# **Microscopic Quantum Interference** in the Theory of Superconductivity

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It is an honor and a pleasure to speak to you today about the theory of superconductivity. In a short lecture one can no more than touch on the long history of experimental and theoretical work on this subject before 1957. Nor can one hope to give an adequate account of how our understanding of superconductivity has evolved since that time. The theory (1) we presented in 1957, applied to uniform materials in the weak coupling limit that defines an ideal superconductor, has been extended in almost every imaginable direction. To these developments so many authors have contributed (2) that we can make no pretense of doing them justice. I will confine myself here to an outline of some of the main features of our 1957 theory, an indication of directions taken since, and a discussion of quantum interference effects due to the singlet-spin pairing in superconductors which might be considered the microscopic analogue of the effects discussed by Professor Schrieffer.

#### **Normal Metal**

Although attempts to construct an electron theory of electrical conductivity date from the time of P. Drude and H. A. Lorentz, an understanding of normal metal conduction electrons in modern terms awaited the development of the quantum theory. Soon thereafter Sommerfeld (3) and Bloch (4) introduced what has evolved into the present description of the electron fluid. There the conduction electrons of the normal metal are described by single-particle wave functions. In the periodic potential produced by the fixed lattice and the conduction electrons themselves, according to Bloch's theorem, these functions are modulated plane waves:

$$\phi_{\mu}(\mathbf{r}) \equiv u_{\mu}(\mathbf{r}) \ e^{i\mathbf{k}\mathbf{r}}$$

where  $u\kappa(\mathbf{r})$  is a two-component spinor with the lattice periodicity. I use  $\kappa$  to designate simultaneously the wave vector  $\mathbf{k}$  and the spin state  $\sigma$ :  $\kappa \equiv \mathbf{k},\uparrow$ ;  $-\kappa \equiv -\mathbf{k},\downarrow$ . The single-particle Bloch functions satisfy a Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2+V_0(\mathbf{r})\right]\phi_{\kappa}=\mathcal{E}_{\mathbf{k}}\phi_{\kappa}$$

where  $2\pi\hbar$  is Planck's constant, *m* is the electron mass, and  $V_0(\mathbf{r})$  is the periodic potential and in general might be a linear operator to include exchange terms.

The Pauli exclusion principle requires that the many-electron wave function be antisymmetric in all of its coordinates. As a result no two electrons can be in the same single-particle Bloch state. The energy of the entire system is

$$W = \sum_{i=1}^{2N} \mathfrak{E}_i$$

where  $\mathcal{E}_i$  is the Bloch energy of the *i*th single-electron state. The ground state of the system is obtained when the lowest N Bloch states of each spin are occupied by single electrons; this can be pictured in momentum space as the filling in of a Fermi sphere (Fig. 1). In the ground-state wave function there is no correlation between electrons of opposite spin and only a sta-

tistical correlation (through the general antisymmetry requirement on the total wave function) of electrons of the same spin.

Single-particle excitations are given by wave functions identical to the ground-state wave function except that one-electron states  $k_i < k_F$  are replaced by others  $k_j > k_F$ . This may be pictured in momentum space as opening vacancies below the Fermi surface and placing excited electrons above (Fig. 2). The energy difference between the ground state and the excited state with the particle excitation  $k_j$  and the hole excitation  $k_i$  is

$$\mathcal{E}_j - \mathcal{E}_i = \mathcal{E}_j - \mathcal{E}_F - (\mathcal{E}_i - \mathcal{E}_F) = \epsilon_j - \epsilon_i = |\epsilon_j| + |\epsilon_i|$$

where  $\epsilon$  is defined to be as the energy measured relative to the Fermi energy

$$\epsilon_i = \mathcal{E}_i - \mathcal{E}_i$$

When Coulomb, lattice-electron, and other interactions, which have been omitted in constructing the independentparticle Bloch model are taken into account, various modifications which have been discussed by Professor Schrieffer (5) are introduced into both the groundstate wave function and the excitations. These may be summarized as follows: The normal metal is described by a ground state  $\Phi_0$  and by an excitation spectrum which, in addition to the various collective excitations, consists of quasi-fermions which satisfy the usual anticommutation relations. It is defined by the sharpness of the Fermi surface, the finite density of excitations, and the continuous decline of the single-particle excitation energy to zero as the Fermi surface is approached.

### Electron Correlations That Produce Superconductivity

For a description of the superconducting phase we expect to include correlations that are not present in the normal metal. Professor Schrieffer has discussed the correlations introduced by an attractive electron-electron interaction, and Professor Bardeen will discuss the role of the electron-phonon interaction in producing the electronelectron interaction which is responsible for superconductivity (6). It seems to be the case that any attractive interaction between the fermions in a manyfermion system can produce a superconducting-like state. This is believed at present to be the case in nuclei, in

Copyright © 1973 by the Nobel Foundation. The author is professor of physics, Brown University, Providence, Rhode Island 02912. This article is the lecture he delivered in Stockholm, Sweden, on 11 December 1972 when he received the Nobel Prize in Physics, a prize which he shared with Dr. J. R. Schrieffer and Dr. J. Bardeen. Minor corrections and additions have been made by the author. The article is published here with the permission of the Nobel Foundation and will also be included in the complete volume of Les Prix Nobel en 1972 as well as in the series Nobel Lectures (in English) published by the Elsevier Publishing Company, Amsterdam and New York. Dr. Schrieffer's lecture appeared in the 22 June issue, page 1243; Dr. Bardeen's lecture will be published in a subsequent issue.

the interior of neutron stars, and has possibly been observed (7) very recently in <sup>3</sup>He. I will therefore develop the consequences of an attractive twobody interaction in a degenerate manyfermion system without enquiring further about its source.

The fundamental qualitative difference between the superconducting and normal ground-state wave function is produced when the large degeneracy of the single-particle electron levels in the normal state is removed. If we visualize the Hamiltonian matrix which results from an attractive two-body interaction in the basis of normal metal configurations, we find in this enormous matrix, submatrices in which all single-particle states except that for one pair of electrons remain unchanged. These two electrons can scatter via the electronelectron interaction to all states of the same total momentum. We may envisage the pair wending its way (so to speak) over all states unoccupied by other electrons. (The electron-electron interaction in which we are interested is both weak and slowly varying over the Fermi surface. This and the fact that the energy involved in the transition into the superconducting state is small leads us to guess that only singleparticle excitations in a small shell near the Fermi surface play a role. It turns out, further, that, because of exchange terms in the electron-electron matrix element, the effective interaction in metals between electrons of singlet spin is much stronger than that between electrons of triplet spin-thus our preoccupation with singlet-spin correlations near the Fermi surface.) Since every such state is connected to every other, if the interaction is attractive and does not vary rapidly, we are presented with submatrices of the entire Hamiltonian of the form shown in Fig. 3. For purposes of illustration I have set all off diagonal matrix elements equal to the constant -V and the diagonal terms equal to zero (the single-particle excitation energy at the Fermi surface) as though all the initial electron levels were completely degenerate. Needless to say, these simplifications are not essential to the qualitative result.

Diagonalizing this matrix results in an energy level structure with M-1levels raised in energy to E = +Vwhile one level (which is a superposition of all of the original levels and quite different in character) is lowered in energy to

$$E = -(M+1)V$$

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Fig. 1 (left). The normal ground-state wave function,  $\Phi_0$ , is a filled Fermi sphere for both spin directions. Fig. 2 (right). An excitation of the normal system.

Since M, the number of unoccupied levels, is proportional to the volume of the container while V, the scattering matrix element, is proportional to the reciprocal of the volume, the product is independent of the volume. Thus the removal of the degeneracy produces a single level separated from the others by a volume-independent energy gap.

To incorporate this into a solution of the full Hamiltonian, one must devise a technique by which all of the electron pairs can scatter while obeying the exclusion principle. The wave function which accomplishes this has been disussed by Professor Schrieffer. Each pair gains an energy due to the removal of the degeneracy as above, and one obtains the maximum correlation of the entire wave function if the pairs all have the same total momentum. This gives a coherence to the wave function in which for a combination of dynamical and statistical reasons there is a strong preference for momentum zero, singlet spin correlations, while for statistical reasons alone there is an equally strong preference that all of the correlations have the same total momentum.

Fig. 3. Submatrices of the entire Hamiltonian. For V = 0, the *M* states all have the energy E =0. For V > 0, M - 1 of the states have the energy E = V; one state has the energy E = -(M - 1)V. In what follows I shall present an outline of our 1957 theory modified by introducing the quasi-particles of Bogoliubov (8) and Valatin (9). This leads to a formulation which is generally applicable to a wide range of calculations in a manner analogous to similar calculations in the theory of normal metals.

We limit the interactions to terms which scatter (and thus correlate) singlet zero-momentum pairs. To do this, it is convenient to introduce the pair operators:

$$b_{\mathbf{k}} = c_{-\kappa} c_{\kappa}$$
$$b_{\mathbf{k}}^{*} = c_{\kappa-\kappa}^{*} c_{\kappa-\kappa}^{*}$$

and, using these, we extract from the full Hamiltonian the so-called reduced Hamiltonian

$$H_{\text{red}} = \sum_{k < k_{\text{F}}} 2 |\epsilon| b_{k} b_{k}^{*} + \sum_{k > k_{\text{F}}} 2\epsilon b_{k}^{*} b_{k} + \sum_{k \, \mathbf{k}'} V_{k' \mathbf{k}} b_{\mathbf{k}'}^{*} b_{\mathbf{k}'}$$

where  $V_{\mathbf{k'k}}$  is the scattering matrix element between the pair states **k** and **k'**.



#### **Ground State**

As Professor Schrieffer has explained, the ground state of the superconductor is a linear superposition of pair states in which the pairs  $(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$  are occupied or unoccupied as indicated in Fig. 4. It can be decomposed into two disjoint vectors—one in which the pair state **k** is occupied,  $\phi_{\mathbf{k}}$ , and one in which it is unoccupied,  $\phi_{(\mathbf{k})}$ :

$$\Psi_0 = u \phi_{\mathbf{k}} + v \phi_{\mathbf{k} \mathbf{k}}$$

The probability amplitude that the pair state **k** is (or is not) occupied in the ground state is then  $v_k(u_k)$ . Normalization requires that  $|u|^2 + |v|^2 = 1$ . The phase of the ground-state wave function may be chosen so that with no loss of generality  $u_k$  is real. We can then write

$$u = (1 - h)^{1/2}$$
  
 $v = h^{1/2} e^{i\phi}$ 

where

$$0 \le h \le 1$$

A further decomposition of the ground-state wave function of the superconductor in which the pair states  $\mathbf{k}$  and  $\mathbf{k}'$  are either occupied or unoccupied (Fig. 5) is:

$$\Psi_{0} = \underbrace{u}_{\mathbf{k}} \underbrace{u}_{\mathbf{k}'} \phi_{(\mathbf{k}),(\mathbf{k}')} + \underbrace{u}_{\mathbf{k}} \underbrace{v}_{\mathbf{k}'} \phi_{(\mathbf{k}),\mathbf{k}'} \\ + \underbrace{v}_{\mathbf{k}} \underbrace{u}_{\mathbf{k}'} \phi_{\mathbf{k},(\mathbf{k}')} + \underbrace{v}_{\mathbf{k}} \underbrace{v}_{\mathbf{k}'} \phi_{\mathbf{k},\mathbf{k}}$$

This is a Hartree-like approximation in the probability amplitudes for the occupation of pair states. It can be shown that for a fermion system the wave function cannot have this property unless the number of particles is variable. To terms of order 1/N, however, this decomposition is possible for a fixed number of particles; the errors introduced go to zero as the number of particles becomes infinite (10).

The correlation energy,  $W_{\rm c}$ , is the expectation value of  $H_{\rm red}$  for the state  $\Psi_0$ 

$$W_{\rm c} \equiv (\Psi_0, H_{\rm red}\Psi_0) \equiv W_{\rm c}[h,\phi]$$

Setting the variation of  $W_c$  with respect to h and  $\phi$  equal to zero in order to minimize the energy gives

$$h = 1/2 \ (1 - \epsilon/E)$$
  
 $E = (\epsilon^2 + |\Delta|^2)^{1/2}$ 

where

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$$\Delta = |\Delta| e^{i\phi}$$

satisfies the integral equation

$$\Delta(\mathbf{k}) = -1/2 \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta(\mathbf{k}')}{E(\mathbf{k}')}$$



If a nonzero solution of this integral equation exists,  $W_{\rm e} < 0$  and the "normal" Fermi sea is unstable under the formation of correlated pairs.

In the wave function that results there are strong correlations between pairs of electrons with opposite spins and zero total momentum. These correlations are built from normal excitations near the Fermi surface and extend over spatial distances typically of the order of  $10^{-4}$  centimeter. They can be constructed, owing to the large wave numbers available, because of the exclusion principle. Thus with a small additional expenditure of kinetic energy there can be a greater gain in the potential energy term. Professor Schreiffer has discussed some of the properties of this state and the condensation energy associated with it.

#### **Single-Particle Excitations**

In considering the excited states of the superconductor, it is useful, as for the normal metal, to make a distinction between single-particle and collective excitations; it is the single-particle excitation spectrum whose alteration is responsible for superfluid properties. For the superconductor excited (quasiparticle) states can be defined in one-toone correspondence with the excitations of the normal metal. One finds, for example, that the expectation value of  $H_{\rm red}$  for the excitation (Fig. 6) is given by

$$E_{\mathbf{k}} = (\epsilon_{\mathbf{k}}^2 + |\Delta|^2)^{1/2}$$

In contrast to the normal system, for the superconductor even as  $\epsilon$  goes to zero E remains larger than zero, its lowest possible value being  $E = |\Delta|$ . One can therefore produce single-par-

ticle excitations from the superconducting ground state only with the expenditure of a small but finite amount of energy. This is called the energy gap; its existence severely inhibits singleparticle processes and is in general responsible for the superfluid behavior of the electron gas. [In a gapless superconductor it is the finite value of  $\Delta(\mathbf{r})$ , the order parameter, rather than the energy gap as such that becomes responsible for the superfluid properties.] In the ideal superconductor, the energy gap appears because not a single pair can be broken nor can a single element of phase space be removed without a finite expenditure of energy. If a single pair is broken, one loses its correlation energy; if one removes an element of phase space from the system, the number of possible transitions of all the pairs is reduced. In both cases the result is an increase in the energy which does not go to zero as the volume of the system increases.

The ground state of the superconductor and the excitation spectrum described above can conveniently be treated by introducing a linear combination of  $c^*$  and c, the creation and annihilation operators of normal fermions. This is the transformation of Bogoliubov (8) and Valatin (9):

$$\gamma_{\mathbf{k}0}^* = u_{\mathbf{k}}c_{\kappa}^* - v_{\mathbf{k}}^*c_{-\kappa}$$
$$\gamma_{\mathbf{k}1} = v_{\mathbf{k}}^*c_{\kappa} + u_{\mathbf{k}}c_{-\kappa}^*$$

It follows that

$$\Psi_{\mathbf{k}i} \Psi_0 \equiv 0$$

so that the  $\gamma_{ki}$  play the role of annihilation operators, while the  $\gamma_{ki}^*$  create excitations

$$\gamma_{\mathbf{k}i}^*, \cdots \gamma_{\mathbf{m}j}^* \Psi_0 = \Psi_{\mathbf{k}i}, \cdots \mathbf{m}j$$

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The  $\gamma$  operators satisfy Fermi anticommutation relations so that with them we obtain a complete orthonormal set of excitations in one-to-one correspondence with the excitations of the normal metal.

We can sketch the following picture. In the ground state of the superconductor all the electrons are in singlet-pair correlated states of zero total momentum. In an m electron excited state the excited electrons are in "quasi-particle" states, very similar to the normal excitations and not strongly correlated with any of the other electrons. In the background, so to speak, the other electrons are still correlated much as they were in the ground state. The excited electrons behave in a manner similar to that of normal electrons; they can be easily scattered or excited further. But the background electrons-those which remain correlated-retain their special behavior; they are difficult to scatter or to excite.

Thus one can identify two almost independent fluids. The correlated portion of the wave function shows the resistance to change and the very small specific heat characteristic of the superfluid, while the excitations behave very much like normal electrons, displaying an almost normal specific heat and resistance. When a steady electric field is applied to the metal, the superfluid electrons short out the normal ones, but with higher frequency fields the resistive properties of the excited electrons can be observed (11).

#### Thermodynamic Properties,

#### the Ideal Superconductor

We can obtain the thermodynamic properties of the superconductor by using the ground-state and excitation spectrum just described. The free enenergy of the system is given by

$$F[h,\phi,f] = W_{c}(T) - TS$$

where T is the absolute temperature, S is the entropy, and f is the superconducting Fermi function which gives the probability of single-particle excitations. The entropy of the system comes entirely from the excitations as the correlated portion of the wave function is nondegenerate. The free energy becomes a function of  $f(\mathbf{k})$  and  $h(\mathbf{k})$ , where  $f(\mathbf{k})$  is the probability that the state **k** is occupied by an excitation or a quasi-particle, and  $h(\mathbf{k})$  is the relative probability that the state **k** is occupied 7 SEPTEMBER 1973 Fig. 6 (left). A single-particle excitation of the superconductor in one-to-one correspondence with an excitation of the normal fermion system. Fig. 7 (right). Variation of the energy gap with temperature for the ideal superconductor.

by a pair given that is not occupied by a quasi-particle. Thus some states are occupied by quasi-particles and the unoccupied phase space is available for the formation of the coherent background of the remaining electrons. Since a portion of phase space is occupied by excitations at finite temperatures, making it unavailable for the transitions of bound pairs, the correlation energy is a function of the temperature,  $W_{c}(T)$ . As T increases,  $W_{c}(T)$  and at the same time  $\Delta$  decrease until the critical temperature  $T_{c}$  is reached and the system reverts to the normal phase.

Since the excitations of the superconductor are independent and in a one-to-one correspondence with those of the normal metal, the entropy of an excited configuration is given by an expression identical with that for the normal metal except that the Fermi function,  $f(\mathbf{k})$ , refers to quasi-particle excitations. The correlation energy at finite temperature is given by an expression similar to that at T = 0 with the available phase space modified by the occupation functions  $f(\mathbf{k})$ . Setting the variation of F with respect to h,  $\phi$ , and f equal to zero gives:

$$h = 1/2 (1 - \epsilon/E)$$
  
 $E = (\epsilon^2 + |\Delta|^2)^{1/2}$ 

and

$$f = \frac{1}{1 + \exp(E/k_{\rm B}T)}$$

where  $k_{\rm B}$  is Boltzmann's constant and

$$\Delta = |\Delta| e^{i\phi}$$

is now temperature-dependent and satisfies the fundamental integral equation of the theory

$$\Delta_{\mathbf{k}}(T) = -\frac{1}{2} \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}(T)}{E_{\mathbf{k}'}(T)} \tanh\left(\frac{E_{\mathbf{k}'}(T)}{2k_{\mathrm{B}}T}\right)$$

The form of these equations is the same as that at T = 0 except that the energy gap varies with the temperature. It is possible to satisfy the equation for the energy gap with nonzero values of  $\Delta$  only in a restricted temperature range. The upper bound of this temperature range is defined as  $T_c$ . For  $T < T_{\rm c}$ , singlet-spin zero-momentum electrons are strongly correlated, there is an energy gap associated with exciting electrons from the correlated part of the wave function, and  $E(\mathbf{k})$  is bounded below by  $|\Delta|$ . In this region the system has properties qualitatively different from those of the normal metal.

In the region  $T > T_e$ ,  $\Delta = 0$  and we have in every respect the normal solution. In particular, f, the distribution function for excitations, becomes just the Fermi function for excited electrons  $k > k_F$ , and for holes  $k < k_F$ 

$$f = \frac{1}{1 + \exp(|\epsilon|/k_{\rm B}T)}$$

If we make our simplifications of 1957 (defining in this way an "ideal" superconductor),

$$V_{\mathbf{k'k}} = -V |\epsilon| < \hbar \omega_{\mathrm{av}}$$
  
 $V_{\mathbf{k'k}} = 0$  otherwise

where  $\hbar \omega_{av}$  denotes the average phonon energy, and replace the energy-dependent density of states by its value at the Fermi surface, N(0), the integral equation for  $\Delta$  becomes

$$1 = N(0) V \int_{0}^{\pi \omega_{av}} \frac{d\epsilon}{(\epsilon^2 + |\Delta|^2)^{1/2}} \times \tanh\left[\frac{(\epsilon^2 + |\Delta|^2)^{1/2}}{2k_{a}T}\right]$$

The solution of this equation, Fig. 7, gives  $\Delta(T)$  and with this f and h. We can then calculate the free energy of the superconducting state and obtain



the thermodynamic properties of the system.

In particular, one finds that at  $T_e$  (in the absence of a magnetic field) there is a second-order transition (no latent heat:  $W_e = 0$  at  $T_e$ ) and a discontinuity in the specific heat. At very low temperatures the specific heat goes to zero exponentially. For this ideal superconductor one also obtains a law of corresponding states in which the ratio

$$\frac{\gamma T_{\rm c}^2}{H_0^2} = 0.170$$

where

$$\gamma = \frac{2}{3} \pi^2 N(0) k_{\rm B}^2$$

The experimental data scatter about the number 0.170. The ratio of  $\Delta$  to  $k_{\rm B}T_{\rm c}$  is given as a universal constant

$$\Delta/k_{\rm B}T_{\rm c}=1.7$$

There are no arbitrary parameters in the idealized theory. In the region of empirical interest all the thermodynamic properties are determined by the quantities  $\gamma$  and  $\hbar \omega_{av} e^{-1/N(0)V}$ . The first,  $\gamma$ , is found by observation of the normal specific heat, while the second is found from the critical temperature, given by

$$k_{\rm B}T_{\rm c} = 1.14 \ \hbar\omega_{\rm av} \ e^{-1/N(0)F}$$

At the absolute zero

$$\Delta = \hbar \omega_{\rm av} \left[ \sinh \left( \frac{1}{N(0)V} \right) \right]^{-1}$$

Further, defining a weak coupling limit  $[N(0)V \ll 1]$ , which is one region of interest empirically, we obtain

$$\Delta\simeq 2\hbar\;\omega_{av}\;e^{-1/N(0)V}$$

The energy difference between the normal and superconducting states becomes (again in the weak coupling limit)

$$W_{\rm s} - W_{\rm n} \equiv W_{\rm c} \equiv {
m ar} \ - 2N(0) \ (\hbar\omega_{\rm av})^2 \ e^{-2/N(0)V} \ {
m tu}$$

Fig. 8. Ultrasonic attenuation as a function of temperature across the superconducting transition as measured by Morse and Bohm (16). [Courtesy of the American Institute of Physics, New York]

The dependence of the correlation energy on  $(\hbar \omega_{av})^2$  gives the isotope effect, while the exponential factor reduces the correlation energy from the dimensionally expected N(0)  $(\hbar \omega_{av})^2$  to the much smaller observed value. This, however, is more a demonstration that the isotope effect is consistent with our model rather than a consequence of it, as will be discussed further by Professor Bardeen.

The thermodynamic properties calculated for the ideal superconductor are in qualitative agreement with experiment for weakly coupled superconductors. Very detailed comparison between experiment and theory has been made by many authors. A summary of the recent status may be found in (2). When one considers that in the theory of the ideal superconductor the existence of an actual metal is no more than hinted at (we have in fact done all the calculations considering weakly



Fig. 9. The two states  $|i\rangle$  and  $\langle f|$  shown are connected by  $c_{\mathbf{k}'\uparrow}^*c_{\mathbf{k}\uparrow}$  with the amplitude  $u_{\mathbf{k}'}u_{\mathbf{k}'}$ .

interacting fermions in a container) so that in principle, with appropriate modifications, the calculations apply to neutron stars as well as metals, we must regard detailed quantitative agreement as a gift from above. We should be content if there is a single metal for which such agreement exists. (Pure single crystals of tin or vanadium are possible candidates.)

To make comparison between theory and experiments on actual metals, a plethora of detailed considerations must be made. Professor Bardeen will discuss developments in the theory of the electron-phonon interaction and the dependence of superconducting properties on the phonon spectrum and the range of the Coulomb repulsion. Crystal symmetry, Brillouin zone structure, and the actual wave function (S, P, or D states)of the conduction electrons all play a role in determining real metal behavior. There is a fundamental distinction between the superconductors which always show a Meissner effect and those (type II) which allow magnetic field penetration in units of the flux quantum.

When one considers, in addition, specimens with impurities (magnetic and otherwise), superimposed films, small samples, and so on, one obtains a variety of situations, developed in the years since 1957 by many authors, whose richness and detail take volumes to discuss. The theory of the ideal superconductor has so far allowed the addition of those extensions and modifications necessary to describe, in what must be considered remarkable detail, all of the experience actually encountered.

#### **Microscopic Interference Effects**

In its interaction with external perturbations the superconductor displays remarkable interference effects which result from the paired nature of the wave function and are not at all present in similar interactions of normal metals. Neither would they be present in any ordinary two-fluid model. These "coherence effects" are in a sense manifestations of interference in spin and momentum space on a microscopic scale, analogous to the macroscopic quantum effects due to interference in ordinary space which Professor Schrieffer has discussed. They depend on the behavior under time reversal of the perturbing fields (12). It is intriguing to speculate that, if one could some-

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how amplify them properly, the time reversal symmetry of a fundamental interaction might be tested. Further, if <sup>3</sup>He does in fact display a phase transition analogous to the superconducting transition in metals as may be indicated by recent experiments (7) and if this is a spin triplet state, the coherence effects would be greatly altered.

Near the transition temperature these coherence effects produce quite dramatic contrasts in the behavior of coefficients which measure interactions with the conduction electrons. Historically, the comparison with theory of the behavior of the relaxation rate of nuclear spins (13, 14) and the attenuation of longitudinal ultrasonic waves in clean samples (15, 16) as the temperature is decreased through  $T_{\rm e}$  provided an early test of the detailed structure of the theory.

The attenuation of longitudinal acoustic waves due to their interaction with the conduction electrons in a metal undergoes a very rapid drop (15) as the temperature drops below  $T_{\rm e}$ . Since the scattering of phonons from "normal" electrons is responsible for most of the acoustic attenuation, a drop was to be expected; but the rapidity of the decrease measured by Morse and Bohm (16) (Fig. 8) was difficult to reconcile with estimates of the decrease in the normal electron component of a twofluid model.

The rate of relaxation of nuclear spins was measured by Hebel and Slichter (13) in a zero magnetic field in superconducting aluminum from 0.94° to 4.2°K just at the time of the development of our 1957 theory. Redfield and Anderson (14) confirmed and extended their results. The dominant relaxation mechanism is provided by interaction with the conduction electrons so that one would expect, on the basis of a two-fluid model, that this rate should decrease below the transition temperature as a result of the diminishing density of "normal" electrons. The experimental results, however, show just the reverse. The relaxation rate does not decrease but increases by a factor of more than 2 just below the transition temperature. This observed increase in the nuclear spin relaxation rate and the very sharp drop in the acoustic attenuation coefficient as the temperature is decreased through  $T_{\rm e}$ impose contradictory requirements on a conventional two-fluid model.

To illustrate how such effects come about in our theory, we consider the 7 SEPTEMBER 1973



Fig. 10. The two states  $|i\rangle$  and  $\langle f|$  are also connected by  $c_{-\mathbf{k}\downarrow}^* c_{-\mathbf{k}'\downarrow}$  with the amplitude  $- v_{\mathbf{k}'\mathbf{k}} v^*$ .

transition probability per unit time of a process involving electronic transitions from the excited state k to the state k' with the emission of energy to or the absorption of energy from the interacting field. What is to be calculated is the rate of transition between an initial state  $|i\rangle$  and a final state |f> with the absorption or emission of the energy  $h_{\omega|\mathbf{k}'-\mathbf{k}|}$  (a phonon, for example, in the interaction of sound waves with the superconductor). All of this properly summed over final states and averaged with statistical factors over initial states may be written:

$$\omega = \frac{2\pi}{\hbar} \frac{\sum_{i,f} \exp(-W_i/k_{\rm B}T) | < f|H_{\rm int}|i>|^2 \,\delta(W_f - W_i)}{\sum_i \exp(-W_i/k_{\rm B}T)}$$

We focus our attention on the matrix element  $< f|H_{int}|i>$ . This typically contains as one of its factors matrix elements between excited states of the superconductor of the operator

where  $c_{k'}^{*}$  and  $c_{k'}$  are, respectively, the creation and annihilation operators for electrons in the states  $\kappa'$  and  $\kappa$ , and  $B_{\kappa'\kappa}$  is the matrix element between the states  $\kappa'$  and  $\kappa$  of the configuration space operator  $B(\mathbf{r})$ 

$$B_{\kappa'\kappa} = \langle \kappa' | B(\mathbf{r}) | \kappa \rangle$$

The operator B is the electronic part of the matrix element between the full final and initial state

$$< f|H_{int}|i> = m_{fi} < f|B|i>$$

In the normal system scattering from single-particle electron states  $\kappa$  to  $\kappa'$  is independent of scattering from  $-\kappa'$  to  $-\kappa$ . But the superconducting states are linear superpositions of  $(\kappa - \kappa)$  occupied and unoccupied. Because of this, states with excitations  $\mathbf{k}\uparrow$  and  $\mathbf{k}\uparrow\uparrow$  are connected not only by  $c_{\mathbf{k}\uparrow\uparrow}^* c_{\mathbf{k}\uparrow}$  but also by  $c_{-\mathbf{k}\downarrow}^* c_{-\mathbf{k}'\downarrow}$ ; if the state  $|f\rangle$  contains the single-particle excitation  $\mathbf{k}'\uparrow$  while the state  $|i\rangle$  contains  $k\uparrow$ , as a result of the superposition of occupied and unoccupied pair states in the coherent part of the wave function, these are connected not only by  $B_{\kappa'\kappa} c_{\kappa'}^* c_{\kappa}$  but also by  $B_{-\kappa^-\kappa}$ ,  $c^*_{-\kappa}c_{-\kappa'}$ . For operators which do not flip spins,

we therefore write:

$$B = \sum_{\mathbf{k},\mathbf{k}'} (B_{\kappa'\kappa} c^*_{\kappa'} c_{\kappa} + B_{-\kappa-\kappa'} c^*_{-\kappa} c_{-\kappa'})$$

Many of the operators, B, we encounter (for example, the electric current, the charge density, or the spin operator) have a well-defined behavior under the operation of time reversal so that

$$B_{\kappa'\kappa} = \pm B_{-\kappa-\kappa'} \equiv B_{\mathbf{k}'\mathbf{k}}$$

Then B becomes

$$B = \sum_{\mathbf{k},\mathbf{k}'} B_{\mathbf{k}'\mathbf{k}} (c^*_{\mathbf{k}'\uparrow} c_{\mathbf{k}\uparrow} \pm c^*_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow})$$

where the upper sign results for operators even under time reversal and the lower sign results for operators odd under time reversal.

The matrix element of B between the initial state,  $\Psi \ldots_{\mathbf{k}\uparrow} \ldots_{\mathbf{k}\uparrow}$ , and the final state  $\Psi \ldots_{\mathbf{k}\uparrow} \ldots_{\mathbf{k}\uparrow}$ 

contains contributions from  $c_{\mathbf{k}'\uparrow}^* c_{\mathbf{k}\uparrow}$  (Fig. 9) and unexpectedly from  $c^*_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow}$  (Fig. 10). As a result the matrix element

squared  $| < f | B | i > |^2$  contains terms of the form

$$|B_{\mathbf{k}'\mathbf{k}}|^2 |(u_{\mathbf{k}'}u_{\mathbf{k}} \mp v_{\mathbf{k}'}v_{\mathbf{k}}^*)|^2$$

where the sign is determined by the behavior of B under time reversal: the upper sign if B is even under time reversal, and the lower sign if B is odd under time reversal.

Applied to processes involving the emission or absorption of boson quanta such as phonons or photons, the squared matrix element above is averaged with the appropriate statistical factors over initial states and summed over final states; subtracting emission probability from absorption probability per unit time, we obtain typically a general attenuation coefficient of the form

$$\alpha = \frac{4 \pi}{\hbar} |m|^2 \sum_{\mathbf{k} \, \mathbf{k}'} |(u_{\mathbf{k}'} u_{\mathbf{k}} \mp v_{\mathbf{k}'} v_{\mathbf{k}}^*)|^3 \times (f_{\mathbf{k}'} - f_{\mathbf{k}}) \, \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} - \hbar\omega_{|\mathbf{k}' - \mathbf{k}|})$$

where  $f_{\mathbf{k}}$  is the occupation probability in the superconductor for the excitation  $\mathbf{k}^{\uparrow}$  or  $\mathbf{k}\downarrow$ . [In the expression above we have considered only quasi-particle or quasi-hole scattering processes (not including processes in which a pair of excitations is created or annihilated from the coherent part of the wave function) since  $\hbar\omega_{|\mathbf{k}'-\mathbf{k}|} < \Delta$  is the usual region of interest for the ultrasonic attenuation and nuclear spin relaxation we shall contrast.]

For the ideal superconductor, there is isotropy around the Fermi surface and symmetry between particles and holes; therefore, sums of the form  $\Sigma$ can be converted to integrals over the superconducting excitation energy, E:

$$\sum_{\mathbf{k}} \rightarrow 2N(0) \int_{\Delta}^{\infty} \frac{E}{(E^2 - \Delta^2)^{1/2}} dE$$

where

$$N(0)[E(E^{2} - \Delta^{2})^{1/2}] = N(0)(E/\epsilon)$$

is the density of excitations in the superconductor (Fig. 11). The appearance of this density of excitations is a surprise. Contrary to our intuitive expectations, the onset of superconductivity seems initially to enhance rather than to diminish electronic transitions, as might be anticipated in a reasonable two-fluid model.

But the coherence factors  $|(u'u \mp v'v^*)|^2$  are even more surprising; they behave in such a way as to sometimes completely negate the effect of the increased density of states. This can be seen if we use the expressions obtained above for u and v for the ideal superconductor to obtain

$$(u'u \mp v'v)_{s}^{2} = \frac{1}{2} \left( 1 + \frac{\epsilon \epsilon' \mp \Delta^{2}}{EE'} \right)$$

In the integration over **k** and **k'**, the  $\epsilon \epsilon'$  term vanishes. We thus define  $(u'u \mp v'v)_{\epsilon}^2$ . In the usual limit where  $\hbar \omega_{|\mathbf{k'-k}|} \ll \Delta$ ,  $\epsilon \simeq \epsilon'$ , and  $E \simeq E'$ , this becomes

$$(u^2-v^2)^2_{s} \rightarrow \frac{1}{2}\frac{\epsilon^2}{E^2}$$

(operators even under time reversal)

$$(u^2+v^2)_{s}^{2} \rightarrow \frac{1}{2}\left(1+\frac{\Delta^2}{E^2}\right)$$

(operators odd under time reversal).

For operators even under time reversal, therefore, the decrease of the coherence factors near  $\varepsilon = 0$  just cancels the increase due to the density of



Fig. 11. Ratio of superconducting to normal density of excitations as a function of  $E/\Delta$ .

states. For operators odd under time reveral the effect of the increase of the density of states is not cancelled and should be observed as an increase in the rate of the corresponding process.

In general, the interaction Hamiltonian for a field interacting with the superconductor (being basically an electromagnetic interaction) is invariant under the operation of time reversal. However, the operator *B* might be the electric current  $\mathbf{j}(\mathbf{r})$  (for electromagnetic interactions), the electric charge density  $\rho(\mathbf{r})$  (for the electron-phonon interaction), or the *z* component of the electron spin operator,  $\sigma_z$  (for the nuclear spin relaxation interaction). Since under time reversal

$$\mathbf{j}(\mathbf{r},t) \rightarrow -\mathbf{j}(\mathbf{r},-t)$$
  

$$\rho(\mathbf{r},t) \rightarrow +\rho(\mathbf{r},-t)$$
  

$$\sigma_{*}(t) \rightarrow -\sigma_{*}(-t)$$

these show strikingly different interference effects.

Ultrasonic attenuation in the ideal pure superconductor for  $ql \gg 1$  (the product of the phonon wave number and the electron mean free path) depends in a fundamental way on the absorption and emission of phonons. Since the matrix elements have a very weak dependence on changes near the Fermi surface in occupation of states other than  $\mathbf{k}$  or  $\mathbf{k}'$  that occur in the transition from the normal state to the superconducting state, calculations within the quasi-particle model can be compared in a very direct manner with similar calculations for the normal metal, as  $B_{\mathbf{k}'\mathbf{k}}$  is the same in both states. The ratio of the attenuation in the normal and superconducting states becomes:

$$\frac{\alpha_s}{\alpha_n} = -4 \int_{\Delta}^{\infty} dE (u^2 + v^2)_s^2 \left(\frac{E}{\epsilon}\right)^2 \frac{df(E)}{dE}$$

Since  $(u^2 - v^2)_s^2 = \frac{1}{2} (\epsilon/E)^2$ , the coherence factors cancel the density of states giving

$$\frac{\alpha_{\rm s}}{\alpha_{\rm n}} = 2f[\Delta(T)] = \frac{2}{1 + \exp\left(\frac{\Delta(T)}{k_{\rm B}T}\right)}$$

Morse and Bohm (16) used this result to obtain a direct experimental determination of the variation of  $\Delta$  with T. A comparison of their attenuation data with the theoretical curve is shown in Fig. 12.

In contrast, the relaxation of nuclear spins which have been aligned in a magnetic field proceeds through their interaction with the magnetic moment of the conduction electrons. In an isotropic superconductor this can be shown to depend upon the z component of the electron spin operator

$$B_{\kappa'\kappa} = B(c^*_{\mathbf{k}\uparrow}c_{\mathbf{k}\uparrow} - c^*_{-\mathbf{k}\downarrow}c_{-\mathbf{k}\downarrow})$$

so that

$$B_{\kappa'\kappa} \equiv -B_{-\kappa-\kappa'}$$

This follows in general from the property of the spin operator under time reversal

$$\sigma_z(t) \equiv -\sigma_z(-t)$$

The calculation of the nuclear spin relaxation rate proceeds in a manner not too different from that for ultrasonic attenuation, resulting finally in a ratio of nuclear spin relaxation rates in superconducting and normal states in the same sample:

$$\frac{R_{\rm s}}{R_{\rm n}} = -4 \int_{\Delta}^{\infty} dE (u^2 + v^2)_{\rm s}^2 \left(\frac{E}{\epsilon}\right)^2 \frac{df(E)}{dE}$$

But  $(u^2 + v^2)^2_{s}$  does not go to zero at the lower limit so that the full effect of the increase in density of states at  $E = \Delta$  is felt. Taken literally, in fact, this expression diverges logarithmically at the lower limit because of the infinite density of states. When the Zeeman energy difference between the spin-up and spin-down states is included, the integral is no longer divergent but the integrand is much too large. Hebel and Slichter (13), by putting in a broadening of levels phenomenologically, could produce agreement between theory and experiment. More recently Fibich (17), by including the effect of thermal phonons, has obtained the agreement between theory and experiment shown in Fig. 13.

Interference effects manifest themselves in a similar manner in the interaction of electromagnetic radiation with the superconductor. Near  $T_c$  the absorp-

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tion is dominated by quasi-particle scattering matrix elements of the type we have described. Near T = 0, the number of quasi-particle excitations goes to zero and the matrix elements that contribute are those in which quasi-particle pairs are created from  $\Psi_0$ . For absorption these latter occur only when  $\hbar \omega > 2\Delta$ . For the linear response of the superconductor to a static magnetic field, the interference occurs in such a manner that the paramagnetic contribution goes to zero, leaving the diamagnetic part which gives the Meissner effect.

The theory developed in 1957 and applied to the equilibrium properties of uniform materials in the weak coupling region has been extended in numerous directions by many authors. Professor Schrieffer has spoken of Josephson junctions and macroscopic quantum interference effects; Professor Bardeen will discuss the modifications of the theory when the electron-phonon interactions are strong. The treatment of ultrasonic attenuation, generalized to include situations in uniform superconductors in which ql < 1, gives a result surprisingly similar to that above (18). There have been extensive developments using Green's function methods (19) appropriate for type II superconductors, materials with magnetic impurities, and nonuniform materials or boundary regions where the order parameter is a function of the spatial coordinates (20). With these methods formal problems of gauge invariance and current conservation have been resolved in a very elegant manner (21). In addition, many calculations (22) of great complexity and deFig. 12 (left). Comparison of ultraobserved sonic attenuation with the ideal theory. The data are due to Morse  $R_s$ and Bohm (16).  $\overline{R_n}$ Fig. 13 (right). Comparison of the observed nuclear spin relaxation rate with theory. () Experimental data of Hebel and Slichter (13); (X) data of Redfield and Anderson (14).

tail for type II superconductors have treated ultrasonic attenuation, nuclear spin re-

laxation, and other phenomena in the clean and dirty limits (few or large numbers of impurities). The results cited above are modified in various ways. For example, the average density of excitation levels is less sharply peaked at  $T_c$  in a type II superconductor; the coherence effects also change somewhat in these altered circumstances but nevertheless play an important role. Overall, one can say that the theory has been amenable to these generalizations and that agreement with experiment is good.

It is now believed that the finite many-nucleon system that is the atomic nucleus enters a correlated state analogous to that of a superconductor (23). Similar considerations have been applied to many-fermion systems as diverse as neutron stars (24), liquid <sup>3</sup>He (25), and elementary fermions (26). In addition, the idea of the spontaneously broken symmetry of a degenerate vacuum has been applied widely in elementary particle theory and recently in the theory of weak interactions (27). What the electron-phonon interaction has produced between electrons in metal may be produced by the van der Waals interaction between atoms in <sup>3</sup>He, the nuclear interaction in nuclei and neutron stars, and the fundamental interactions in elementary fermions. Whatever the success of these attempts, for the theoretician the possible existence of this correlated paired state must in the future be considered for any degenerate many-fermion system where there is some kind of effective attraction between fermions for transitions near the Fermi surface.

In the past few weeks my colleagues and I have been asked many times: "What are the practical uses of your



theory?" Although even a summary inspection of the proceedings of conferences on superconductivity and its applications would give an immediate sense of the experimental, theoretical, and developmental work in this field as well as expectations, hopes, and anticipations-from applications in heavy electrical machinery to measuring devices of extraordinary sensitivity and new elements with very high switching speeds for computers-I, personally, feel somewhat uneasy responding. The discovery of the phenomena and the development of the theory is a vast work to which many scientists have contributed. In addition, there are numerous practical uses of the phenomena for which theory should not rightly take credit. A theory (although it may guide us in reaching them) does not produce the treasures the world holds. And the treasures themselves occasionally dazzle our attention; for we are not so wealthy that we may regard them as irrelevant.

But a theory is more. It is an ordering of experience that both makes experience meaningful and is a pleasure to regard in its own right. Henri Poincaré has written (28):

I e savant doit ordonner; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison.

One can build from ordinary stone a humble house or the finest chateau. Either is constructed to enclose a space, to keep out the rain and the cold. They differ in the ambition and resources of their builder and the art by which he has achieved his end. A theory, built of ordinary materials, also may serve many a humble function. But when we enter and regard the relations in the space of ideas, we see columns of remarkable height and arches of daring breadth. They vault the fine structure constant, from the magnetic moment of the electron to the behavior of metallic junctions near the absolute zero; they span the distance from materials at the lowest temperatures to those in the interior of stars, from the properties of operators under time reversal to the behavior of attenuation coefficients just beyond the transition temperature.

I believe that I speak for my colleagues in theoretical science as well as myself when I say that our ultimate, our warmest pleasure in the midst of one of these incredible structures comes with the realization that what we have made is not only useful but is indeed a beautiful way to enclose a space.

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## The Future of the Americas

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and anxieties. In the truest sense, this

I cannot undertake to speak about the future of the Americas without speaking about the future of the world. You scientists, of all people, know that today all of us human beings are intertwined with one another. We are all together in a world of alarm and strife -a world that appears not to have quite made up its mind whether it is too primitive for peace or too advanced for war.

Our age has been called both the Age of Science and Technology and the Age of Anxiety. Both are accurate. Indeed, one feeds upon the other: as our scientific and technological competence has increased, so have our fears is a time of paradox-a time of unparalleled affluence and unprecedented need. It is a time in which there have been great advances in science and technology, and yet these are overshadowed by incredible advances in instruments of destruction. It is a time when man seems to have learned how to achieve most and to fear most, when he seems to know much more about how to make war than how to make peace, more about killing than he does about living. It is also a time in which the world fears, not the primitive or the ignorant man, but the educated, the scientifically trained, the technically competent man, who has it in his power to destroy civilization. It is a time in which we seem to know almost everything about know-how and very little about know-why. It is a time in which we can send men to walk the moon, yet witness the timeliness of

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Santayana's observation that men have come to power who, "having no stomach for the ultimate, burrow themselves downward toward the primitive."

In such a time, there can be no escape from facing front and asking hard questions. For in this nuclear age we can't hide, and we can't drop out. We can only choose where best to take our stand.

Irish poet Arthur O'Shaughnessy wrote: "Each age is a dream that is dying, or one that is coming to birth." The age that is coming to birth-indeed, the one that is with us alreadyis so changing and dynamic that no one can really know how it will be to live in it. We know that the habits of the past will not suffice for the challenges of the future. We also know that it has never been more important to reach for a world of peace and freedom -a world made safe for people.

If we are to move toward that goal, then we must realistically confront the terrible disparity in living standards between the so-called developed North and the underdeveloped South-between the world's "haves" and "havenots"-a gap described by Barbara Ward as "inevitably the most tragic and urgent problem of our day." The tragedy is in the economic despair and emptiness that mark the lives of all too many in the developing countries; the urgency is in preventing a political re-

The author is former ambassador to the Or-ganization of American States and is chairman of The National Urban Coalition. This article is ganization of American States and is chairman of The National Urban Coalition. This article is adapted from an address given at the Inter-American meeting Science and Man in the Ameri-cas, sponsored jointly by the American Associa-tion for the Advancement of Science and the Consejo Nacional de Ciencia y Tecnologia de Mexico, Mexico City, 27 June 1973.