attempt to say "There exists numerical utility, $U(X^A) = U(x_1^A, x_2^A, \ldots)$, which is greater than $U(X^B)$."

But now what happens to our commonsense notions: "Coffee and tea are *substitute* goods. Tea and lemon are *complementary* goods." The old test fails: "If the sum of increments of utility that I get from experiments that increase but one good at a time exceeds the increment of utility I get from increasing both goods *together*—then (like tea and lemon) they are complementary goods."

Hicks ingeniously proposed an alternative behavioristic test: "Raise the price of coffee and raise the consumer's income just enough to leave him as well off as before. If the amount bought of tea now goes up, tea and coffee are *substitutes*; if the amount of cream goes down, cream and coffee are *complements*." Then, just as Clerk Maxwell had proved reciprocity relations in thermodynamics (dependent on $\partial^2E/\partial V\partial S \equiv \partial^2E/\partial S\partial V$), Hicks proves that

$$(\partial x_i/\partial P_j)_U \equiv (\partial x_j/\partial P_i)_U$$

that is, if tea is a substitute for coffee, coffee must be a substitute for tea.

All this he applies to bonds and stocks as well as consumption goods, contributing to the revolutionary advances in business cycle control that we associate with Keynes's *General Theory of Money, Interest and Employment.*

Existence of General Equilibrium and Its Dynamic Stability

Before describing what I regard as Arrow's two greatest analytical contributions, let me connect some of his work with that of Hicks. Hicks reduced the general equilibrium of production and exchange of n goods to the following homogeneous-of-degree-zero net demand functions involving prices, $P = (p_1, \ldots, p_n)$

$$\begin{aligned} 0 = -F(P) &= -f_j[p_1, \ldots, p_n] \\ &\equiv -F[\lambda P] \end{aligned}$$

He demonstrated that a unique solution to price ratios, P^*/p_1^*, would be assured if everyone always spent each extra dollar of income in the same way.

In that case, I and others proved that the system would be dynamically stable, in the sense that the following algorithm of price formation would converge to P^*/p^*

$$\dot{P} = (\dot{p}_j) = (-k_jf_j[P] = -KF[P]$$
$$\lim_{t\to\infty} P(t) = P^*/p_1^*, \text{ for any } P(0) > 0$$

Here K is a diagonal matrix with positive, but arbitrary, k_j elements.

Arrow, in collaboration with Leonid Hurwicz of the University of Minnesota, explored global stability when the Jacobian matrix $[\partial f_i/\partial p_j] = F'[P]$ is not symmetric but does have positive off-diagonal elements.

Arrow, in accordance with the new tradition stemming from topological work by A. Wald and J. von Neumann, went beyond the mere counting of equations and unknowns in $F[P] = 0$. The question of the existence of at least one equilibrium solution, P^*, had to be explored in terms of the use of inequalities, usually involving delicate fixed-point theorems of the type developed by Brouwer and Kakutani. Collaborating with G. Debreu, Arrow not only established such existence and

Speaking of Science

A Black Hole in Our Galaxy?

Last week several well-respected astronomers publicly stated that there is good evidence for a black hole in the constellation Cygnus.

For more than a year evidence has been mounting that a certain source of x-rays is a black hole, a region left behind after a very massive star has collapsed and where the gravitational forces are so strong that nothing, not even light, can escape (*Science*, 28 January 1972). Very little new evidence was presented last week for christening the x-ray source named Cygnus X-1 as the first black hole, but on the basis of prior evidence Jeremiah Ostriker of Princeton University said at a meeting of the American Astronomical Society in Pasadena, California, that, if the x-ray source were associated with a certain blue star, "I'd be hard put to say it isn't a black hole."

Although many scientists at the Pasadena meeting were skeptical about the publicity given to an unproven idea, most agreed that a betting man would bet on a black hole.

Apparently Cygnus X-1, like most x-ray sources, is part of a binary star system. Although the x-ray source cannot be seen, it appears to be orbiting about a large blue star. If the blue star is very massive, as its spectral type would indicate, then the x-ray source must be massive and almost certainly must be a black hole. Gravitational theory predicts that almost any compact object with more than twice the mass of the sun must be a black hole. (Lighter compact objects can be neutron stars and white dwarfs.)

But the x-ray source may not be part of the blue star system, because the identification depends on a rather detailed argument about the linkage of three observations: radio, optical, and x-ray. The x-ray observation is crucial because it alone indicates a very compact object, but it is not as precise as the other two measurements. During the last year a shift in the x-ray emissions coinciding with a shift of the radio signal tended to corroborate the linkage. However, there is still some uncertainty whether the blue star and the x-ray source are really companions or not.

Another uncertainty is the true mass of the blue star; because it is not well separated from other stars, the spectral type may not be a good indicator of its mass.

The various announcements at Pasadena certainly did not indicate a consensus that a black hole had been discovered, but participants at the conference noted that many astronomers are investing their energies in further research on the Cygnus X-1 system. Preliminary data on the blue star system may indicate the direction of future investigations. Jerome Kristian of the Mount Wilson and Palomar Observatories and Harding Smith of the Berkeley Space Sciences Laboratory reported helium emission lines coming from the blue star, or possibly from the x-ray source. Emissions from the x-ray source could establish its mass directly, and verify its tentative identification as the first black hole.—W.D.M.