Strategies of Mutual Deterrence

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Advances in weapons technology and the possibilities of nuclear proliferation threaten the stability of world politics. Moreover, the burdens of maintaining an effective defense posture have increased to the point where military expenditures have become a major public concern. These problems have caused heated debates (*1*) about the need for and the advisability of various weapons systems—antiballistic missiles (ABM's), new bombers, submarines, and so forth. Unfortunately, criteria for evaluating the effects of such systems are elusive, highly complex, or nonexistent.

This article presents some simple mathematical models useful for judging the merits of stereotypical weapons systems in achieving mutual strategic deterrence among nuclear powers. Because the models treat stereotypes and do not reflect the diversity of actual weapons systems, we take no positions with respect to existing or proposed weapons systems.

In analyses of this sort, it is customary to assume that a national government is acting in a calculated fashion to achieve some specific set of strategic objectives. But military policies are also in part functions of organizational goals and procedures and of bureaucratic politics (*2*). Comprehensive understanding of strategic behavior would require considerable insight into the policy-making process (*3*). This article concentrates on one aspect of this subject to provide techniques for estimating bounds and sensitivities in the design of strategic forces. Accordingly, we make the assumption that each nation's primary strategy is to maintain retaliatory forces sufficient to deter attack by other nations.

The policy of maintaining such retaliatory forces has been called an "assured destruction" policy (*4*). The name stems in part from former Secretary of Defense McNamara's specification that the retaliatory capability after attack should be sufficient to assure the destruction of aggressor nations as viable societies. The ability to absorb an all-out attack and still maintain sufficient retaliatory forces is the principal means of deterrence (*5*).

We have studied hypothetical situations in which opposing nations feel secure from attack because each believes that no feasible attack would reduce its retaliatory capability to a level that potential attackers could accept. This article summarizes an analysis of such mutual deterrence relationships (*6–8*). We first compute the force required to assure particular retaliatory threat. (This is a function of various technological parameters and of the number of nations involved.) Next, we examine the effects of introducing various defenses of retaliatory forces and explore the dynamics of force growth for two nations, each attempting to achieve a deterring posture. Finally, we explore the effects of conflicting strategic objectives and find some quantitative bounds on the comparative values of land-based missile forces, bomber forces, and submarine forces.

A Basic Model

We assume that each nation (party) attempts to maintain retaliatory forces it believes sufficient to deter attack by any or all of the others. (A reliable alliance can be regarded as a single party.) Since its forces could be reduced by an attack, each party must acquire more weapons than it would otherwise deem necessary to deter attack. This attempt to achieve mutual deterrence could result in each party being able to destroy the other by a strike against urban centers but none being confident that he can deter the others from a strike directed against his retaliatory forces.

It is easy to imagine that this situation must lead to an unlimited "arms race." Even if stability of armaments is held to be achievable it is widely viewed as a fragile situation (*9*) dependent on some sort of parity being maintained between the parties—the avoidance of a "missile gap." As a simple instance, consider three nations, each fearing attack by the others. Suppose each believes that two surviving weapons are sufficient to deter attack. If, on the average, it takes two weapons to neutralize one, it is easy to convince oneself that an endless arms race is in prospect. If each nation buys two weapons, each is faced with the possibility of attack by four, which could eliminate its force. It must then buy two more, and so on. This intuitive description of an arms race can be quite misleading. The fallacy lies in the phrase "on the average, it takes two weapons to neutralize one," which does not account for the diminishing threat posed by additional weaponry.

We have developed a simple model that illustrates the "diminishing returns" phenomenon. It is an idealization of the situation presented by multi-warhead missiles located in sites dispersed and hardened against nuclear attack. Using this model we have found that unlimited arms races need not occur in the mutual attempt to achieve deterrent forces. Moreover, this stability of the mutually deterrent postures is not dependent on any particular parity being maintained between the parties.

In constructing a mathematical prototype (*10, 11*) of a force of multi-warhead missiles attacking a field of missile sites, we have assumed that attack of a site by a warhead has a fixed probability of destroying the attacked missile and that the attacks are independent (attacks are Bernoulli trials). If the number of attacking warheads is not an exact multiple of the number of missiles being attacked, then the remainder are targeted on missiles chosen at random, the choices being independent among members of an attacking alliance (*12*). Party j has M_j missiles each carrying μ_j warheads, each of which has (in the view of country i) probability p_{ij} of destroying one of party i's missiles. (For notational convenience, $p_{jj} = 0$.) Let $[x]$ and $\langle x \rangle$ denote the integer and fractional parts of x, respectively. The probability of any one of i's missiles surviving an all-out attack by j is

$$(1 - p_{ij})^{[\mu_j M_j / M_i]} \; (1 - p_{ij} \langle \mu_j M_j / M_i \rangle) \tag{1}$$

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so the number of missiles that i can expect to survive the attack is M_i times this quantity. If i feels he needs an expected force γ_{ij} to deter j, he must adjust his force so that this number is at least γ_{ij}. Under attack by a coalition the survival probability is the product of those resulting from individual attacks. To deter a coalition, we assume that i deems the sum of the γ_{ij}'s for all members j of the coalition to be required. We call this sum γ_i. The conditions for mutual deterrence (*13*) are then

$$M_i \Pi_j \, (1 - p_{ij})^{[\mu_j M_j / M_i]} \cdot (1 - p_{ij} \langle \mu_j M_j / M_i \rangle) \geqq \gamma_i \qquad (2)$$

for all i. (Π_j denotes the product over all j.)

We observe that this system of inequalities always has (positive) solutions M_i for $p_{ij} < 1$. For any fixed set of positive $\overline{M}_i$'s, if they are multiplied by a postive number α, the product terms on the left remain unchanged but the factors $\alpha \overline{M}_i$ can be made arbitrarily large. Thus, by the choice of a sufficiently large α the inequalities can be satisfied (*14*). The existence of a mutual assured-destruction relationship is thus independent of the parameters and of the number of parties. Moreover, solutions exist for any desired ratio of weapons between the participants—no fixed parity between nations is necessary.

Although, theoretically, mutual deterrence can always exist, computations indicate that the size of the required retaliatory force grows so rapidly with the number of parties that it may be economically infeasible for more than two nations to maintain mutual deterrence.

We speak of parties being equal when we assume that the relevant parameters μ_i, p_{ij}, and γ_{ij} are the same for all $j \neq i$ as i varies over the parties. (If all parties are equal we write $\gamma_{ij} = \gamma$, $p_{ij} = p$, $\mu_i = \mu$ for $i \neq j$.) For n equal parties, the number of missiles required by each nation, $M(n)$, may be compared with the requirements in the corresponding two-party case, $M(2)$:

$$\frac{M(n)}{\gamma} = \frac{(n-1)}{(1-p)^{\mu(n-1)}} = (n-1)\left(\frac{M(2)}{\gamma}\right)^{n-1} \qquad (3)$$

If mutual deterrence between two equal parties requires each to maintain three times the retaliatory force expected to survive an attack, then the existence of a third equal party would expand the number of weapons each requires by a factor of 6. If each requires five times the expected retaliatory force in a two-party situation, then the addition of a third equal party would require a tenfold expansion of forces.

Computations indicate that the expansion of armaments required by the introduction of a third power is not critically dependent on the assumption of equality of the third power. For example, if the third power were to use more vulnerable missiles, the greater vulnerability must be compensated by greater numbers. This causes an expansion in armaments of the two equal powers comparable with that found for three equal powers. Unless specifically excepted, the equal-party case is the basis of computations in the rest of this article (*6, 15*).

Defense of Retaliatory Forces

The preceding section showed that the weaponry required for multilateral deterrence is critically dependent on the number of countries and the warhead-kill probabilities. On the other hand, changes in the desired deterrent levels (the γ_{ij}'s) only affect the forces linearly. For example, in the equal-party case with $p = 1/2$, $\mu = 3$, and $\gamma = 100$, we find that the force level required for each of two countries is 800 missiles. If each nation anticipates a technological advance in the accuracy of missiles, leading to $p = 3/4$, then the number of missiles required would increase to 6400. Thus, any mechanism that would reduce p from 3/4 to 1/2 would be worthwhile if its cost did not exceed that of 5600 missiles. For three equal parties, the case $p = 1/2$ would require 12,800 missiles per nation; this would become 819,200 if p were 3/4. In this case, a defense that would reduce p to 1/2 would be worthwhile if the cost did not exceed that of 806,400 missiles. These numbers are, of course, extraordinarily large. They are intended to emphasize the point that antidotal technology is indispensable if an offensive-technology race (increasing p) cannot be curtailed by other means.

(Clearly, the least expensive means of reducing the kill probability, p, would be for each nation to degrade the guidance systems of its own missiles; but mutual deterrence is based on mutual distrust. No country would trust the others to degrade their systems.)

The effect of a system designed to decrease p will depend on the particular method chosen. Not only do different systems have different initial costs, but some can be more provocative than others. In this section we explore three representative techniques—the "shell game" and "unambiguous" and "ambiguous" ABM systems. (Other methods for reducing the vulnerability of land-based missile systems must also be considered in evaluating actual forces—mobile systems and "superhardened" silos, for example.)

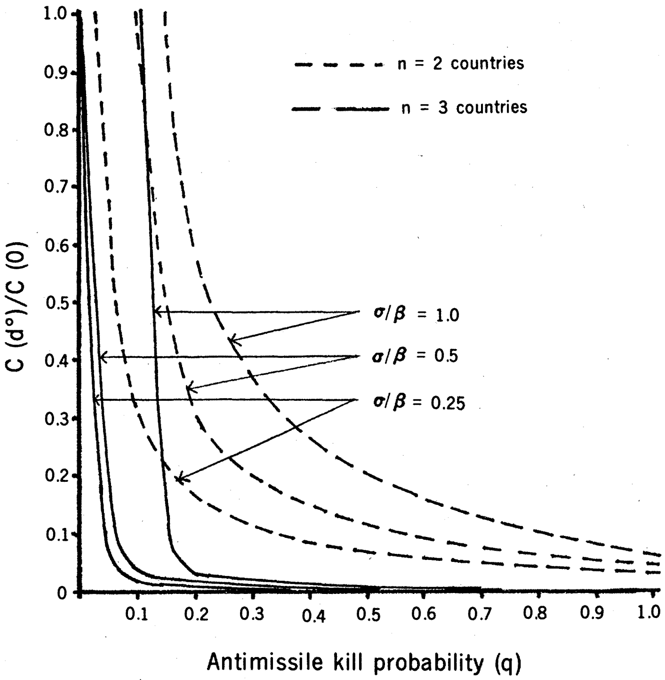
For the shell game each nation would have a relatively small stock of weapons, but each would build many additional silos and rotate missiles among them in the hope that an aggressor would not know which contained the missiles. In this way the effective kill probability could be reduced substantially.

To be conservative, assume that missiles are costless—a missile-bearing silo and an empty silo cost the same (β). The objective is to determine how many empty silos, S, and missile-bearing silos, M, should be purchased to minimize the total cost of maintaining deterrence among n nations. The mathematical problem is to determine $S^o \geqq 0$, and $M^o \geqq (n-1)\gamma$, so that $C(S) \equiv \beta(M + S)$ is minimized subject to

$$M(1-p)^{\left[\frac{(n-1)\mu M}{M+S}\right]} \left(1 - p\left\langle \frac{(n-1)\mu M}{M+S} \right\rangle\right) = (n-1)\,\gamma \qquad (4)$$

This computation yields surprising results. In particular, consider the reduction in cost effected by the shell game in the illustrative cases at the beginning of this section. Let $C(S)$ be the cost of maintaining deterrence with S empty silos, $M(S)$ the force level required by each nation purchasing S shells, and $p(S) = pM(S)/(M(S) + S)$ the probability that a missile chosen at random would be destroyed by a warhead. For the two-country case, with $\mu = 3$ and $p = 3/4$, $C(S^o)/C(0) \cong .14$ and $p(S^o) = 1/6$; for the corresponding three-country case, $C(S^o)/C(0) \cong .004$ and $p(S^o) = 1/12$.

Even with these cost reductions, the economic possibility of multination mutual deterrence postures may be doubtful. For the three-country case, for example, with $\gamma = 100$, $C(S^o) = 3600\ \beta$—a substantial expenditure. Moreover, the shell game is critically dependent on the intelligence apparatus of each nation. If the number of missile-bearing silos is small, and if an opponent can discern which ones are empty, a disarming first strike becomes possible. If, on the other



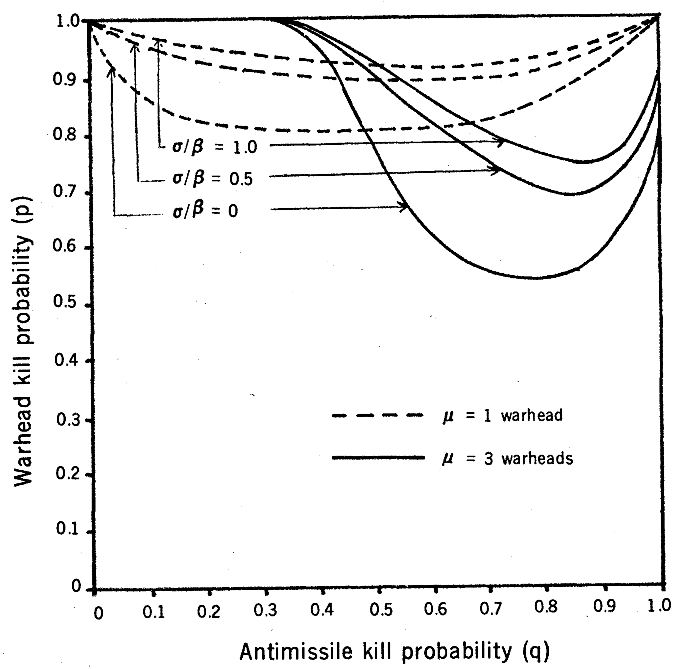


Fig. 1 (left). Cost reduction resulting from the use of unambiguous ABM to maintain multilateral deterrence for $n = 2$ and $n = 3$ countries. The ordinate $C(d^0)/C(0)$ is the ratio of the costs of defended and undefended postures. The quantity σ/β is the ratio of the unit cost of an antimissile to the unit cost of a missile. An attacking missile has $\mu = 3$ warheads, each with a kill probability $p = 3/4$. Fig. 2 (right). Indifference curves showing regions of preference for ambiguous ABM defense or for no defense for $\mu = 1$ and $\mu = 3$ warheads, where $n = 2$ countries. The area above each curve represents conditions under which the cost of maintaining mutual deterrence is less with ambiguous ABM than without it; the area below each curve represents conditions under which the cost of maintaining mutual deterrence is less without defense than with ambiguous ABM. The quantity σ/β is the ratio of the unit cost of an antimissile to the unit cost of a missile.

hand, the nations fear that their opponents are filling the empty silos, the strategy fails and the optimum policy is to abandon the shell game. (There would, of course, be a great temptation to fill the empty silos because of the relatively small incremental costs and the diverse nature of strategic objectives.)

Antiballistic missile systems are another method of reducing the probability, p. We distinguish two modes of antimissile defense, ambiguous and unambiguous. The ambiguous defense can defend the urban targets of a retaliatory second strike as well as defending the retaliatory force from a first strike. The unabiguous defense, on the other hand, can only defend the retaliatory force from a first strike. The effects of these two forms of antimissile defense were analyzed in terms of the economics of maintaining mutual deterrence between two equal parties. They were found to be drastically different. Very modestly effective unambiguous defense was found to produce substantial savings, even if the missiles were relatively costly. Indeed, since unambiguous defense is de-escalatory it was found in many cases to be economical to spend much more on defenses than on the retaliatory missiles themselves (7). On the other hand, ambiguous defense has a twofold effect. By rendering retaliatory forces less vulnerable it tends to reduce force requirements, but by reducing the effectiveness of the deterrent forces (hence increasing the γ_{ij}'s) it tends to increase the force requirements.

Ambiguous defense is economical only in cases of high vulnerability of the retaliatory force (7). Since an interceptor force is more effective against the reduced deterrent forces (the γ_{ij}'s) than against the full forces used in a counterforce first strike, ambiguous defense would appear to be basically escalatory. Over a wide range of parameters it was found that the escalatory effects of employing substantial ambiguous defense could increase the cost of maintaining mutual deterrence manyfold even if the defense were costless.

Our analysis of antimissile defense assumes a fixed probability of q of each of d antimissiles destroying one of the $(n-1)\mu M$ warheads. In the case of unambiguous defense, each country needs a force of

$$M = \frac{(n-1)\gamma}{\{1 - p(1-q)^{[r]}(1 - q\langle r\rangle)\}^{(n-1)\mu}} \quad (5)$$

where $r = d/(n-1)\mu M$ is the ratio of defensive missiles to attacking warheads. Unit costs β and σ are assigned to the M missiles and the d antimissiles, respectively, and the total cost is minimized by varying d (16). In evaluating ambiguous defense the factor γ in the above expression must be replaced by

$$\hat{\gamma} = \frac{\gamma}{(1-q)^{[d/\mu\hat{\gamma}]}(1 - q\langle d/\mu\hat{\gamma}\rangle)} \quad (6)$$

reflecting the reduction in effectiveness of the retaliatory force.

The exact analysis of these cases is rather complicated (7), so we present only some sample results here. Denote by $C(d^0)$ the cost of maintaining multilateral deterrence by defending missile sites with an unambiguous ABM force d^0 which minimizes the total cost, and by $C(0)$ the cost of the undefended posture. For the two-country case, with $\mu = 3$, $p = 3/4$, $\sigma = \beta$, and $q = 3/5$, computations reveal that $C(d^0)/C(0) \cong .16$. For the corresponding three-country case, $C(d^0)/C(0) \cong .006$. These results are comparable with the dramatic results obtained above for the shell game, but they do not have the same critical dependence on intelligence capabilities. Figure 1 illustrates typical cost savings parametrically.

For ambiguous ABM, cost reduction may be nonexistent. If $\mu = 1$, $n = 2$, $p = ½$, and $q = ⅘$, for instance, installing an ambiguous defense system consisting of one antimissile for each missile would triple the cost to each party of maintaining its deterrent even if the antimissiles were costless. Figure 2 depicts the regions of the *p-q* plane where ambiguous ABM defenses can result in cost reduction (*17*).

"Warfighting"—

A Conflicting Strategic Objective

Conservative military planning suggests the possibility that an attacker might not commit his entire force to a counterforce strike. The reserves could be used as a counterthreat to retaliation. Urban centers spared by the initial counterforce attack could then be held hostage against retaliation. A surviving counterforce capability which could reduce the threat of the attackers' reserved weapons would appear a reasonable answer to this strategy. This leads to consideration of multistage missile duels, frequently called "warfighting." We have found that if the attacked party could know rather accurately which weapons in the attackers' forces had been reserved, then even this policy (held multilaterally) would not lead to an unlimited arms race. Failing a means for providing such knowledge, however, an unlimited arms race could result. To adopt the second-strike counterforce policy multilaterally and avoid unlimited escalation, it would be necessary for all parties to maintain information systems that are capable of telling reliably which weapons an attacker had reserved.

In the case of two equal parties, suppose each party estimates that its opponent would reserve h weapons and that $\bar{h}$ additional surviving weapons would suffice to reduce the h reserved missiles to x in expectation. Without much loss in generality we can assume that $\mu h < M$, that is, the reserved warheads do not exceed the total number of missiles. In that case, if the locations of the h reserved missiles can be discerned, we find that the number of missiles each party needs is increased by

$$\Delta M = \frac{\bar{h}}{(1-p)^{\mu}} - \frac{\mu p h}{(1-p)} \qquad (7)$$

For example, if $p = ¾$, $\mu = 3$, $h = 100$, and $x = 10$, we must have $\bar{h} = 60$ and $\Delta M = 2940$.

On the other hand, if it is not known which of the M missiles have been reserved, no solution to the problem exists if x is too small. For example, for $\mu = 3$ and $p = ¾$, if each party anticipates the other would reserve 20 percent of its force, no solution exists if $x/h < .9$ (*8*).

Dynamics and Uncertainty

Although mutual deterrence is stable in a static situation in which each party knows exactly the size of the others' forces, one might suspect that an unlimited arms race could result from each party overestimating the others' forces or from each extrapolating current rates of deployment to estimate the future balance of forces. We have found, however, that for the case of two (not necessarily equal) parties this cannot happen. (It appears that similar conclusions hold also for more than two parties, but we have not pursued the analysis.) An arms race that occurs in the mutual pursuit of deterrent postures —even when it is based on erroneous data and on projections of current deployment rates—will eventually be damped out by the diminishing threat to those postures posed by additional weapons.

Consider two (not necessarily equal) parties each extrapolating the other's rate of deployment to adjust its own deployment rate. The objective of each is always to assure that a deterrent posture will be attained after a specified lapse of time (possibly different for each party). The first party, for instance, projects at each time t that at time $t + \tau_1$ the armaments of each side will be

$$\left.\begin{aligned}\hat{M}_1(t+\tau_1) &= M_1(t) + \tau_1\dot{M}_1(t)\\ \hat{M}_2(t+\tau_1) &= M_2(t) + \tau_1\dot{M}_2(t)\end{aligned}\right\} \qquad (8)$$

and adjusts the rate of growth $\dot{M}_1$ so that deterrence will be assured. (Suitable inequality constraints $\hat{M}_1 \geqq \gamma_1$ and $\hat{M}_2 \geqq \gamma_2$ should also be imposed on the prediction, of course. One may also impose the restriction that disarmament is not allowed.) The second party behaves similarly, using a time lag τ_2. This yields a system of nonlinear differential equations which we have shown to have bounded solutions (*7*).

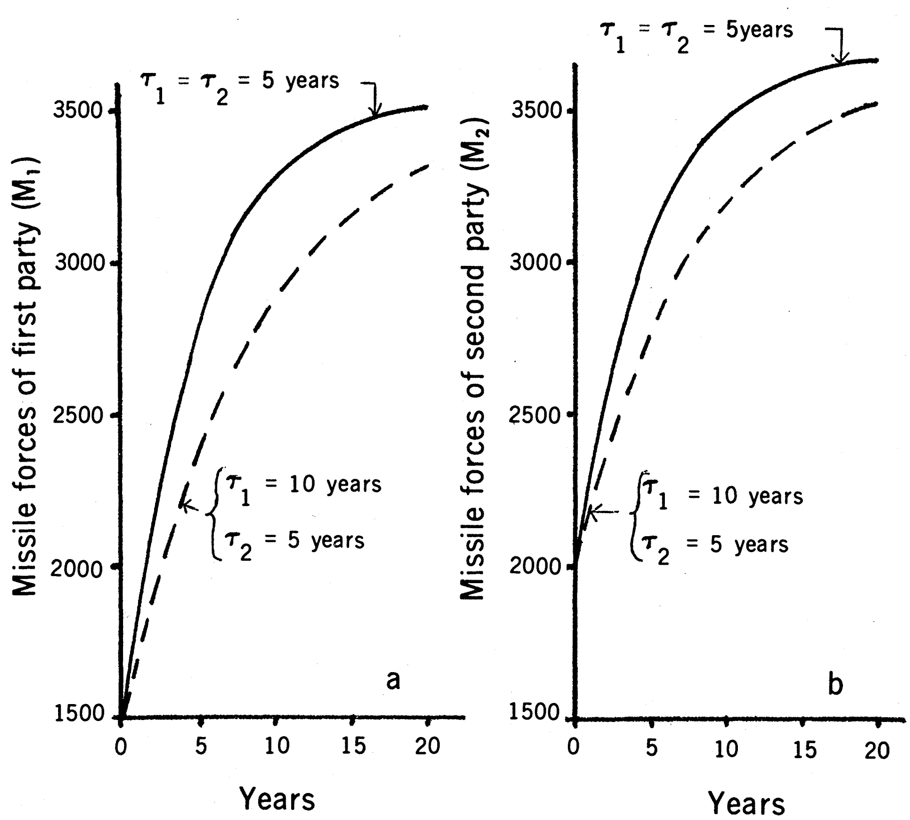


Fig. 3. Missile growth comparisons, illustrating two phenomena in a two-party case. (a) Stable dynamics of force growth (note that the curves asymptotically approach static solutions); (b) effects of a difference in projection-time intervals (compare solid with broken curves). The deterrent force for the first party, γ_1, is 400 and for the second party, γ_2, is 500. The respective initial force sizes are $M_1(0) = 1500$ and $M_2(0) = 2000$. The attacking missiles have $\mu = 3$ warheads, each with a kill probability $p = ½$.

Some typical graphs illustrating the types of solutions obtained are given in Fig. 3.

Alternate Forces (Bombers and Submarines)

Although our investigations have concentrated on a mathematical model for missile forces, we have given some consideration to other types of forces to obtain conditions under which one type of force would be more economical than another (*18*). We have studied the economy of maintaining mixed forces of bombers and missiles with a fraction of the bombers constantly airborne and assumed invulnerable to a first strike (but not, of course, to bomber defenses). The most economical forces were usually unmixed—all missiles or all bombers—and we established inequality conditions involving costs and technical parameters under which bombers would be more economical. An important point is that the airborne fraction of the bomber force is assumed invulnerable to a first strike. This makes the requirements for maintaining mutual deterrence much less sensitive to the number of parties—bombers are more economical, relatively, when more parties are involved.

The inequality that determines when a force of bombers is more economical for each of two parties than a force of missiles on each side is illustrated in Fig. 4. In that figure it is assumed that each missile carries three warheads yielding 250 kilotons each and that each warhead has probability ¾ of destroying a missile at which it is targeted. An example may serve to put Fig. 4 in perspective. Assume that each bomber carries a payload equivalent to ten 1-megaton weapons. If only 5 percent of the bomber force were expected to deliver its weapons, then the critical cost ratio, from Fig. 4, is about 26. This means that if such a force could be maintained at an average unit cost (cost per bomber) of no more than 26 times the unit cost (cost per missile) of the missile force then the bomber force would be more economical. If this example is extended to the three-party case, however, bombers are more economical than missiles if the unit cost of a bomber is not more than 1664 times that of a missile. (This striking effect results from the fact that the number of bombers required depends only linearly on the number of parties while the number of missiles involves both linear and exponential factors.)

Currently, submarine-based missile forces are widely regarded as invulnerable. The principal reason for the invulnerability of submarines, however, is the difficulty of locating them. As with missiles and bombers, we have been able to formulate inequalities relating costs, technical parameters, and the random error of an attacker in locating a submarine.

Basically, the model we have used for a counterforce strike against missiles can be applied to submarine forces where each is subject to attack by the missiles of the other. For this purpose, we use M as the number of submarines and μ as the number of warheads aboard each submarine. The probability, p, of a submarine being destroyed by a warhead and the dependence of p on the random error in locating the submarine are the factors of greatest interest. It appears that the point at which submarines and land-based missiles become economically competitive is reached when the submarines can be localized within tens of miles rather than hundreds—if the probable error in locating a submarine is of the order of miles then land-based systems would be more economical, while if it is of the order of hundreds of miles submarines would be more economical (*8*). With accurate vulnerability data, these estimates can be improved.

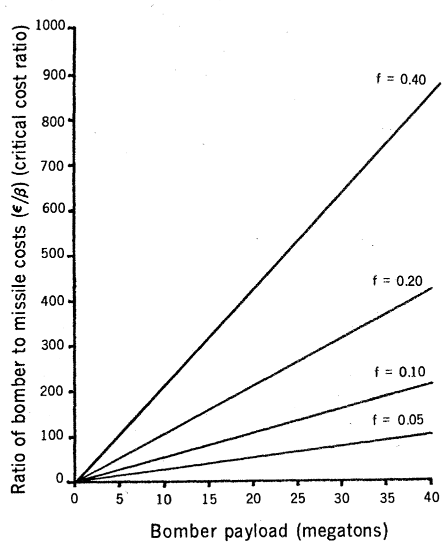


Fig. 4. Indifference lines showing regions of preference for bombers or for missiles in maintaining mutual deterrence between $n = 2$ parties. For each line, bombers are more economical than missiles if the cost ratio ϵ/β lies below the line (critical cost ratio); missiles are more economical than bombers if ϵ/β lies above the line; ϵ/β is the ratio of the unit cost of a bomber to the unit cost of a missile. Each missile carries three 250-kiloton warheads, each with a kill probability $p = 3/4$ of destroying a targeted missile. The quantity f is the expected fraction of the bomber force that can deliver its weapons.

Summary

As missile forces grow in size, the incremental threat of a disarming first strike diminishes. This furnishes a strong stabilizing mechanism in an arms race based on an attempt by each party to maintain an assured-destruction posture toward the others. Despite this, the number of missiles required by each party grows rapidly as the number of parties increases—so rapidly that the economic feasibility of more than two powers maintaining such postures independently seems questionable. Moreover, this number depends critically on the vulnerability of the missiles, the effects of vulnerability being amplified by mutual interactions. Some examples of "antidotal" technology to counter the effects of increased vulnerability are considered. A shell game using empty silos could produce dramatic savings, but it would be crucially dependent on each party's intelligence apparatus. "Ambiguous" ABM systems capable of defending either missiles or urban centers are found to be undesirable in most cases (even if they are costless), but "unambiguous" ABM systems capable only of protecting missile forces can avoid escalation and hence be less expensive than increasing the missile force. (It may be economical to spend several times the cost of a missile in defending it.)

Although pure assured-destruction policies produce stability—albeit at high armament levels—a policy that is also directed at "warfighting capability" can lead to an unlimited arms race. Whether or not such a policy does produce an unlimited arms race depends on the intelligence systems available and the first-strike capability of the weapons used to implement it.

Inequalities for the relative economy of bomber systems and missile systems have been established. The use of bombers becomes more advantageous relative to the use of missiles as the number of nations maintaining deterrent postures increases. Similar inequalities measure the relative economy of missiles and submarines in terms of the probable error with which a submarine can be located by a potential attacker.

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3. The relevance of mathematical modeling to "real-world" decision-making is often tenuous. No attempt will be made in this article to discuss the relevance or implications of the methodology used herein.
4. Senate Committee on Appropriations and Senate Committee on Armed Service, *Deparmtent of Defense Appropriations* (Government Printing Office, Washington, D.C., 1965), part 1.
5. The term "deterrence" has broad ramifications. This article deals with a restricted usage of the term, namely, for policies of "assured destruction" or "warfighting," or both. Other conceivable usages of deterrence are not discussed in this article, for example, for a first-strike capability or an extensive civil defense posture.
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12. It is well known that the optimum attack deployment is to target all missile sites as uniformly as possible [for example, see Afheldt and Sonntag (*11*)].
13. Reliability and other statistical effects can be handled by appropriate adjustment of the parameters. For example, see (*7*) for a discussion of the increased force requirements to maintain mutual deterrence at any confidence level.
14. It suffices to choose α such that

$$\alpha \geqq \max_i \left\{ \gamma_i \left(\bar{M}_i \Pi_j (1 - p_{ij})^{[\mu_j \bar{M}_j / \bar{M}_i]} \cdot (1 - p_{ij} \langle \mu_j \bar{M}_j / \bar{M}_i \rangle) \right)^{-1} \right\}$$

15. In the case of two equal parties maintaining mutual deterrence with the introduction of a weaker third party not attempting to maintain such a relationship, the number of additional missiles required by each of the major powers is proportional to the third power's forces.
16. The effects of countermeasures to ABM, such as radar attacks or decoys, should be treated by game-theoretic methods. However, these effects can be reflected by appropriate adjustment of the intercept probability q. Costs β and σ are overall program costs (in current dollars) of the total weapons systems, if coincident program time-frames for both systems and sizable procurements are assumed.
17. These results are reinforced by the effects of ABM countermeasures.
18. A complete analysis of the triad of forces—missiles, bombers, and submarines—leading to the optimal mix involves nonlinear mixed-integer optimization problems. The solution of these problems can involve detailed simulations and were not treated.
19. The views expressed in this article are the authors' and do not necessarily reflect the views of the United States government. We are grateful to Drs. Lee R. Abramson and Felix E. Ginsberg and Col. Alfred C. Herrera for their critical reading of this article. We also thank George A. Lincoln for his critical review and thoughtful observations.

Population Density and Pathology: What Are the Relations for Man?

Evidence from one city suggests that high population density may be linked to "pathological" behavior.

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Studies of various animal populations suggest that high levels of population density frequently produce "pathological" behavior. The results of these studies, coupled with an increased concern about high rates of growth in the human population, have led to speculations about the implications of high levels of density for human populations. We begin this article with a review of some of these studies, noting the implications of possible animal-human similarities, and then take the animal studies as a serious model for human populations and devise a test case.

In 1962, Calhoun published an article detailing the ways in which overcrowding affects the behavior of rats. In his experiment, he gave the rats sufficient food and water, but the density of the population was substantially higher than it is in the rats' natural habitat. Calhoun observed the following "pathological behaviors" under these conditions: increased mortality, especially among the very young; lowered fertility rates; neglect of the young by their mothers; overly aggressive and conflict-oriented behavior; almost total withdrawal from the community (the "somnambulists"); and sexual aberrations and other "psychotic" behavior (*1*). It should be noted that these aberrations were much more common in the central pens, where the rats *voluntarily* congregated.

In recent years it has become clear that rats are not alone in being adversely affected by high density (*2, 3*). A study by Susiyama (*4*) of wild monkeys indicated that high density led to a general breakdown in the monkeys' social order and resulted in extremely aggressive behavior, hypersexuality, the killing of young, and so on. High density appears to cause death in hares (*5*) and shrews (*6*). Morris (*7*) has found that high density causes homosexuality in fish. Probably the most frequently demonstrated effect of density is in the area of natality. For example, under conditions of high density the clutch size of the great tit decreases (*8*), as does the number of young carried by shrews (*6*). It appears likely that high density reduces the fertility of elephants (*9*). Female house mice abort if they smell a strange male mouse (*10*), as do shrews (*11*).

In sum, high population density appears to have a serious inhibiting effect on many animals. It must be noted, however, that the effect of density is not uniform among different species; different species react to density in different ways. It is probably inevitable that increasing knowledge of the effect of density on animal behavior leads to concern about the effect density may have on human behavior. By now, the idea that density has, or at least may have, serious consequences for man appears to have fairly wide acceptance. Such acceptance is obvious in much popular writing (*12*) as well as in work specifically aimed at behavioral scientists (*13*).

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