cana of mathematical pyramidolatry. Turn, instead, to Gillings's exposition of what is reliably inferable about the mathematics of the Nile valley in the Pharaohs' times.

Actually, the surviving papyri of the Middle Kingdom (inscribed almost a millennium after the erection of the great pyramid) indicate that π was then taken to be $(4/3)^4$, which is correct to 0.6 percent, and therefore good—but not mysteriously good. Other notable accomplishments that the author discusses are: the solving of problems in proportion; the summing of certain arithmetic and geometric progressions; the computation of various square roots; the solution of simple equations of the second and third degrees; and the finding of many nonsimple regular areas and volumes—including, it seems, the surface area of the hemisphere. His researches also support what other scholars have averred but nonspecialists are always surprised to hear, namely, that Pythagoras' theorem remained unknown during the entire era.

Gillings assesses the material in an original way. He emphasizes, on the one hand, that the mathematics at issue was based on two elementary multiplication processes, binary and "two-thirds"; and, on the other hand, that it was innocent of abstraction and the concept of proof. Within these constraints the achievement was massive, and is worthy of more respect than it is usually accorded. Gillings suggests that we glibly associate neglect of proof in mathematics with logical shallowness. The ancient Egyptians were not concerned with "a priori symbolic argument that would show clearly and logically their thought processes. What they did was to explain and define in an ordered sequence the steps necessary in the proper procedure. . . . This was science as they knew it, and it is not proper or fitting that we of the twentieth century should compare too critically their methods with those of the Greeks. . . ." Which is fair comment, and it might be added that Egyptian arithmetic, unlike its Greek counterpart, was never bedeviled by number mysticism and contempt for calculation.

This is a pioneer full-length study. It is also wide-ranging, covering incidentally the calendar, and quantity surveying, and units of measurement. The important matters of notation and translations are handled very well.

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More Recent Mathematics

Mathematical Thought from Ancient to Modern Times. MORRIS KLINE. Oxford University Press, New York, 1972. xviii, 1238 pp., illus. $35.

General histories of mathematics have appeared fairly regularly for many years, and all of them have suffered from the vastness of the subject and the paucity of reliable historical studies on which to draw. In particular, they often peter out with brief sections on the 19th century, and do not regard 20th-century mathematics as historical at all.

By contrast, nearly half of Kline's new book deals with developments after 1800, with a previous 200 pages for the 18th century. It is the first general history which begins to reflect the actual development of mathematics, and is by far the best yet to appear. Its size has made inevitable a price that will confine its sales to libraries, but one hopes institutions that offer courses in the history of mathematics will purchase several copies for the use of their students. It is excellently printed and furnished with comprehensive name and subject indexes.

In a review of this kind I can only indicate general features and note selected points of detail. Though the book is large, it does not cover all branches of mathematics, as Kline notes in his preface (p. vii). But I cannot fully understand how the criteria described there have determined the selection of topics for the main text following. The book is strong on the calculus, mathematical analysis, differential equations, set theory and foundations, the various geometries (including differential geometry), and calculus of variations. But it is much less detailed on complex variables, linear algebra, algebraic structures, number theory, rational mechanics, and numerical analysis and is silent about probability and statistics.

A pleasant feature of the book is three interludes (chapters 18, 26, and 43) where Kline reflects on the conception of mathematics at the beginning of the 18th, 19th, and 20th centuries and provides information on the support of mathematicians and the changing media of mathematical publication. Sociological matters of this kind are of considerable importance, especially for the professionalization of mathematics during the last 300 years.

Kline is obviously most at home in these later periods, for the early chapters seem too brief, and perhaps derivative, for the material covered. It is noticeable that there are barely a dozen footnotes in the first dozen chapters (which take the story from Mesopotamia to the Renaissance), whereas many later chapters have more than 40 each. The standard of reference citation is high; works that have appeared in collected editions of their authors' work are also cited in their original appearances, and each chapter ends with a bibliography of the principal primary and secondary sources used.

Impressive though the book is, there do seem to be significant matters for criticism. I shall give a few examples of mistakes or questionable interpretations from those bits of the history of mathematics with which I have some acquaintance.

1) On p. 677 Kline gives the diagram of a Fourier series over several periods of its representation. But he leaves blank the jumps of the graph over its discontinuities, whereas Fourier joined such jumps by vertical lines. This point could have motivated several historical problems, such as the naive envelope argument on which Fourier seems to have drawn and its relation to multiple limit techniques, the interpretation of a function as a curve rather than as an expression, the extension of continuity to incorporate this geometric connectedness and its relation to differentiability, and so on. Fourier also ought to have appeared on p. 270 for the inductive proof of Descartes' rule of signs, on p. 715 with his analysis of the cooling cylinder as inspiration for his admirers Sturm and Liouville, and much more prominently on p. 710 for his comprehensive treatment of the particular "Bessel function" $J_0(x)$, years before Bessel, en route to the cylinder solution. Further, Poisson's enthusiasm for Fourier's methods was much more muted than Kline states on p. 678; indeed, an important "political" battle over Fourierian versus other methods has been missed here.

2) Cauchy's 1814 paper on definite integrals seems to me to contain much clearer indications for the development of complex variable function theory than Kline allows on p. 635; indeed, the seeds of the whole basic theory are there. It was delayed in publication until 1825 (rather than submitted then, as Kline claims), at which time Cauchy published as a pamphlet a marvelous sequel paper. Kline is quite wrong in saying that it was not published until 1874 (p. 637); the 26 volumes of Cauchy's works mentioned on p. vii

still await their final volume, which is to contain all of Cauchy's pamphlets and lithographs.

3) Page 988 contains some doubtful remarks concerning Dedekind and Peano. Dedekind's *Was sind und was sollen die Zahlen?* was only first-drafted during 1872–1878, and it attracted far more interest after its publication in 1887 than Kline allows. But, as opposed to the claim often made elsewhere and repeated here, it does not seem to have been used by Peano when writing his 1889 *Arithmetices Principia*, for in a later paper Peano said quite explicitly that his pamphlet had been prepared independently of Dedekind's.

4) Kline discusses at length Cantor's theory of real numbers (pp. 984–85), but he overlooks the difficulty in Cantor's theory of interpreting the equality of two numbers defined by different fundamental sequences. Later he misrepresents Cantor's second number class (p. 1001), giving it a last term. He also asserts that Klein "was by no means in sympathy" with Cantor's ideas (p. 1003), whereas Klein had Cantor's papers published in *Mathematische Annalen* after the opposition from Kronecker.

5) Kline's discussion of Russell's theory of types merges its "simple" and "ramified" parts (p. 1195) and so renders enigmatic the remarks on the axiom of reducibility. The discussion following of the construction of mathematics by logicist means omits mention of ordering and relation arithmetic, whose techniques are vital to such developments.

6) The discussion of measure and its applications (chapter 44) astonishingly ignores W. H. and G. C. Young, who did as much as anyone in this area.

One could continue in this vein; but nothing can, or should, dispel the fine impression that this book leaves. I am still amazed by the amount that Kline has achieved.

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Lore of an Element

Mercury. A History of Quicksilver. LEONARD J. GOLDWATER. York, Baltimore, 1972. xiv, 318 pp., illus. $15.

Is this the first case history of a chemical element? Quite possibly. It sets a pattern worthy of emulation for each of the other elements. However, few elements have been of such fascination to man over so many centuries as has mercury, liquid yet with the mysterious properties of metals. Chinese alchemists in the fourth century prescribed cinnabar, red mercuric sulfide, as the elixir for attainment of immortality. In later eras mercury in the ear was a method of murder—suggesting a possible cause of death of Hamlet's father. Mercury it seems even when swallowed in amazingly large quantities often passes through the system with little harm. Yet mercury breathed or mercury in the blood has awful consequences. Mercury was one of the three elements (the others being sulfur and salt) of which all substances were believed by Paracelsus and others to be constituted. It even had a planet named after it. It played a major role in the battle against syphilis and still there is disagreement whether treatment did more harm or good. All this and much more is contained in this book, including the questioning of one thing we did think we knew! Was Alice's Mad Hatter afflicted with the "hatter's shakes," the occupational disease of the felting industry, in which mercury compounds were used, or was "mad as a hatter" a corruption of "mad as an adder"?

Here is a work of broad-ranging scholarship delving into the earliest discovery and the uses of mercury in every corner of the globe. Part 1 deals with aspects of mercury other than its effects in man, tracing the history of its use in the occult arts, its extraction, its importance in trade and finance, and knowledge and use of it through the ages, including its role in chemistry and its uses in scientific instruments. Part 2, the medical aspects, represents the author's special field, for he was studying the hat industry in 1936, was involved in World Health Organization studies of mercury, lead, and arsenic pollution in 1956 (ten years before the public alarm), and has been developing analytical procedures for large-scale human studies.

The book carries extensive lists of references, though unfortunately they are not always specific, occasionally omitting page numbers. One piece of mercury lore not found in the book would interest the author and maybe others. Spinach is reported to have been used by the early Chinese as an antidote against mercury poisoning no doubt produced by their attempts to become immortal (E. H. Shafer, *The Golden Peaches of Samarkand*, University of California Press, 1963, p. 147). Is the oxalate in spinach responsible for the removal of mercury?

This is an absorbing book of interest to a wide range of readers. Historians, teachers, ecologists, researchers dealing with mercury (and who doesn't?), doctors, dentists, and investigators of occupational diseases all will find material to ponder or to be amused by.

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Health Care and Social Policy

The Case for American Medicine. A Realistic Look at Our Health Care System. HARRY SCHWARTZ. McKay, New York, 1972. xiv, 240 pp. $6.95.

The American Medical Machine. ABRAHAM RIBICOFF, with Paul Danaceau. Saturday Review Press, New York, 1972. vii, 212 pp. $6.95.

Health Care: Can There Be Equity? The United States, Sweden, and England. ODIN W. ANDERSON. Wiley, New York, 1972. xxii, 274 pp., illus. $11.95.

The movement toward an improved health care system in America proceeds haltingly but persistently. It is a centipedic movement and its forward pace is sometimes hampered by separate limbs reaching in different directions. Sometimes there seems to be no motion at all.

Impetus for the movement comes from many sources, some internal and others external to the system. Advances in biomedical science constantly challenge medical practitioners to alter treatment practices. A range of personal and professional reasons contribute to a discernible trend toward multi-specialty group practice and away from solo practice, and to a related if somewhat vaguer attention to preventive care. Demonstrated needs occasionally prompt new governmental policies aimed at eliminating severe medical hardships for special groups of citizens. High costs of medical care bring popular cries for controls, or at least for financial help. Politicians recognize the citizenry's continuing preoccupation with health, and are prop-