Reports

The Earth's Mantle: Evidence of Non-Newtonian Flow

Abstract. *Recent information from experimentally deformed dunite coupled with a reanalysis of data on the Fennoscandian postglacial rebound suggest that the rheological behavior of the upper mantle is distinctly non-Newtonian, and that the shear strain rate is proportional to the shear stress raised to about the third power.*

During the last decade, it has been conclusively shown that flow on a truly massive scale is occurring in at least the earth's upper mantle. Evidence for this flow is the global pattern of continental drift and sea floor spreading. If a quantitative understanding of this flow is to be achieved, the rheological behavior of mantle rock must be known. The rheological behavior is characterized by a flow law relating the strain-rate field to the stress field and to environmental variables, notably temperature, hydrostatic pressure, and water content.

A key assumption almost universally made by investigators seeking quantitative models of flow in the mantle is that the earth's mantle behaves like a Newtonian fluid (*1–4*). This report challenges the assumption that the mantle is Newtonian. The data on postglacial rebound and experiments on hot creep of dunite both suggest non-Newtonian flow in the mantle.

The melting of the ice cap in Fennoscandia (Finland, Sweden, Norway, and Denmark) about 10,000 years ago left a surface depression that could be only partially removed by immediate elastic rebound. The history of the return toward isostatic equilibrium is shown in Fig. 1a, and the present rate of return in Fig. 1b. The depression has kept a nearly constant shape and the present rate of uplift is everywhere proportional to the amount of remaining depression. This leads us to believe that postglacial rebound belongs to a special class of behavior: "proportional relaxation" of stressed bodies. The ratio of the effective shear stress at any point to that at any other point remains constant with time, while the amplitudes of both stresses diminish asymptotically to zero. The validity of applying this concept to Fennoscandia is discussed below.

We assume that the mantle obeys a general flow law of the form

$$\dot{\varepsilon}_s = B(z)\tau^n \qquad (1)$$

where $\dot{\varepsilon}_s$ is the steady-state effective strain rate, τ is the effective shear stress, B depends only on depth (z), and n is a constant exponent which is to be determined from postglacial rebound (*5, 6*). The effective viscosity is

$$\eta_{eff} = 1/[2B(z)\tau^{n-1}]$$

which reduces to: $\eta_{eff} = \eta = 1/[2B(z)]$ for a Newtonian fluid ($n = 1$). It is assumed that the shear stress resulting from the depression is greater than shear stresses in the mantle due to other causes (for example, plate tectonics). If the latter stresses were much greater the effective n would be 1, no matter what the intrinsic value (*7*). Modern plate (and plume) theory yields small relative motion of Fennoscandia and the deep mantle in recent times, so in this locale it is believed that the assumption is justified.

In a material undergoing proportional relaxation and obeying Eq. 1, where B is a function of position but not of time, it can be shown that

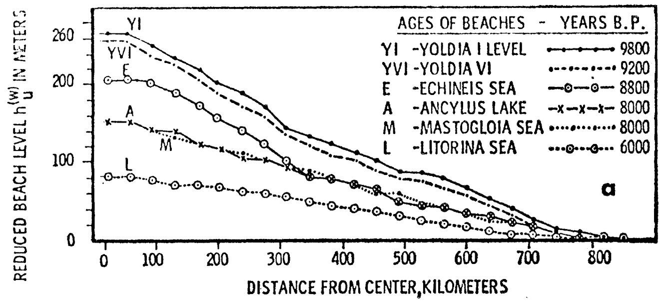
$$\langle \dot{\varepsilon}_s \rangle = A \langle \tau \rangle^n \qquad (2)$$

where $\langle \dot{\varepsilon}_s \rangle$ and $\langle \tau \rangle$ are the averages over any finite space of the effective shear strain rate and stress, respectively, and A is a constant (*8*). It follows immediately for the case of postglacial uplift that $\dot{\zeta} \propto \langle \dot{\varepsilon} \rangle$ and $\zeta \propto \langle \tau \rangle$, so that

$$\dot{\zeta} = C\zeta^n \qquad (3)$$

where ζ is the amount of remaining uplift at the center of the uplifting area, $\dot{\zeta}$ is the rate of uplift at that point, and C is a constant which depends on $B(z)$ and the areal extent of the mass deficiency. It will be noted that Eqs. 1, 2, and 3 are equally valid for Newtonian flow ($n = 1$) and for flow of the type characteristically observed in crystalline solids at high temperatures where Eq. 1 is obeyed and $n \geq 3$.

We now consider the validity of proportional relaxation in postglacial uplift. If the mantle is assumed to be incompressible (with no elastic component), the stress distribution in the mantle must be consistent with the surface depression and also subject to the constraint that the total viscous dissipation be a minimum at all times. Provided that the flow law in the mantle has the form of Eq. 1, the shape of the stress distribution in the mantle will depend on the shape of the surface depression (including its horizontal extent) but not on the magnitude of the actual surface elevations. (This would not be



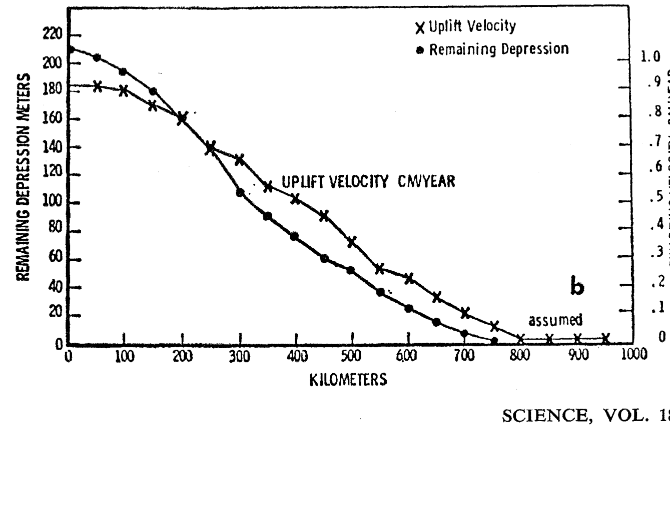


Fig. 1. (a) Beach levels and (b) remaining depression and present rate of uplift in Fennoscandia. Reproduced with permission of McConnell (*2*).

true if n were dependent on stress.) If a particular stress distribution in the mantle results in minimizing the total viscous dissipation for a particular shape of the surface elevation, the stress distribution in the mantle resulting from a multiplication of the surface elevations by a constant factor F will result in a minimum of viscous dissipation if the original stresses are also multiplied by the same factor F. The viscous dissipations ($\tau \dot{\epsilon} dV$, where dV is a volume element) will all be multiplied by F^{n+1}. In other words, if the surface depression keeps a constant shape with time, as observed in Fennoscandia, the stress distribution in the mantle will also keep a constant shape, and proportional relaxation is achieved. Consequently, Eq. 3 should be valid for the Fennoscandian uplift, if an incompressible mantle is assumed (*9*).

To apply Eq. 3 to Fennoscandia, ζ is obtained by adding the remaining uplift to the elevation of past beach levels from Fig. 1a. The remaining central uplift has been estimated as 180 m (*3*), 210 m (*10*), and 150 m (*11*). For 180 m, the values of ζ are given in Table 1. In order to check these against Eq. 3, we integrate the latter

$$\zeta^{1-n} = \zeta_{\max}^{1-n} + C(1-n)t, \quad n \neq 1 \qquad (4)$$

$$\ln\zeta = \ln\zeta_{\max} + Ct, \quad n = 1 \qquad (5)$$

where $\zeta_{\max}$ is the central depression at $t = 0$, the time of glacier retreat (446 m).

Equation 5 applies for Newtonian viscous flow, in which case a plot of the logarithm of ζ against time should yield a straight line. Figure 2 shows that this is not true. Fitting Eq. 4 to these data to minimize the root-mean-square deviations yields

$$\zeta^{-2.21} = \zeta_{\max}^{-2.21} + 9.629 \times 10^{-10} t \qquad (6)$$

which on differentiation yields

$$\dot{\zeta} = -4.36 \times 10^{-10} \zeta^{3.21} \qquad (7)$$

A comparison of Eq. 6 with the observations is shown in Table 1 and Fig. 2. The present central uplift rate from Eq. 7 is 7.6 mm/year, compared to Kääriäinen's (*12*) estimate of 9 mm/year from geodetic leveling measurements over the last few decades.

Altering the estimate of the amount of uplift remaining from 180 m to 210 or 150 m does not affect the quality of the fit, but the preferred exponent n is changed from 3.21 to 3.58 and 2.85, respectively. The calculated rate of present uplift remains the same within 2 percent because the change in ζ is offset by the change in n. It thus appears that this proportional relaxation model of the Fennoscandian postglacial uplift requires non-Newtonian flow with a stress exponent of 3.2 ± 0.3 (*13*). Newtonian flow seems to be excluded by the curvature of the data in Fig. 2.

Table 1. Observed and calculated amounts of central uplift (ζ) remaining at various times.

Time (years ago)	ζ (m) Observed (*2*)	ζ (m) Calculated
9320*	446	446
9200	437	430
8800	385	388
8000	331	333
6000	261	260
0	180	180

* Corrected from McConnell's (2) value of 9800 years. Subsequent uplift indicates that removal of the ice cap was not complete 9800 years ago.

Postglacial rebound data also exists for a number of areas in Canada (*14*). An interpretation by the above method is, however, severely hampered by the lack of knowledge of the remaining uplift and lack of data at the centers of the uplifting regions. A preliminary analysis indicated non-Newtonian flow with the stress exponent $n \cong 3.5 \pm 1.0$ (*15*).

We now examine the consistency of the above finding for mantle flow with experimental evidence on the flow of presumed mantle rock. In recent years, the "plastic" flow behavior of a number of such rocks has been investigated at high temperatures and pressures (*16, 17*). Most experiments have involved compression, employing three types of deformation techniques: (i) constant strain rate; (ii) creep, or constant compressive stress; and (iii) relaxation (the advance of a deformation piston is stopped, and the nonhydrostatic stress in the sample is relieved through conversion of the elastic strain, in both the apparatus and the sample, into plastic strain in the sample). The deformational behavior of dunite (composed of olivine plus minor accessory minerals) is of particular interest, as olivine is likely to be the primary constituent of the upper mantle. Dunite, experimentally deformed at high temperatures and relatively low stresses, develops textures similar to those found in olivine-rich xenoliths believed to have come from the mantle (*18*).

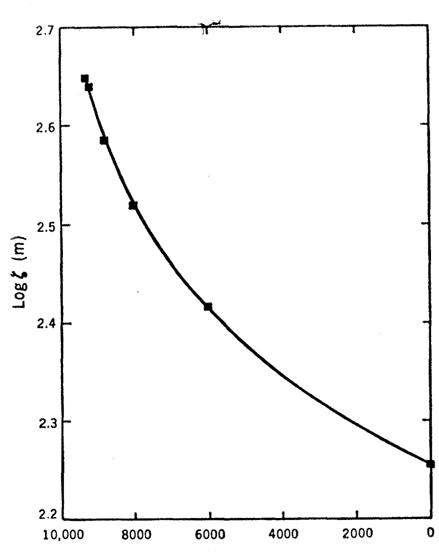


Fig. 2. Fennoscandia postglacial rebound: remaining uplift (ζ) versus time. (■) Observed values from Table 1 (*7*); (solid line) Eq. 6, where $n = 3.21$. Note that Newtonian flow would give a straight line on this plot for proportional relaxation.

Creep and relaxation experiments on Mt. Burnett dunite in both hydrous and anhydrous environments were performed by Post (*15*). Creep experiments in which steady-state flow was attained at a particular stress and relaxation tests at temperatures (T) of 1000° to 1600°K were fitted best by the following equation, when the nonhydrostatic compressive stress was less than 3 to 4 kbar

$$\dot{\epsilon}_s = 1.7 \times 10^9 \exp(-31.8 T_m/T) \times \tau^{3.18 \pm 0.18} \qquad (8)$$

where T(°K) is the sample temperature; T_m(°K) is the estimated melting temperature, as influenced by varying water content and hydrostatic pressure; and τ(kbar) and $\dot{\epsilon}_s$ (sec^{-1}) are defined in Eq. 1. The activation energy of undried samples of Mt. Burnett dunite deformed in jackets of dehydrated talc is 93.1 ± 2.5 kcal/mole. In a completely anhydrous environment, the activation energy is 130 kcal/mole (*15, 17*).

The stress exponent of 3.18 ± 0.18 in Eq. 8 is in good agreement with the 3.2 ± 0.3 determined from the analysis of the uplift data. Thus, the available experimental and geophysical data agree in predicting a non-Newtonian mantle where $\dot{\epsilon}_s$ is proportional to τ^3. With an assumed profile of T/T_m in the mantle, and the assumption that the elastic stress field is similar to the actual field, Eq. 8 can be used to estimate the rheological behavior. The elastic stress distribution beneath the Fennoscandian region can be calculated from the observed depression. With that stress field the resulting central uplift rates predicted from Eq. 8 are within a factor of about 2 of the observed central uplift rates obtained from the raised beachlines and survey measurements (*15*). While agreement as close as this must be fortuitous, considering the number of assumptions involved in predicting the uplift rates, the experimental and geophysical data appear consistent with each other, and support the contention that the mantle, at least beneath shield areas, is non-Newtonian (*19*).

In conclusion, recent evidence indicates a non-Newtonian mantle with $n \simeq 3$, at least under Fennoscandia. This suggests that all studies of mantle motion, notably the search for the driving force for plate tectonics, and all inferences about the viscosity of the earth should endeavor to incorporate non-Newtonian flow of this kind.

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5. Equation 1 describes the creep behavior of most metals and ceramics at high temperatures and low to intermediate stresses (*6*). The value *B* varies directly with the diffusion coefficient for the rate-controlling atomic (or molecular) species. The stress exponent *n* ranges from 3 to 5 for most coarse-grained polycrystalline materials. Equation 1 is, however, equally valid for Newtonian flow.
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8. The proof of Eq. 2 reduces to proving that $\langle\tau^n\rangle = k\langle\tau\rangle^n$, where k is a constant. The definition of proportional stress relaxation is: $\tau = \tau(r,t) = \tau(r,0)f(t)$, where r is a vector defining position and $f(t)$ is the appropriate function of time, invariant with position. Consequently

$$\frac{\langle\tau^n\rangle}{\langle\tau\rangle^n} = \frac{\langle[\tau(r,t)]^n\rangle}{\langle\tau(r,t)\rangle^n} = \frac{\langle[\tau(r,0)]^n\rangle[f(t)]^n}{\langle\tau(r,0)\rangle^n[f(t)]^n} = k$$

9. The inclusion of an elastic component into a model of a non-Newtonian mantle is definitely a complicating factor. When the elastic component is taken into account, the viscosity values calculated from the 6000- to 9000-year-old raised beachlines (for an assumed Newtonian mantle) are about half the values that result when the elastic component is ignored [D. P. McKenzie, *Geophys. J. Roy. Astron. Soc.* **14**, 297 (1967)]. The main reasons for concluding that the elastic component should not seriously affect the validity of Eq. 3 are that (i) the stress distribution resulting from a surface load on an elastic half-space is generally quite similar to that resulting from the same load on a nonelastic half-space [H. Jeffreys, *The Earth* (Cambridge Univ. Press, Cambridge, England, ed. 2, 1952), chapter 6]; (ii) the stress distribution in the elastic case keeps a constant shape if the surface load keeps a constant shape, as in the purely viscous case; and (iii) the stress distribution in a viscoelastic mantle should be intermediate between the purely elastic and purely viscous (both Newtonian and non-Newtonian) cases.
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17. N. L. Carter and H. G. Avé Lallemant [*Geol. Soc. Amer. Bull.* **81**, 2181 (1970)] found $n = 2.4$ and 4.8 for wet and relatively dry dunite, respectively. The corresponding activation energies were 80 and 120 kcal/mole. It is believed that the current results are more reliable because the effect of transient (primary) creep was completely evaluated and the transition from low stress ($n = 3$) to high stress ($\dot{\epsilon} \propto \sinh \tau$) was determined by Post (*15*), but not by Carter and Avé Lallemant. Moreover, their experiments were done at constant strain rate rather than in creep or relaxation. When these factors are taken into account, the two sets of experimental data are consistent within the experimental error.
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19. B. Gjevik [*Phys. Earth Planet. Interiors* **5**, 403 (1972); *ibid.*, in press] has considered the possibility that mantle transitions might govern the rate of return to isostatic equilibrium. To first order, a mantle transition does not affect the shape of the flow field, but can exert a "back pressure" if the transition is significantly displaced from its equilibrium level by the flow. We believe that in this circumstance the back pressure would be proportional to the surface displacement, so that the conditions of proportional relaxation would be maintained. We estimate that the maximum back pressure would reduce the flow velocity by about 30 percent. As Gjevik points out, however, the dynamics of such transitions are difficult to analyze except in certain simple cases, so this effect will bear more examination.
20. The simple proof of Eq. 2 given in (*8*) was suggested by W. G. McMillan. N. L. Carter, B. Gjevik, and R. I. Walcott critically read the manuscript. We thank R. K. McConnell for permitting reproduction of his figures. Discussions with R. L. Shreve were stimulating and helpful. Publication No. 1191 of the Institute of Geophysics and Planetary Physics, University of California at Los Angeles. Supported by NSF grants GA-1394, GA-26027, and GA-36077x.

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Response of the Equatorial Countercurrent to the Subtropical Atmosphere

Abstract. *The strength of the equatorial countercurrent of the North Pacific and associated variations in sea surface temperatures at its eastern extremity off Central America are related to the zonal wind flow in the remote subtropical atmosphere with lags of as much as 8 months between wind and temperature.*

Wyrtki (*1*) showed that an index of transport of the equatorial countercurrent is the difference in sea level between islands situated north and south of the countercurrent. Constructing a time series of this index for 21 years, he demonstrated a dramatic relationship between this index and sea surface temperatures off Central America, indicated a time lag of 3 months, and suggested that increased countercurrent transport leads to the El Niño, or anomalously warm water mass, off Peru. Wyrtki cautions, "One should not, however, disregard the possibility that extremes in both the sea surface temperature and the countercurrent transport are simultaneously associated with an anomaly of oceanic and atmospheric circulation over a much larger part of the Pacific." In this report I attempt to amplify this suggestion and extend Wyrtki's "teleconnections" into the subtropical midtroposphere.

The north equatorial countercurrent, flowing in a narrow zone between 4°N and 10°N, is influenced by the stress of the northeast trade winds. Accurate, long-range measurements of the trade winds are lacking in these latitudes, but we may assume that these tropical trade winds are related to those in the subtropics farther north. The index used is from the 700-mbar level (~ 10,000 feet or 3 km) rather than the surface. This contains less of the "noise" introduced by smaller-scale features such as cyclones and anticyclones, and experience has shown it to be a more reliable indicator than surface indices for many purposes. Its