and perspectives. It also indicates a conscious decision to touch only sketchily those topics which have been discussed exhaustively elsewhere and deal critically and in detail with those for which the information has heretofore been scattered through the journals. (The comprehensive discussion of photospheric and chromospheric oscillations is an outstanding example of the latter.) This technique has greatly increased the general usefulness of the work as a starting point for the study of solar physics and makes it an excellent supplement to previous compilations on the sun. It will not replace these compilations, however. A newcomer to the field, impressed by its general excellence and apparent comprehensiveness, might, for example, conclude from it that the optical emission corona has yielded no useful information, even that it can be observed only at an eclipse. He would be unaware that optical monochromatic observations, next to sunspots, constitute the most complete synoptic record that we have of the sun; or that monochromatic photographs still give the most detailed pictures available, prior to Skylab at least, of coronal structure.

A central theme in the book is the idea that many of the phenomena of the solar atmosphere should be explained in terms of small- and larger-scale convective processes made evident by granulations and supergranulations. Another recurring idea is that the appearance of a feature in the chromosphere—whether bright or dark, for example—can be explained only through a nonlocal-thermodynamic-equilibrium computation of how the source function of the line in which the feature is observed varies with distance above the photosphere.

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Geophysics Etc.

Annual Review of Earth and Planetary Sciences. Vol. 1. FRED A. DONATH, FRANCIS G. STEHLI, and GEORGE W. WETHERILL, Eds. Annual Reviews, Palo Alto, Calif., 1973. viii, 350 pp., illus. $12.

Inaugurating a new series, this volume discusses a fascinating array of current problems and shows Annual Reviews' customary high level of editorial and authorial competence. The topics of the 14 papers are: geophysics state of the art (Jeffreys), glacioisostatic rebound (Walcott), the origin of red beds (Van Houten), rock fracture (Mogi), Jupiter's and Saturn's interiors (Hubbard), magnetospheric electrons (Coroniti and Thorne), the origin of mammals (Crompton and Jenkins), subsurface water chemistry (Barnes and Hem), the origin of mineral deposits (Skinner and Barton), earthquake strain release (Kanamori), Cenozoic plankton paleontology (Riedel), rock magnetism (Hargraves and Banerjee), lower-atmosphere electrical balance (Vonnegut), and silicate mineral order-disorder (Burnham).

The topics are really too scattered for a single volume. Do the editors seriously think that there are many people working on both mammal tax-