

## Thermal Conductivities of the Elements

Results of a critical evaluation.

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If different portions of a mass of matter are at different temperatures, thermal energy is transported from the higher-temperature region to the lower-temperature region. The thermal conductivity is the property of a material which provides a quantitative measure of the rate at which thermal energy is transported along the thermal gradient.

The ubiquitousness of temperature gradients ensures that a knowledge of thermal conductivity will continue to be important for an understanding of a wide range of phenomena. In 1753 Benjamin Franklin (1) explained in a letter to a friend in England that the metal lock of his desk felt colder than the nearby wood because of the different powers possessed by wood and metal for conducting heat. He observed that a silver dollar held in a flame would soon be dropped but not so a similarly shaped piece of wood, and that teapots had wooden handles because metal ones would be too hot to hold. On a different scale, the thermal conductivity of the earth's crust was an important factor in determining the rate of cooling of the earth and hence in determining the time at which sur-

face temperatures became suitable for the development of living organisms. At the present time it helps define the extent of permafrost. Finally, it should be emphasized that there are countless situations in modern technology where the flow of heat must be encouraged or restricted. In these situations there are almost always additional factors that must be considered. Certain chemical or mechanical criteria must be met and the ambient conditions are constrained by the nature of the application. From all of the materials available to us, that one must be selected which comes closest to providing an ideal set of properties to fill the need.

The Thermophysical Properties Research Center (TPRC) of Purdue University, under the sponsorship of the Office of Standard Reference Data of the National Bureau of Standards, has just completed a comprehensive compilation and critical evaluation of the data (up to late 1970) on the thermal conductivities of the elements (2). The provision of critically evaluated data for science and technology may be the primary result of such a project, but it is not the only one. At the end of the project the researcher is afforded an astronaut's view of his field. The high points and low points, strengths and weaknesses are laid out before him. It is the purpose of this article to present

some of the unusual results uncovered during the evaluation and to discuss the data on thermal conductivity as they stand today, with some suggestions as to measurements that would be particularly useful in bringing about improvements.

Before starting, some word might be said on the magnitude of the task just finished. In keeping with the importance of thermal conductivity, its literature is voluminous. The comprehensive volume which will be published shortly is more than 950 pages long and has 185 tables, 179 figures, and 1658 original references. The determined scientific "do-it-yourselfer" who insists on searching the literature for himself and making his own evaluations faces a formidable challenge.

### A Look in Retrospect

To chronicle some of the more dramatic results uncovered in the course of the evaluation, it is useful to have a baseline for the state of the art before the evaluation. The *Metals Handbook* (3) of the American Society for Metals (ASM) provides such a baseline. The eighth edition was published in 1961, a short time before the TPRC evaluation was begun, and contained, among other things, a table of the physical properties of the elements, including the thermal conductivity at room temperature. This edition was well received, is still widely used, and is representative of the state of handbook reference data at the time. The eighth edition contained data on 64 elements. Many of these data were identical with the values in the seventh edition (1948), which covered 45 elements and drew heavily from a publication of 1943 (4), which covered 43 elements.

By 1970 experimental data were found in the literature for 82 elements. Furthermore, many of the values recommended in the current compilation differ substantially from those listed in 1961; the 22 elements for which the change is 18 percent or more

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Table 1. Thermal conductivities of a number of elements at room temperature for which there are notable differences between the data in the *Metals Handbook* (3) and the TPRC compilation (2).

Element	Thermal conductivity ( $\text{W cm}^{-1} \text{K}^{-1}$ )		Difference (%)
	TPRC (cutoff date December 1970)	<i>Metals Handbook</i> (1961)	
Antimony	0.247	0.19	30
Beryllium	2.04	1.46	40
Calcium	2.02*	1.26	60
Carbon (graphite)	0.057 to 19.6	0.24	— 76 to 8000
Chlorine (gas)	0.000086	0.000072	19
Chromium	0.944	0.67	41
Cobalt	1.01	0.69	46
Erbium	0.143	0.096	49
Gadolinium	0.105	0.088	19
Indium	0.821	0.24	242
Iridium	1.48	0.59	151
Lithium	0.850	0.71	20
Neodymium	0.165*	0.130	27
Plutonium	0.067	0.084	— 20
Rhenium	0.481	0.71	— 32
Rhodium	1.50	0.88	70
Silicon	1.53	0.84	82
Tellurium	0.0343† 0.0199‡	0.059	— 58
Thallium	0.463	0.39	19
Thorium	0.543	0.377 (373.2 K)	44
Titanium	0.220	0.167	32
Yttrium	0.172	0.146	18

\* Estimated value. † Parallel to *c*-axis. ‡ Perpendicular to *c*-axis.

are given in Table 1. For one of the commonest elements on the list, calcium, it is still only possible to give an estimated value.

The thermal conductivity of carbon is a subject that could occupy a complete volume. The thermal conductivity at room temperature can range from  $0.0157 \text{ W cm}^{-1} \text{K}^{-1}$  for the amorphous form of carbon to  $23.8 \text{ W cm}^{-1} \text{K}^{-1}$  for a type IIa diamond. Graphite occupies an intermediate position, the TPRC values given in Table 1 being for pyrolytic graphite in directions normal ( $0.057 \text{ W cm}^{-1} \text{K}^{-1}$ ) and parallel ( $19.6 \text{ W cm}^{-1} \text{K}^{-1}$ ) to the layer planes. Samples can be found which have almost the complete range of intermediate values. No single number can convey the wide range of possible values, and hence of possible uses, for graphite. The TPRC compilation refers to over 1100 sets of data for graphites; those for seven different types of graphites have been divided and treated separately; the rest are grouped together for the remaining miscellaneous graphites. A significant comment on this vast quantity of data is that no graphite sample has yet had its thermal conductivity measured over the full temperature range for the solid state.

Now consider in some detail the two elements iridium and rhodium. In 1919 the Physikalisch-Technischen Reichsanstalt (now the PTB), the standards

laboratory of Germany, published thermal conductivity values of  $1.37$  and  $1.25 \text{ W cm}^{-1} \text{K}^{-1}$  for iridium and rhodium, respectively (5). In 1954 workers in two other national laboratories published reviews (6) dealing with the thermal and electrical conductivities of metals and alloys at low temperatures. One review drew attention to the German work, which appeared to have been generally overlooked, and recommended use of the German values in place of the commonly tabulated values of  $0.59$  and  $0.89 \text{ W cm}^{-1} \text{K}^{-1}$  for iridium and rhodium, respectively. The other review did overlook the German values and gave the lower values, which were then repeated in the 1961 handbook. Subsequent measurements have led to recommended values more nearly in agreement with the 1919 values, as can be seen from Table 1. It is apparent that even the work of a major national laboratory may be unwittingly overlooked, a fact which underscores the need for systematic data retrieval.

The change proposed for indium in Table 1 is even greater than those just mentioned, an increase by a factor of 3.5. Although the thermal conductivity of indium at room temperature has been listed in handbooks since 1936 and perhaps even earlier, the earliest measurement found dates from 1962 (7). The lower value, which was used

by handbook compilers until about 1966, is probably still used in many calculations; it is identical with the specific heat, and it seems probable that it initially became identified with the thermal conductivity through an error in transcription. One is led to wonder, for how many heat transfer problems, over a 30-year span, might indium have provided a ready solution had its thermal conductivity been known to be 3.5 times greater than listed, and higher than that of iron?

Another simple but nonetheless serious source of error, which occurs rather frequently and may well go undetected unless data are systematically evaluated, is mistakes made in transcription, including errors in drafting, typing, and printing (8).

In addition to errors in the data there are also omissions, some of them surprising. Table 2 is a list of the elements for which no experimental thermal conductivity data have been found. About one-third of these are transplutonium elements with short half-lives, for which knowledge of this property can scarcely be expected to be available. For radon (saturated liquid and saturated vapor and gas at 1 atmosphere) only data obtained from generalized correlations are tabulated (9). The big surprise is to find that calcium is included in Table 2, since it is one of the most abundant elements. Nevertheless, no source that gives an experimental measurement for the thermal conductivity of calcium was found. The origin of the value given in the *ASM Metals Handbook* (see Table 1) cannot be traced, and the value is low compared with the value now estimated from the theoretical Lorenz number,  $L_0$ , and the lowest value reported for the electrical resistivity. Experimental values covering a wide temperature range are long overdue for this important element. Its neighbors in group IIA of the periodic table, barium and strontium, also require experimental study.

In Fig. 1, the thermal conductivity of each element at room temperature is plotted against the atomic number. The systematic variations associated with the positions of the elements in the periodic table are shown in Fig. 2, which also shows the thermal conductivity at the Debye temperature. The upper entry relates to room temperature, the lower to the Debye temperature, which appears in parentheses below the thermal conductivity entry. The variations revealed in figures of

this type can often serve to provide additional information on properties, although exceptions do occur. A fairly independent check is furnished for the estimated values obtained by other means for some of the elements of Table 2. The value derived for the short-lived nonmetallic element astatine seems reasonable. This was obtained from a plot of thermal conductivity against melting point for other nonmetals. The values for barium and strontium derived from their electrical resistivities also conform well, but the value for calcium appears to be high. This is one more example of the urgent need for experimental measurements on these three elements of group IIA. The probable form of Fig. 1 for elements of atomic numbers 94 to 103 is conjectural, and the proposed thermal conductivity values are very uncertain; those for atomic numbers 104, 105, and 106 have been obtained by extrapolation of the data for earlier members of their respective groups.

Figure 3 is intended to illustrate the total number of experimental data sets and estimations found in the literature for each element. Figure 4 is a similar plot to indicate the temperature ranges

Table 2. Elements lacking experimental thermal conductivity data.

Element	Atomic number	Element	Atomic number	Element	Atomic number
Actinium	89	Einsteinium	99	Nobelium	102
Americium	95	Element 104	104	Polonium	84
Astatine	85	Element 105	105	Promethium	61
Barium	56	Europium	63	Protactinium	91
Berkelium	97	Fermium	100	Radium*	88
Calcium	20	Francium	87	Radon	86
Californium	98	Lawrencium	103	Strontium	38
Curium	96	Mendelevium	101		

\* The one value reported may be estimated; details are not known.

covered for each element. While Fig. 4 helps by showing where there are gaps in the temperature ranges for certain elements, these gaps are not necessarily the only ranges requiring further attention. For instance, beryllium appears to be adequately covered from about 2 to 1400 K, but this is far from true. The values from approximately 4 to 120 K were obtained with impure samples. No set of values exists for a sufficiently pure sample to allow the maximum to be obtained with certainty. In the TPRC evaluation, the maximum has been placed at about 36 K on a curve fitting just two values at 23 and 91 K. Again, the only set of measure-

ments spanning the range 100 to 300 K is the very unlikely set first made for this metal, which has a strong positive temperature coefficient (10). All other indications are that in this range the thermal conductivity decreases when the temperature increases.

Melting points have been indicated for the elements in Fig. 4, and it is clear that for several important metals, such as iridium, osmium, and ruthenium, the greater portion of their solid-phase temperature range still lacks experimental values. Indeed, perhaps one of the most important outcomes of the compilation has been to show the haphazard nature of much earlier work.

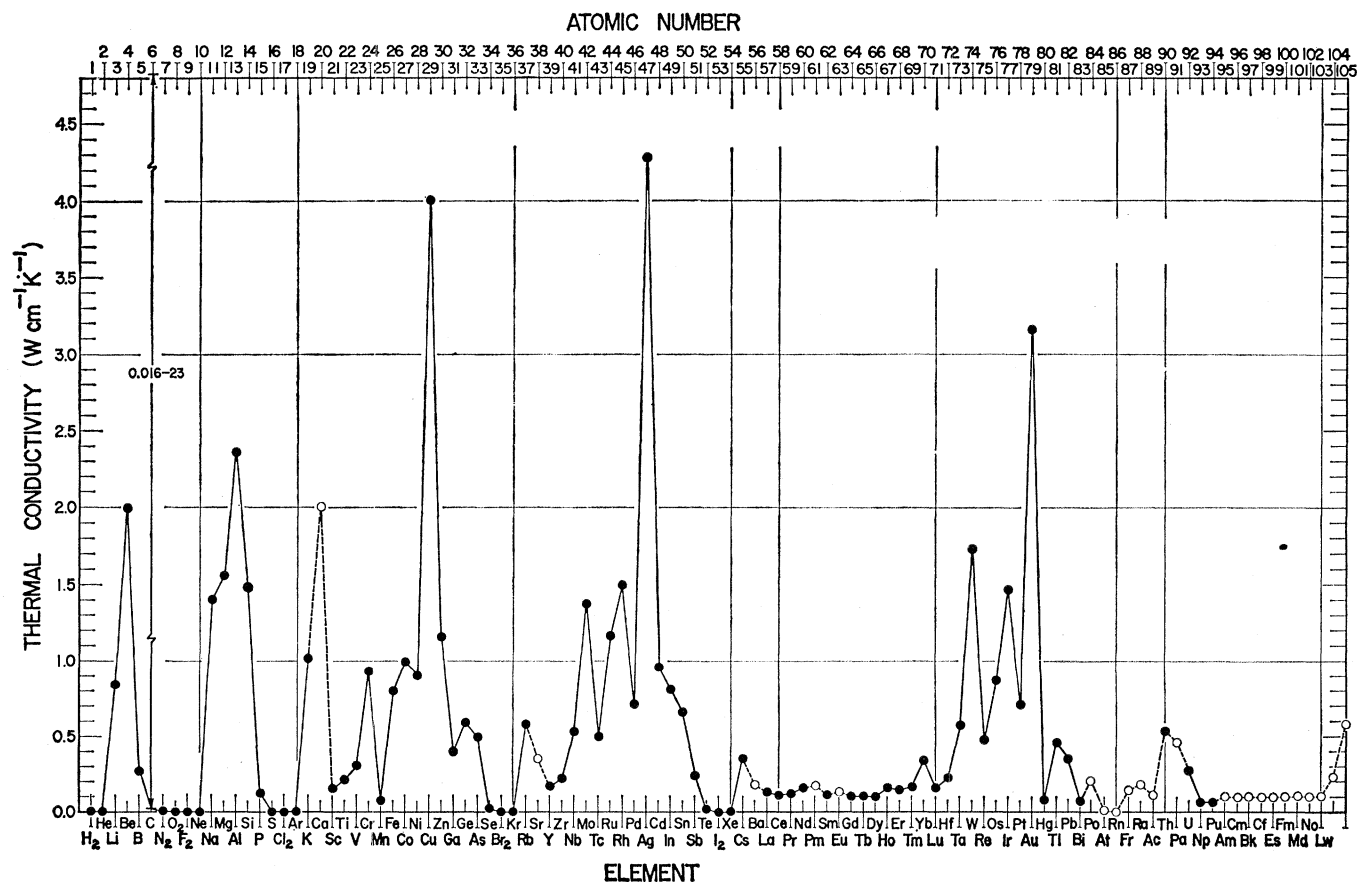


Fig. 1. Thermal conductivities of the elements at 300 K, plotted against atomic number. Open circles indicate estimated values for the elements for which no experimental data are available.



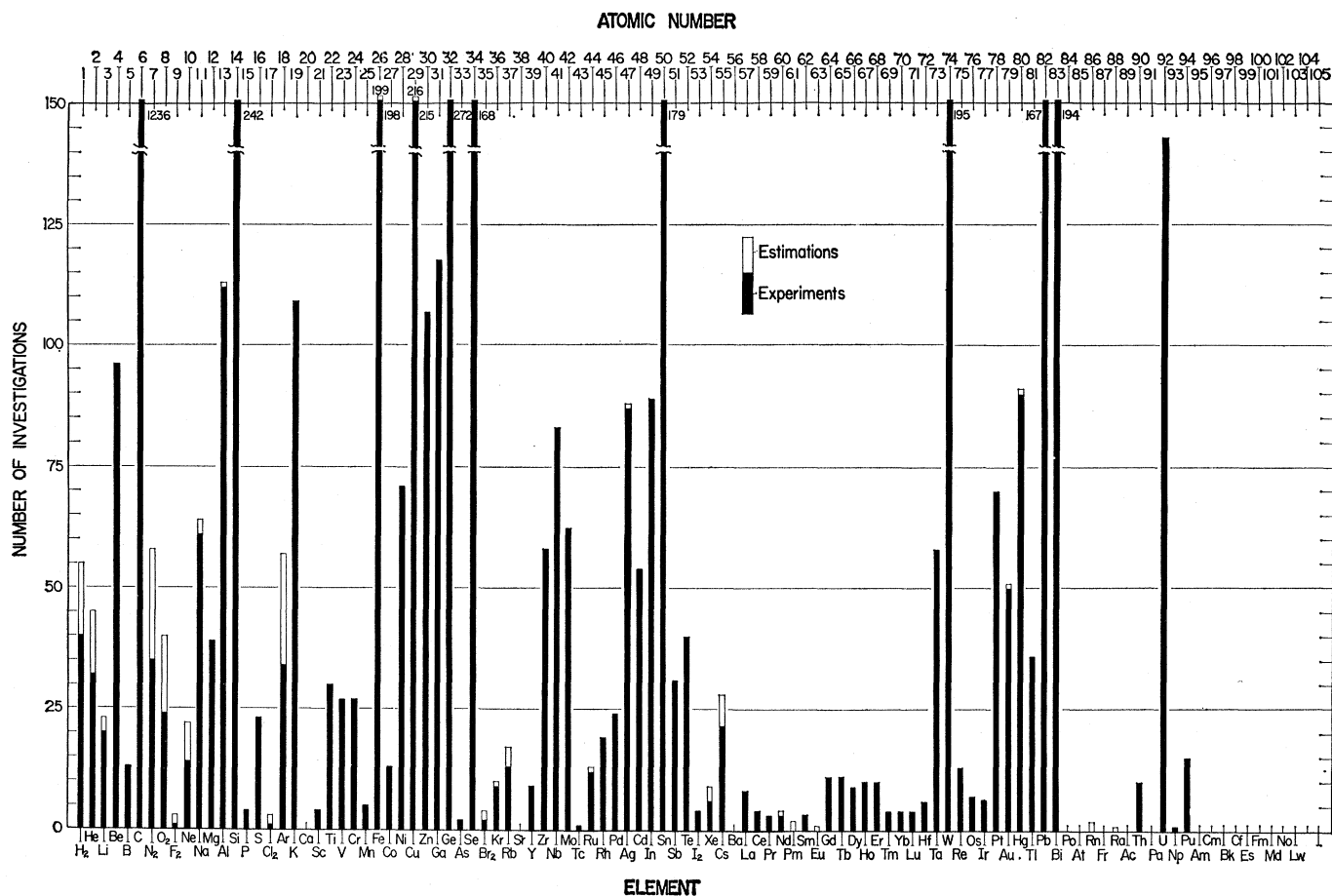


Fig. 3. Number of investigations on the thermal conductivity of each element.

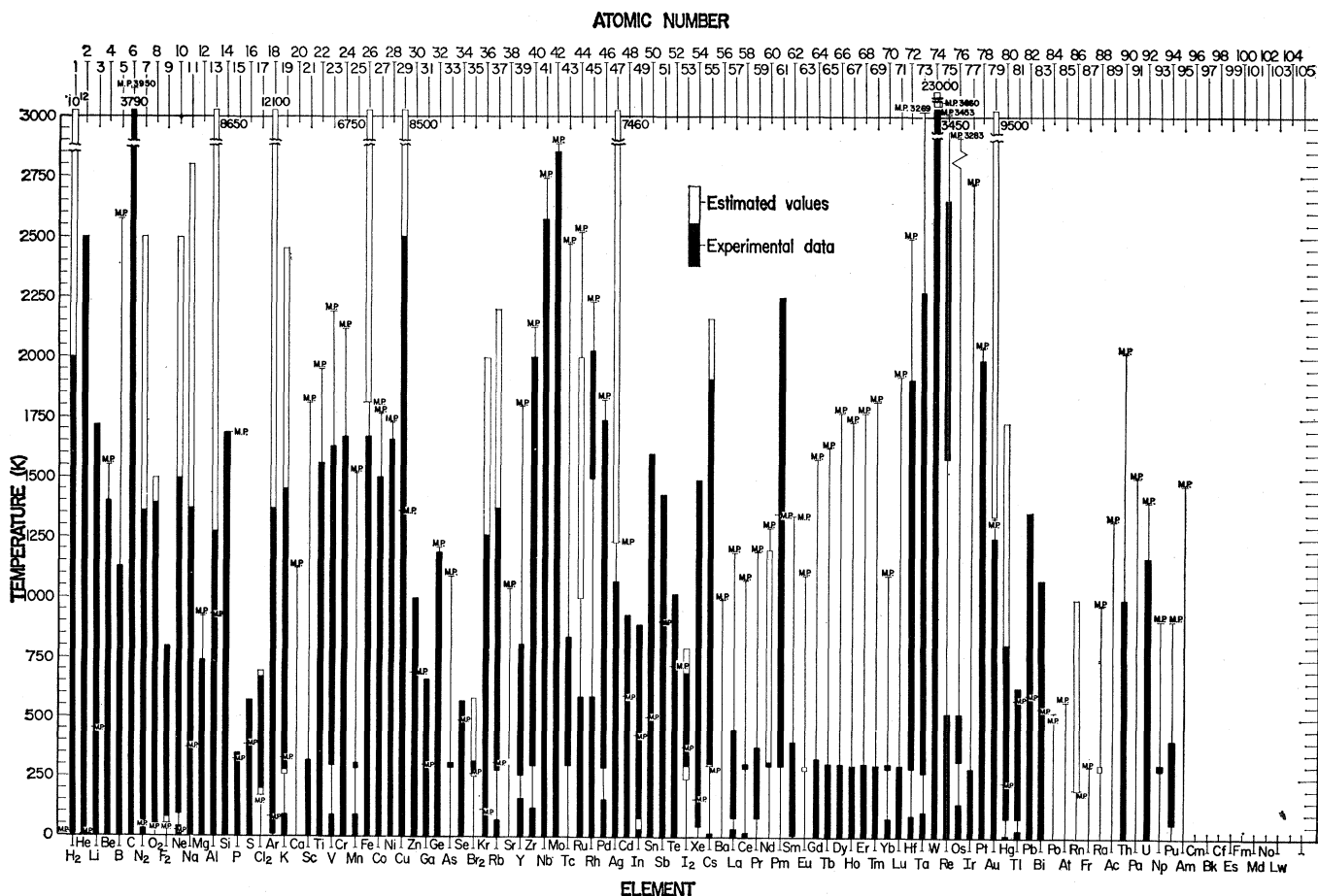


Fig. 4. Temperature range of available thermal conductivity data for each element; *M.P.*, melting point.

assessment of the data for metals in the TPRC evaluation has been facilitated by use of the electrical resistivity,  $\rho$ , and temperature variations of the ratio  $\lambda\rho/T$  (or strictly  $\lambda_e\rho/T$ ), which we term the Lorenz function.

To illustrate the general nature of the dependence of the thermal conductivity of metals on temperature a plot for aluminum is given in Fig. 5. This shows that at low temperatures the thermal conductivity has a maximum, which is greater in value and occurs at a lower temperature as the purity increases. In this temperature range the thermal conductivity is very strongly dependent on structural and chemical purity. Thus, at low temperatures it is not possible to quote a specific value for the thermal conductivity of a metal. Fortunately, in the lowest temperature region the WFL law frequently holds good. Also, the electrical resistivity has usually decreased to a constant value,  $\rho_0$ , the so-called residual resistivity.

If  $\rho_0$  is known and  $\lambda_g$  is assumed negligible, the equation

$$\lambda = L_0 T \sigma_0 = 2.443 \times 10^{-8} T / \rho_0 \quad (1)$$

can be used to estimate the lower end of the curve for  $\lambda$  plotted against  $T$ , or alternatively it provides a check on experimental determinations made in this region.

In Fig. 5 the recommended curves are labeled with different values of  $\rho_0$ . At higher temperatures the differences between samples decrease. Earlier theoretical investigators of the thermal conductivity of metals predicted a low-temperature minimum. As none had been reported, successive improved theories practically eliminated this minimum. Then in 1965 measurements made on aluminum in the rather neglected subnormal temperature region revealed a small but distinct minimum followed by a maximum (15) and the results have since been confirmed. The recommended curve of Fig. 5 now shows such a minimum. Also, the transition to the liquid state is marked by a sharp discontinuity.

At low temperatures it has become customary to specify  $\rho_0$  for a metal when giving its thermal conductivity, and this has been done in our evaluation. Purity and size have corresponding effects on the thermal conductivity of nonmetals, but so far no simple means have been found for adequately detecting and specifying them.

For metallic conductors, considerable use has been made of the results of

an earlier investigation (16) according to which the thermal conductivities of most metals at low temperatures can be correlated to come within a few percent of a single curve. The equation of this curve is

$$\lambda^* = \left[ \frac{1}{3} (T^*)^2 + \frac{2}{3T^*} \right]^{-1} \quad (2)$$

where  $\lambda^* = \lambda/\lambda_m$ ,  $T^* = T/T_m$ , and  $T_m$  is the temperature of the maximum of the low-temperature thermal conductivity,  $\lambda_m$ .

Below  $1.5 T_m$ ,  $\lambda$  can often be derived from

$$\lambda = (\alpha' T^m + \beta T^{-1})^{-1} \quad (3)$$

where

$$\alpha' = \alpha'' \left( \frac{\beta}{n\alpha''} \right)^{\frac{m-n}{m+1}} \quad (4)$$

$$\beta = \rho_0 L_0^{-1} \quad (5)$$

The coefficients  $\alpha'$  and  $\beta$  depend on purity, but the other quantities,  $m$ ,  $n$ , and  $\alpha''$  are constants for a particular metal. Physically,  $m$  is the absolute value of the slope of the straight line passing through the thermal conductivity maxima in a logarithmic plot such as Fig. 5.

From about  $T_m$  to the Debye temperature,  $\theta$ , considerable departure from the WFL law occurs, but above  $\theta$  it often holds to within a few percent. For tungsten, platinum, and palladium, however, the Lorenz function at high temperatures has values of 15 to 25 percent above  $L_0$ . Such departures are capable of explanation and are considered to be valid for these and possibly for other transition metals (17).

On the other hand, departures of this order or even greater, but in the direction to yield values below  $L_0$ , have been reported by a few workers for some molten metals, including lead, tin, and gallium, and for earlier measurements on copper and potassium (18, 19). Major changes in the theoretical treatments appear to be required to explain these results, and this has developed into an area of major concern (20). Very carefully planned experiments are needed for electrical resistivity as well as for thermal conductivity, in order to confirm these results and possibly to examine other liquid metals. Should the low values of  $L$  be definitely confirmed, not only will some of the recommended values for liquid metals need to be reduced, but theoretical physicists will have another phenomenon to explain.

## Need for Further Work

Several elements or particular aspects of work which require further study have been mentioned already; for example, the elements for which no experimental data were found were listed in Table 2. There are other elements for which there are scarcely any data, or for which data are totally lacking in one or another important temperature region. These include arsenic, cerium, cesium (solid), iodine, manganese, neodymium, neptunium, white phosphorus, praseodymium, and ytterbium.

Two things have been done to show systematically where measurements are needed. First, the elements are listed below alphabetically, with a brief statement of the needs for each element. (The elements listed in Table 2 are not repeated.) Second, there is a brief discussion of needs by temperature range. The second listing corresponds more naturally with the theoretical background given in the preceding section and with the fact that different equipment is needed for the different temperature ranges. The first listing should satisfy those who have an interest in a specific element or class of elements.

Since the cutoff date of the literature covered in the TPRC study was December 1970, it was felt desirable to ascertain how much new literature has appeared since. Through the TPRC it was found that 171 additional articles appeared on the thermal conductivity of the elements in 1970, 83 articles in 1971, and 49 articles in 1972. These papers were briefly examined for new information pertinent to the comments presented below. In the listings that follow, an asterisk indicates that new information has become available, even though it may be inadequate. In undertaking any new investigation the researcher should check the recent publications, either through TPRC or personally.

## Measurements Needed by Element

**Aluminum.** Molten state, both  $\lambda$  and  $\rho$ . Particularly, check the finding of Kononenko *et al.* (21) that preheating in a vacuum at 1300 K lowers  $\rho$  for the liquid. Liquid helium range, check findings in (22).

**Antimony.** Values of  $\lambda$  and  $\rho$  for pure samples from room temperature upward, including the liquid state, and for single crystals.

**Argon.** High-precision studies of the solid\* to account for the effect of impurity, of the critical region for the liquid and vapor, and above 2000 K for the gas.

**Arsenic.** Data for the full temperature range, including the liquid state.

**Beryllium.** Single crystals and polycrystals at low temperatures to fix the maximum, and also up to 300 K. There are no data near the melting point (1550 K) or for the liquid state.

**Bismuth.** Low-temperature work\* for well-characterized samples of high purity;  $\lambda$  and  $\rho$  for the solid near the melting point and well into the liquid state.

**Boron.** Low-temperature work for well-characterized samples of high purity; also above 750 K to the melting point and into the liquid state.

**Bromine.** No measurements have been made on the solid. The only measurements for the liquid are of low accuracy. No measurements have been made on the vapor. New data for the gas are also desirable.

**Cadmium.** Unusual low-temperature behavior, with  $n$  varying from 2.2 to 4.5 as  $\beta$  decreases, requires investigation\*. Do single crystals fit one family of curves? There may be a slight minimum at about 100 K. Values of  $\lambda$  and  $\rho$  needed for the liquid state.

**Carbon.** For diamond, results are available from only two laboratories (23–25), and none have been obtained above 300 K. Graphite (Acheson), no measurements below 90 K; graphite (ATJ), no measurements below 100 K; graphite (AWG), no measurements above 520 K; graphite (875S), no measurements below 400 K; graphite (890S), no measurements below 800 K; data on many other graphites are either scarce or nonexistent.

**Cerium.** Data for purer samples of solid cerium for the full temperature range and for the liquid\*.

**Cesium.** Data for the full temperature range of the solid state.

**Chlorine.** No experimental data exist for the solid, liquid, or vapor, and only one set of measurements exists for the gas.

**Chromium.** Some uncertainty near room temperature (at the Néel temperature)\*. The decreases in  $\lambda$  at high temperatures reported by Zinov'ev *et al.* (26) should be checked. There are no data for the liquid state.

**Cobalt.** Room temperature upward, including the phase change\* (690 K) and the Curie point (1400 K). No data exist for the liquid state.

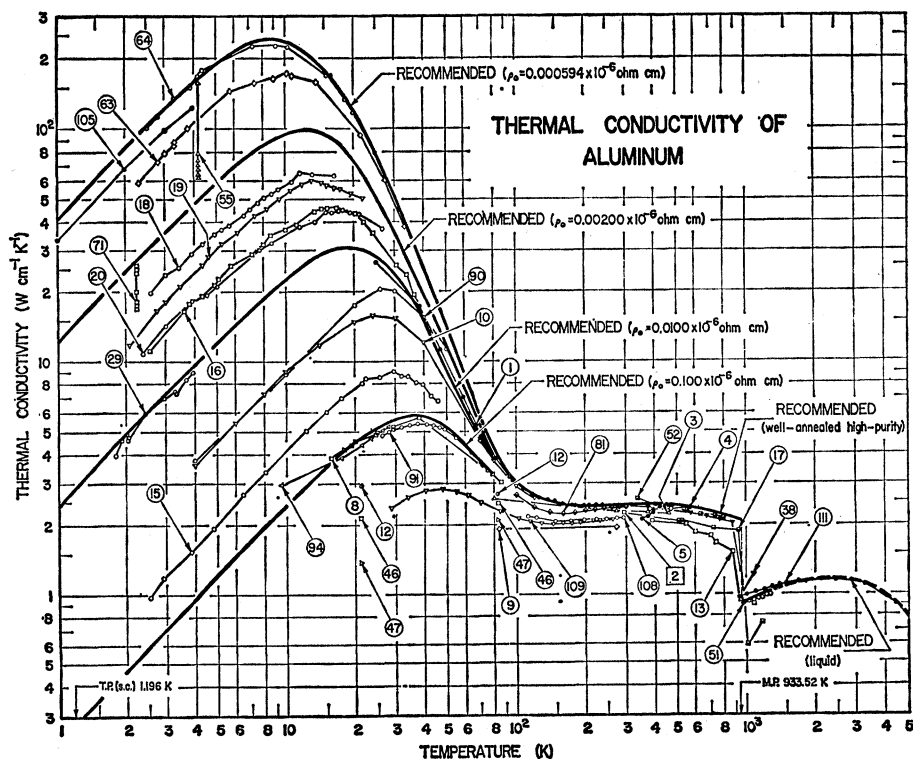


Fig. 5. Thermal conductivity of aluminum.

**Copper.** Liquid state,  $\lambda$  and  $\rho$ . Is  $L$  low as reported (19)?

**Dysprosium.** Check whether  $L$  is normal or high at low temperatures. No measurements have been made above room temperature. Include single crystals.

**Erbium.** Check whether  $L$  is normal or high at low temperatures. No measurements have been made above room temperature. Include single crystals.

**Fluorine.** No experimental data for the solid, liquid, or vapor, and only one set for the gas.

**Gadolinium.** Check whether  $L$  is normal or high at low temperatures. No measurements have been made above room temperature. Include single crystals.

**Gallium.** The outstanding question is how the polycrystal data for  $\lambda$  and  $\rho$  (if obtainable) will be related to the single crystal data. Liquid phase\*, check  $\lambda$  and  $\rho$  to determine  $L$ .

**Germanium.** There are no data for the liquid state.

**Gold.** There are no data for the liquid state.

**Hafnium.** Data at low, moderate, and high\* temperatures for much purer sample and for single crystals.

**Helium.** For solid  $^3\text{He}$  and  $^4\text{He}$  it is just possible that a meaningful correlation of thermal conductivity with sample size, orientation, density, structure,

and so forth, could be made\*. Studies on the liquid are still needed near the critical point. For the gas, no experimental data were located below 2 K, and gaps occur from 4 to 14 K and from 21 to 50 K. Further measurements are also needed above 1500 K.

**Holmium.** Data for  $\lambda$  and  $\rho$  at low temperatures to resolve questions about  $L^*$ , and also above room temperature and for single crystals. No information for the liquid state.

**Hydrogen.** For normal hydrogen there have been few investigations for the solid\* and liquid and none for the vapor. Further experimental studies are also recommended for the gas state below 100 and above 500 K\*. Few measurements for any state of the para form\* exist, and a complete measurement is desirable.

**Deuterium.** Data for solid\*, saturated liquid, saturated vapor, and gas\*.

**Tritium.** Data for all states.

**Indium.** Data from about 30 to 80 K, and for the liquid state\*.

**Iodine.** Data for the solid, for the full temperature range; also for the saturated liquid and vapor and the gas.

**Iridium.** Data from 500 K upward\* and for the liquid state.

**Iron.** Data for the gamma-delta phase transition, the solid-liquid phase transition, and the liquid state.

**Krypton.** High-accuracy measurements for the solid, for the liquid and



vapor near the critical point, and for the gas above 1500 K\*.

*Lanthanum.* Data for most of the low-temperature range and above 450 K; also for single crystals and for the liquid state.

*Lead.* Thermal and electrical data for pyrometric standard lead 49c and 49e from about 343 and 450 K (27) are sufficiently interesting to warrant confirmation\* and extension to higher temperatures. Values of  $\lambda$  and  $\rho$  for liquid lead are needed to high temperatures.

*Lithium.* The temperature range 90 to 450 K and the Lorenz function for the liquid state need further study.

*Lutetium.* Data above 300 K for single crystals and polycrystalline samples.

*Magnesium.* Data for the liquid state.

*Manganese.* Full temperature range needs investigation for a purer material.

*Mercury.* Single crystals and polycrystalline solids require measurements over the full temperature range.

*Molybdenum.* More high-temperature data\* required; should include single crystals\* to justify ignoring the data of Khusainova and Filippov (28). No data on liquid state are available.

*Neodymium.* Data for low, moderate\*, and high\* temperatures. Single crystals and the liquid state\* require study.

*Neon.* High-accuracy data for the solid, liquid, and vapor, and for the gas above and about 1500 K.

*Neptunium.* Data for the full temperature range.

*Nickel.* Further studies of samples of very high purity at low temperatures. No data exist for the liquid state.

*Niobium.* More data for the range 90 to 293 K. At high temperatures data on specific heat as well as thermal conductivity. No data exist for the liquid state.

*Nitrogen.* Only one experimental study for the solid was located and none for the vapor. Further experimental measurements are also suggested for the liquid near the critical point, and for the gas, especially above 1200 K\*.

*Osmium.* Data for samples of high purity (single crystals and polycrystals) over a wider range of low temperatures. The value of  $m$  is high and such a study would give a better value. Measurements also needed above 500 K\* and for the liquid state.

*Oxygen.* No experimental data exist for the solid or the vapor. Further

studies of high accuracy needed for the liquid near the critical point and for the gas above 750 K.

*Palladium.* Data required for the liquid state.

*Phosphorus.* No data exist for red phosphorus. Data needed for black phosphorus above 300 K and into the liquid state, and for white (or yellow) phosphorus for a purer sample and the full temperature range.

*Platinum.* The liquid state has not been studied and data for the solid state are uncertain above 1600 K.

*Plutonium.* Only measured over the range 60 to 413 K, needs extending to lower and higher temperatures, including the liquid state.

*Potassium.* Measurements needed from about 50 to 336 K (the melting point) and into the liquid state, where  $\lambda$  and  $\rho$  need careful checking.

*Praseodymium.* Measurements for the full temperature range.

*Rhenium.* More data for polycrystalline samples and single crystals at low temperatures and from about 400 K upward\*, and for the liquid state.

*Rhodium.* Data from about 500 K to the melting point, and for the liquid state.

*Rubidium.* Data from about 70 to 312 K and for both  $\lambda$  and  $\rho$  throughout the liquid range.

*Ruthenium.* The value of  $m$  is high and additional samples, including single crystals\*, should be measured to confirm it. Data also needed from 600 K to the melting point and for the liquid state.

*Samarium.* Further measurements for the full range of temperatures. Include single crystals and the liquid state.

*Scandium.* Further measurements for purer samples for the full range of temperatures. Include single crystals and the liquid state.

*Selenium.* Measurements for the liquid state\* and for amorphous selenium above 320 K.

*Silicon.* Several of the low-temperature curves for both  $n$ - and  $p$ -types have maxima that are almost temperature independent, while others show the normal behavior. This needs further investigation. Determinations are required for the liquid state.

*Silver.* Measurements of  $\lambda$  and  $\rho$  for the liquid state.

*Sodium.* Measurements of  $\lambda$  and  $\rho$  for the liquid state to check the value of  $L$ .

*Sulfur.* The high values of Ioffe and Ioffe (29) at 300 K for two mutually

perpendicular single-crystal directions have been ignored, but should be checked; single crystals need to be studied for the full temperature range since no measurements have been made between 92 and 273 K. Measurements are required for amorphous sulfur below 80 K and above 300 K.

*Tantalum.* Measurements are required from about 60 to 300 K. At high temperatures better specific heat data for samples in thermal diffusivity experiments would improve the accuracy of the derived thermal conductivity values. No data exist for the liquid state.

*Technetium.* Data below room temperature and above 840 K.

*Tellurium.* There is much scatter in the results for the liquid state, and further measurements are desirable\*.

*Terbium.* Low-temperature values of  $L$  required to resolve differences between results; single crystals and polycrystals should be studied. Measurements needed above 300 K, including the liquid state.

*Thallium.* Data for single crystals and polycrystals, from about 60 K upward. Measurements also needed for the liquid state.

*Thorium.* Values are rather uncertain, and pure thorium needs further measurement from about 100 K upward. No values exist for the liquid state.

*Thulium.* Measurements on single crystals and polycrystals from 300 K upward. No values exist for the liquid state.

*Tin.* Careful measurements of  $\lambda$  and  $\rho$  to give  $L$  for the liquid state. No values known for gray tin.

*Titanium.* Measurements for single crystals and at high temperatures\*; include the phase transformation at 1155 K\*. Measurements also needed for the liquid state.

*Tungsten.* No values are reported for the liquid state.

*Uranium.* At low temperatures no maximum has yet been obtained. The suggestion of Fischer and Dever (30) to remove the possible retained  $\alpha$ -phase should be tried. No data exist for the liquid state.

*Vanadium.* Measurements at low temperatures on purer samples, and for a wider temperature range, to give a maximum. No information available for  $\lambda$  or  $\rho$  in the liquid state.

*Xenon.* No experimental data exist for the vapor. Measurements are also needed for the liquid near the critical



point and for the gas above 500 K\*.

**Ytterbium.** Data for the full temperature range, including the liquid state, and particularly for  $L$  at low temperatures.

**Yttrium.** Data for single crystals and polycrystals for purer samples from low temperatures upward, particularly above about 150 K\*. No data exist for the liquid state.

**Zinc.** More information needed on the crossing of the low-temperature curves for single crystals and for zinc of very high purity in the probable minimum region (80 to 200 K). More determinations of  $\lambda$  and  $\rho$  in the liquid state.

**Zirconium.** Data needed on single crystals, which have yet to be studied; the range 80 to 300 K; and the influence of the phase transformation at 1135 K\* on  $\lambda$ . Greater accuracy needed at high temperatures; this includes a better knowledge of  $\rho$  and of the specific heat at constant pressure ( $C_p$ ) if  $\lambda$  is derived from thermal diffusivity determinations. No data exist for the liquid state.

## Measurements Needed by

### Temperature Range

**Low temperatures.** A group of non-metallic materials requiring increased study at cryogenic temperatures includes iodine and the solid phases of the elements which normally occur as gases. Oxygen and nitrogen do not appear to have been studied through the region of maximum thermal conductivity, nor have measurements been made for single crystals of the members of this group likely to have anisotropic properties. More information on the influence of sample size and purity, and particularly a parameter that could indicate the total effective purity, are needed for this class of materials.

For the metallic elements, as samples of much higher purity become available further measurements will be needed. There will be academic interest in investigations of the influence of sample size (31), changes in the location of the maxima affecting the value of  $m$  and indicating the need for an additional scattering term in Eq. 3 (32), and possible unexpectedly large values of  $\lambda_g$  for aluminum in the range 1.2 to 4.2 K (22).

Among the metals for which low-temperature data are lacking or inadequate are cesium (gap from 16 to

295 K), mercury (gap from 4 to 80 K), plutonium (no data below 62 K), beryllium, hafnium, and vanadium. Then, there are several rare earth metals—dysprosium, erbium, gadolinium, holmium\*, terbium, and ytterbium—for which more measurements are required to resolve a major uncertainty (33) regarding the value of the Lorenz function. Measurements on much purer samples are required for cerium, manganese, and uranium, for which no low-temperature maximum has so far been reported. Presumably, purer samples should give a maximum, but for uranium the TPRC work (2) refers to a note by Fischer and Dever (30) and suggests that some retained second phase might be acting as an impurity. Apparently, this can be reduced by heating in pulses to temperatures below 22.5 K, so the effect of such treatment on the thermal conductivity might well be investigated.

**Subnormal temperatures.** One temperature range requiring further study is that between the upper limit of apparatus designed to operate from liquid-helium temperatures up, and room temperature, which is the lower limit for another class of apparatus—that is, from about 100 to 300 K. Often, the substitution of liquid nitrogen for the water frequently used as the coolant will allow the inclusion of this interesting temperature range (34). Elements for which new data are required at these subnormal temperatures include cadmium, cesium, indium, lithium, potassium, rubidium, samarium, tantalum, thallium, thorium, vanadium, and zirconium; several of these might prove, like aluminum, to have a thermal conductivity minimum in this range.

**Intermediate temperatures, above room temperature.** Metals requiring study above room temperature include most of the rare earths; the refractory metals iridium\*, osmium\*, rhenium\*, rhodium, ruthenium, and scandium, for which the measurements should be extended to the highest temperatures possible; manganese and plutonium, for which phase transitions have been reported; and antimony above 360 K, where there is only one rather questionable set of data. Diamond has not been studied above 320 K, and measurements are needed at least for theoretical interest. Single crystals of sulfur should be studied to ascertain whether there is any support for the unusually high values reported by Ioffe and Ioffe (29),

and no data exist for black phosphorus to the melting point, 1300 K.

**High temperatures, above 1000 K.** Black phosphorus, as just mentioned, is also included in this section. Boron is another nonmetal which should be studied to well above 1000 K. Petrov *et al.* (35) found that the thermal conductivity, which had been decreasing nearly as  $T^{-2}$  up to 400 K, became almost constant by 700 K. A second conducting mechanism was proposed by these authors, but they gave no indication of the possible course of the curve to higher temperatures.

Titanium\* and zirconium\* have phase transformations at about 1155 and 1135 K, and cobalt has a Curie temperature at 1400 K. The thermal conductivity of cobalt has been reported to undergo a pronounced change in this region, but for titanium and zirconium the recommended curves do not show any discontinuity. All the data show considerable scatter, and reliable measurements should prove interesting to theorists. Other metals that could usefully be studied include iridium\*, molybdenum\*, niobium\*, platinum, rhenium\*, and tantalum\*. In the methods now commonly used to determine the thermal diffusivity, the specific heat cannot also be determined, and difficulties arise as to the appropriate specific heat to be used for the derivation of thermal conductivity. A method which also permits accurate determinations of the specific heat is required. For electrical conductors the multiproperties apparatus involving direct electrical heating seems the most promising of the methods at present available (36).

## Conclusions

Those who will have an opportunity to examine the comprehensive TPRC report (2) will recognize that the systematic compilation and evaluation of the experimental data in this field has been a momentous task. As a result of this effort the authors have reached the following conclusions:

1) The consistency and depth of analysis provided make such an evaluated set superior to any individual set of measurements.

2) The availability of reliable data over a wide temperature range is valuable, particularly if extensive calculations involving several materials are contemplated.

3) A concise, comprehensive review of the status of the field is now available. In the absence of such a work coverage of the field is spotty and there are undesirable omissions and unnecessary duplications. The review provides a firm base for further experimental measurements, and, for that matter, theoretical deductions.

4) The tedious and time-consuming process of compilation itself provides a valuable resource. It should never again be necessary for workers in the field to search the literature published before the cutoff date (December 1970) of the search made for the review. If the bibliography is insufficient, requests can be addressed to TPRC for the articles themselves, particularly those which are difficult to obtain. It is believed the inadvertent omissions are few and will be rectified in time. In any event, the coverage is far beyond what an individual researcher would be apt to reach in any reasonable time.

The work has been time consuming and costly because it needs trained specialists. The cost is, however, minute when compared with the total initial cost of obtaining and publishing the more than 5200 sets of raw data that have been evaluated. We submit that until such data have been compiled and evaluated their full meaning and usefulness have not been realized.

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