Reports

Capture Resonance of the Asteroid 1685 Toro by the Earth

Abstract. *The asteroid 1685 Toro has its perihelion inside the Earth's orbit and a period which is 8/5 of that of the Earth. A calculation of Toro's orbit covering 200 years shows that this asteroid at present is captured in resonance with the Earth. The capture is due to the gravitational interaction at close encounters between the bodies.*

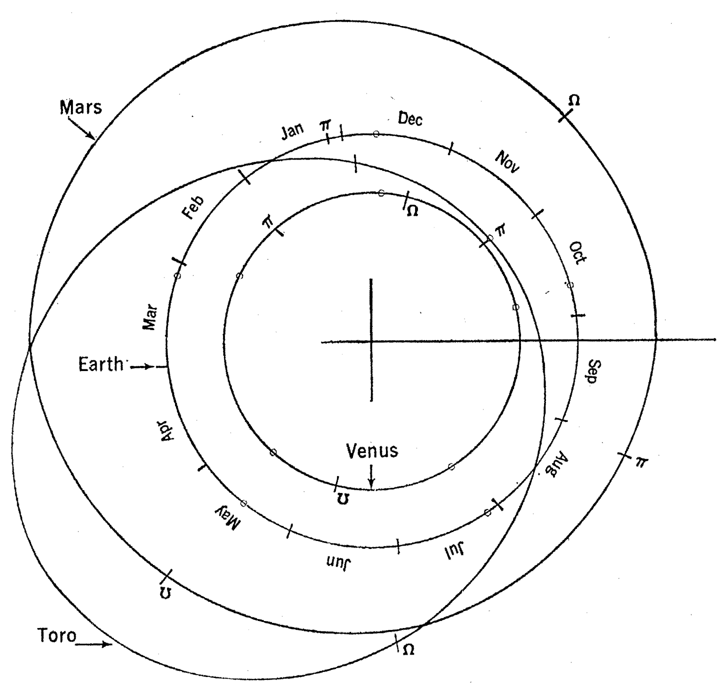
In the solar system there are many resonance phenomena due to different mechanisms. The motions of two planets or satellites can couple together so that the ratio of their orbital periods has the value of p/q where p and q are small integers (*1*). This kind of coupling also occurs between the spin of one body and the orbital motion of another. The most remarkable example is the orbital coupling between Neptune and Pluto. Due to its large eccentricity, the orbit of Pluto intersects the orbit of Neptune. In spite of this the two bodies can never collide because, as found by Cohen and Hubbard (*2*), these two planets are in a 3/2 resonance such that the distance between them is always larger than 18 astronomical units (A.U.). This effect implies that Neptune and Pluto may have coexisted since the beginning of the solar system.

This result leads to the question whether stray bodies could also be stored in the inner part of the interplanetary space by similar effects, namely, by resonances with the Earth or Venus, or both. In this report we discuss the asteroid 1685 Toro.

The present orbital elements of Toro show that its period is almost exactly 1.6 years. Further, the relative positions of Toro and the Earth exhibit a certain symmetry, including close approaches. This capture resonance, if we may call it so, has been investigated in more detail by integrating the perturbed orbit of Toro for 100 years backward and forward, that is, for a total of 200 years. The perturbations by Venus, Earth, Mars, Jupiter, and Saturn are included in the orbit integration according to Cowell's method. The integration step size is 5 days per step. If there are close encounters between Toro and Venus, Earth, or Mars, the step size can be divided to be 2.5, 1.25, and so on, days per step so that the accuracy is retained.

The orbits of Toro and the terrestrial planets are at present oriented as shown in Fig. 1 (projection on the ecliptic plane). The possible positions of the Earth and Venus when Toro is at perihelion are marked. The orbit of Toro crosses the orbits of both Mars and the Earth; the orbit of the Earth is crossed at the longitudes that the Earth will have during the months of January and August.

In a coordinate system rotating with the Earth, the orbit of Toro is as shown in Fig. 2. This particular loop is pro-



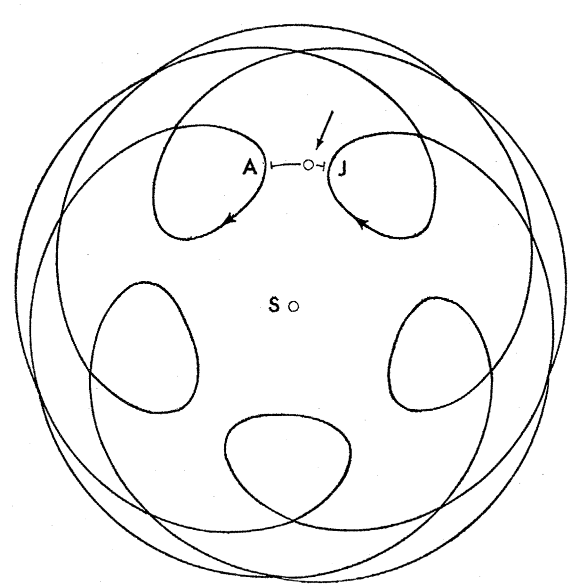


Fig. 1 (left). Projection of the orbits of the terrestrial planets and Toro onto the ecliptic plane. The positions of Venus and Earth when Toro passes its perihelion are marked. The data used are from the perihelion epoch of December 1967 and four following perihelions. Fig. 2 (right). Projection of Toro on the ecliptic plane in a coordinate system rotating with the Earth. The fact that Toro's period oscillates around 1.6 years is shown by superimposing an oscillation on the rotation of the coordinate system. This makes the Earth oscillate along a 16° arc with a period of 144 years. The arrow indicates the position of the Earth with respect to the orbital pattern of Toro in the year 2020.

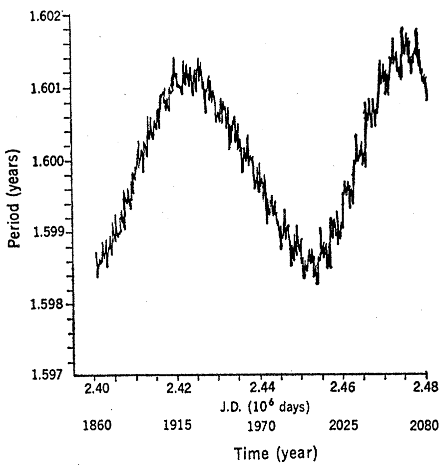


Fig. 3. Variation of Toro's period. The time is in Julian days and years.

duced during 8 years around the year 2020. The loop is not exactly closed; instead it can be said to oscillate around its center, the Sun, with an amplitude of about 8° and a period of 144 years. In Fig. 2 this is best illustrated by keeping the 8-year loop of Toro fixed and allowing for oscillations of the Earth along a 16° arc.

At each close encounter (every eighth year) in the vicinity of its ascending node, Toro is ahead of the Earth. Thus, due to the gravitational atttraction by the Earth, Toro is retarded, the angular momentum as well as the period being decreased. This tends to make the subsequent encounters farther from the Earth. Instead, Toro will approach the Earth in the vicinity of its descending node (in January every eighth year). At the January encounter Toro is behind the Earth. It is then accelerated, its period is increased, and again it tends to move away from the Earth to approach it in the vicinity of the ascending node at the August encounter.

The variation of the period is shown in Fig. 3. The upward and downward slopes of the curve can be explained in terms of the alternating perturbations from the Earth during the January and August encounters. Due to the oscillation of the position of the Earth relative to the loop of Toro (Fig. 2), the closest distances at the January and August encounters also vary. For example, around the year 1964 the Earth is closer to Toro during the August encounter than during the January encounter. The decelerating force is larger than the accelerating force, hence the period is decreased in this section. The curve has a negative slope until the year 2004 is reached. Then the relative position of the Earth has moved to the midpoint, which is equidistant from the points of the January and August encounters. In this region, the accelerating force cancels the decelerating force and a zero slope appears. As soon as the midpoint is passed the relative distances at the January encounters are smaller than those at the August encounters, and as the accelerations dominate the decelerations of Toro, an upward slope in the plot of the period results.

The phase variations in Fig. 4 are based on the longitudinal distances from Toro to the Earth and Venus when Toro is at perihelion. The data are reduced so that the positions of the planets always refer to the points around 13° longitude (October) in Fig. 1; that is, $n \times 72°$ has been added, where n is 0, 1, 2, 3, or 4. The equilibrium phase angle would be expected to be 36° (for zero inclination), and the resonance is expected to disappear if the amplitude (relative to equilibrium) of the libration exceeds 36° (that is, if the phase angle passes 0° or 72°). From Fig. 4 it is then concluded that the phase variation relative to the Earth is compatible with a capture resonance during the period studied.

The configuration described would be favorable for a stable resonance, but two circumstances make it possible that Toro could lose its coupling to the Earth. First, Toro's orbit is at a rather large distance from the ecliptic plane at the close encounters. At the August encounter Toro is always at least 2.5° ahead of the Earth and 6° above the ecliptic plane; hence, the perpendicular distance is about three times the longitudinal distance. Secular variations or other perturbations might bring Toro to the "wrong side" of the Earth there. Second, Toro also has close encounters with Venus. Within the time interval we have studied, the closest approach is about 0.15 A.U. As secular variations change the node and inclination, Venus might either "capture" Toro from the Earth or perturb it out of resonance altogether.

Naturally, a time span of 200 years is insufficient for determining whether Toro is caught in a permanent resonance. Since we know that Toro comes at least as close as 0.15 A.U. to the Earth, we consider an investigation based on the secular variation of the elements rather unreliable. Further calculations are being made in order to study the stability of the resonance.

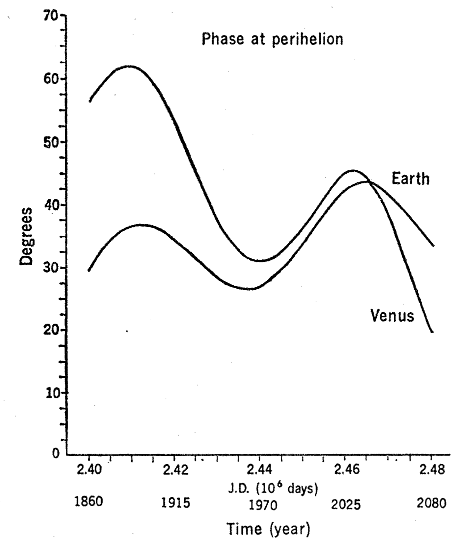


Fig. 4. The difference in longitude between Toro, Venus, and the Earth at perihelions every eighth year.

One of the objects of studying orbits like that of Toro in detail is that a resonance orbit such as this is one of the possible precapture orbits for the Moon (*3*). Another is that the existence of orbits like Toro's may be important for determining the lifetimes of the Apollo asteroids (*4*).

In summary, a calculation of the orbit of the asteroid 1685 Toro over 200 years shows that it is in at least a temporary 8/5 resonance with the Earth. The amplitude of the libration around the equilibrium position is about 8°, and the period of libration is about 144 years. Toro is also very close to 13/5 resonance with Venus.

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References and Notes

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