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The System of Planetary Masses

New results show that Pluto's mass cannot be determined reliably from existing data.

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Space exploration has quickened interest in the determination of planetary masses. Why should anyone be interested? The reasons are manyfold. In combination with the radius, the mass yields the average density—perhaps the most fundamental planetary characteristic because of its many implications for the formation, chemical composition, and evolution of the planetary system. Mass values are also crucial for precise tests of gravitational theories that involve solar system dynamics. As an example on a more practical level, the mass of a planet can be essential in planning certain spacecraft missions: to play celestial billiards successfully by the use of a gravitational assist from one planet to reach another requires precise mass data for the usual fuel-limited situation. Present plans call for use of this technique on "grand tour" missions to the major planets later in this decade and on the 1973 Venus-Mercury flyby mission. The latter may even be enhanced by the use of Mercury's gravity field to insure a reencounter with Mercury a half year later (*1*).

For some purposes, of course, extreme precision in mass determination is not required. Yet for the outermost planet, Pluto, even the first significant figure is in doubt. The situation for the innermost, Mercury, is only slightly better.

How have planetary masses been estimated? The technique is centuries old, but some precision variants have become available only recently. The basic method involves measurement of the effects produced by the planet whose mass is to be estimated on the trajectory of an orbiting body. The affected body can be:

1) A spacecraft such as a Mariner, or an asteroid such as Eros, that makes a close approach to a planet and undergoes a noticeable deflection of its trajectory.

2) A satellite for which measurements of its period and mean distance from the planet, through Kepler's third law, yield estimates of the parent planet's mass. Difficulties arise especially for the outermost planets because the fractional error in the determination of mean distance becomes quite large.

3) Another planet or an asteroid in a distant orbit. Here three cases can be distinguished: (i) a resonance effect, where, for example, an asteroid has a mean motion commensurable with Jupiter's, and the latter's relevant perturbations on the former increase monotonically with time; (ii) a long-period effect such as "the great inequality" in the mutual perturbations of Jupiter and Saturn; and (iii) the short-period perturbations produced by one planet on another. This last was especially useful for classical estimates of the inner planet masses but has been largely superseded by the far more precise results available from Mariner flybys. Another method, applicable only for the mass of the earth-plus-moon, involves both items 1 and 2 and is discussed below.

A summary of "accepted" and recent determinations of inverse planetary masses is given in Table 1 with appropriate references. The unit, conventional in astronomy, is the inverse solar mass. Our results, to be described later, are not completely independent since in some cases the same data were utilized either in whole or in part.

We combined, for the first time, modern radar data obtained from observations of the inner planets with almost all existing optical observations of the sun and planets made between 1750 and 1970. We also included the available observations of two special asteroids, Eros and Icarus. The total number of separate measurements involved is about 300,000. The radar data are from M.I.T.'s Millstone Hill and Haystack facilities. The optical data, consisting of right ascensions and declinations, were culled mostly from the original observatory reports and transformed into machine-readable form in a common format—a chore that occupied about 6 years of part-time effort (*2*). Not all data known to exist were obtained, despite requests directed to the relevant observatories; the uncovering of these we leave to more enterprising archeologists.

Our results for the planetary masses are in reasonable accord with prior estimates with one notable exception: for Pluto, the data seem to allow only an upper bound of about 5×10^{-7} solar masses to be set. There is thus insufficient basis for the widespread conclusion that Pluto's average density is greater than, or even comparable to, the earth's.

Data Analysis Procedures

Our scheme to deduce planetary masses from these data can be described as follows. We performed several linearized least squares analyses based on

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successively larger portions of the data to obtain better estimates of the initial conditions and some of the relevant astronomical constants. The masses were generally fixed, but at values not differing appreciably from the last entries in Table 1. This preliminary solution gave a sufficiently good representation of the data to indicate that one additional differential correction would achieve de facto convergence in the final least squares analysis.

With the new values for initial conditions and constants, the Encke-type equations of motion of general relativity, expressed in harmonic coordinates, and the corresponding variational equations for all relevant parameters were integrated numerically from 1970 back to 1750 (*3*) for each planet and asteroid individually. The precision in these integrations was controlled by the equations of motion, and the results were expressed as a function of coordinate time in a Cartesian coordinate system centered at the sun and with axis directions determined in the usual manner by the mean equinox and equator of 1950.0. The orbit of the moon, which both determines the motion of the earth about the earth-moon barycenter and perturbs the motion of the latter, was adapted from standard sources (*4*). The "perturbing planet" coordinates were obtained from a nine-body, double precision, Cowell-type integration (*3*) and utilized initial conditions and constants that were generated in the preliminary analysis.

The variational equations for each body included those for the six initial conditions and, for each of the inner planets and asteroids, also included those for the masses of all planets out through Saturn. The partial derivatives with respect to the remaining three planets were not considered since the inner planet and asteroid data contribute little to the estimates of the masses of the three outermost planets. Similarly, for each outer planet, partial derivatives with respect to masses were integrated only for outer planet masses.

The results for the single-body integrations of the equations of motion of the planets agreed more than well enough for our purposes with the nine-body integration. For Mercury, the fastest moving planet, agreement between the two integrations, after corrections were applied for the slight differences in initial conditions and constants, was found to at least the ninth significant figure throughout the 220-year interval. A similar comparison yielded 8 place agreement over the almost 80-year interval spanned by the Eros data. For Icarus (eccentricity $\approx$ 0.83) many different checks were made that established the position accuracy as more than sufficient. All of the individual computations were performed with 16 decimal place accuracy.

From the planetary, lunar, and asteroidal positions and velocities, the provisional values for the relevant constants, the standard expressions for the precession and nutation matrices (*5*), and published values for the differences between universal and coordinate time (*6*), we calculated the theoretical value corresponding to each observation. The residuals, observed minus computed values (O − C), disclosed that about 1 percent of the meridian circle measurements had been incorrectly transcribed (*7*). The deletion of these data had no significant effect on our results.

The observations were divided into about 50 separate series depending on the data type, the observatory, and the time period involved. With only three exceptions (*8*), no more than a 30-year span of data was placed in a single series and no data from different observatories were placed in the same series. To account partially for the use of different star catalogs by different optical observatories and for possible distortions in these reference systems, each series of optical data was parametrized as follows: Two parameters were introduced to describe possible small differences in the inclinations of the equatorial reference planes and in the equinox positions, and one parameter was introduced to allow for a bias in the declination observations. The division of data into series according to time period was designed to prevent the accumulation of errors that would accompany the use of incorrect values for stellar proper motions in a given reference catalog. (In principle, the individual observations should have been re-reduced with the use of modern values for the positions and proper motions of the relevant reference stars. But even less grandiose procedures, such as attempts to determine the systematic differences between star catalogs for each small region of the sky separately, were beyond the scope of our study.) For the inner planets, allowance was also made for possible errors in reduction from the center of light to the center of mass. Thus, possible phase corrections were introduced in the form of a different Fourier series for each inner planet, for each series of optical data.

The O-C values, together with the assumed standard errors and the corresponding partials for each observation with respect to each parameter, were used to form the normal equations, which were then stored on magnetic tape for each observation series separately, for each planet separately, and also in toto. Such a procedure gives enormous flexibility and efficiency in carrying out computer experiments. Any subset of the total of more than 500 parameters—predominantly from the equator, equinox, declination, and phase-bias models—can be estimated simultaneously, with the data selected from any appropriate subset among the observed planets and observation series. The initial conditions for each observed body as well as those for the earth are always included in the parameter subset. A typical solution in which 50 parameters are estimated along with the standard errors and correlations of the estimates requires less than 30 seconds of computer time on the IBM 360

Table 1. Summary of recent planetary mass determinations.

Inverse mass* ($M_\odot^{-1}$)	Quoted error ($M_\odot^{-1}$)	References
	Mercury	
6,000,000		(*38*)†
6,020,000	50,000	(*11*)
5,983,000	25,000	(*39*)
	Venus	
408,000		(*38*)
408,250	120	(*11*)
408,522	1	(*12*)
	Earth + moon	
329,390		(*38*)
328,900	60	(*11*)
328,895	20‡	(*40*)
328,900.1	0.4	(*39*)
	Mars	
3,093,500		(*38*)
3,111,200	9,000	(*11*)
3,098,708	9	(*19*)
	Jupiter	
1,047.355		(*38*)
1,047.356	0.03‡	(*41*)
	Saturn	
3,501.6		(*38*)
3,498.6	1‡	(*22*)
	Uranus	
22,869		(*38*)
22,934	9	(*42*)
22,692	33	(*43*)
	Neptune	
19,314		(*38*)
19,296	28	(*44*)
19,349	31	(*45*)
	Pluto	
360,000		(*38*)
400,000	59,000	(*46*)
1,812,000	50,000	(*26*)

* The units are inverse solar masses. † For comparison, the first value given for each planet is the IAU accepted value. ‡Represents half-width of spread in results quoted in references and for most cases is far larger than the mean errors given by the individual analyses. The inverse mass shown is approximately the average of the values obtained in the separate studies.

model 67 computer. The only time-consuming part is the matrix inversion, which scales in computer time approximately as the cube of the matrix dimension.

The possiblities for computer experiments are nearly innumerable. One can study mass estimates from radar data or from optical data, from post-1900 optical data or from pre-1900 optical data, from one observatory's data or from another's, and so on. Clearly one must be selective. Our selections were governed mainly by a priori judgments of the sensitivity of the estimates to the data and parameter selections.

But why does one need to perform computer experiments? Why doesn't a single "grand" solution suffice? These are obvious questions; the answers involve the fact that the measurement errors do not conform to our model of Gaussian statistics with zero means and no correlations. Or, put another way, our theoretical model for the actual observable is incomplete. The estimates and formal errors obtained from a grand solution are therefore likely to be seriously misleading. The computer experiments, involving as they do different subsets of the data and of the parameters, provide an ad hoc means of studying the sensitivity of the conclusions to the assumptions implicit in our model. Of course, the proper allowance to make for these modeling errors in arriving at reliable values for estimate uncertainties is nigh impossible to determine precisely. Our choices are based primarily on (i) the reliability of the values obtained for other similar parameters for which independent and more accurate data are available for comparison; (ii) the aforementioned sensitivity of our results to variations in the data and parameter sets; and (iii) an analysis of the postfit residuals. Thus, while essential, analysis of residuals is in practice not sufficient (*9*).

Estimates of Planetary Masses

Having described our technique for data analysis—less briefly than the reader might have anticipated but far more briefly than a complete description would have required—we proceed to a discussion of the results. In Table 2 are given our best estimates for the inverse masses and their corresponding uncertainties. An attempt was made to be conservative, yet realistic, in the determination of appropriate values for the uncertainties. We discuss each entry

Table 2. New planetary mass estimates.

Planet	Inverse mass ($M_{\odot}^{-1}$)	Uncertainty ($M_{\odot}^{-1}$)	Density (g/cm^3)
Mercury	6,025,000	15,000	5.42
Venus*	408,520	100	5.25
Earth + moon†	328,900‡	20	5.51‖
	328,900§	1	
Mars*	3,098,000	4,000	3.96
Jupiter¶	1,047.4	0.1	1.33
Saturn	3,498.5	0.5	0.68
Uranus	22,900	200	1.60
Neptune	19,400	100	1.65
Pluto	4,000,000	2,000,000	3 (assumed)

* The spacecraft tracking data mass results are consistent with these, but far superior [see appropriate entries in Table 1 and (*13*)]. † The earth-moon mass ratio estimate is 81.301 ± 0.002; see also (*18*). ‡ Method 1 (see text). § Method 2 (see text). ‖ Earth alone. ¶ The results from analyses of minor planet orbits (*41*) give a somewhat superior result.

in turn and compare it with the corresponding entries in Table 1. For Mercury, our result $M_{☿}^{-1} = 6{,}025{,}000 \pm 15{,}000$ inverse solar masses, is determined primarily by the earth-Venus radar data through the short-period perturbations introduced by Mercury in the orbit of Venus. The formal standard error in this inverse mass determination is only about 3,000. The true uncertainty is substantially higher because of the possible systematic errors in the interpretation of the radar echo delays. Such systematic errors are caused by surface-height variations and inhomogeneities in the radar scattering law for Venus (*10*). The value given in Table 2 for Mercury's mass and its uncertainty were based on a large number of computer experiments in which various subsets of the data were deleted and various different combinations of parameters estimated, including low-order coefficients of a harmonic expansion of Venus' surface topography. The values obtained from the optical data of the inner planets alone, although not so accurate [typically $M_{☿}^{-1} \approx 6{,}000{,}000 \pm 65{,}000$ (formal standard error)], are in good agreement with the radar result and with Newcomb's 1895 value (first entry in Table 1). The close agreement between the results we obtained with different theoretical models, disjoint data sets, and different data types lends credence to the estimated uncertainty. Using the radar data value of 2439 ± 1 km for Mercury's average equatorial radius, we conclude that Mercury's average density is within 1 percent of that of the earth (*11*). Thus, in view of Mercury's smaller size and correspondingly lower central pressure, it likely has a bulk composition substantially richer in the heavier elements than the earth's.

The inverse mass of Venus, for which we obtained 408,520 ± 25 (formal standard error) from radar data alone, is in good agreement with the superior Mariner 5 value (*12*, *13*). Our Table 2 determination is not a serious competitor to the Mariner value; we consider the comparison useful primarily as an indication of the reliability of our results for, say, Mercury, for which an independent and more accurate estimate is not available.

The optical data are relatively ineffective for the determination of Venus' mass, the accuracy obtainable being only several parts per thousand. The density of Venus, determined from its mass and average equatorial radius 6050.0 ± 0.5 km (*14*), is only 4 percent less than the earth's (*11*), thus enhancing Venus' somewhat tarnished reputation as our sister planet.

The mass of the earth-moon system presents a special case in that we have two different methods for estimation. The first is based primarily on the observations of Eros, which makes periodic close approaches to the earth. The formal standard error in terms of inverse solar masses is less than 3. However, computer experiments of the type described above show that values between 328,885 and 328,915 (*15*) could be obtained when using all of the Eros data. With the major portion—obtained during the 1930–31 opposition—deleted, values as high as 328,950 were found. A realistic estimate of the uncertainty is therefore about 20 as shown.

The Eros result was first used to estimate the astronomical unit of length in terms of a terrestrial unit through the theoretical relation (*16*):

$$(M_{\oplus} + M_{☾})^{-1} = A^3 F_1 k^2 [R_e^2 g_e (1 + \mu)]^{-1}$$

where $M_{\oplus}$ is the mass of the earth and $M_{☾}$ the mass of the moon, A denotes the astronomical unit, F_1 a constant close to unity, k the Gaussian constant, R_e the earth's mean equatorial radius, g_e the earth's mean equatorial surface gravity, and μ the moon-earth mass ratio. By turning this procedure around and using our radar result for the astronomical unit (accurate to about 1 part in 10^8 in light seconds), we obtain the second earth-plus-moon entry in Table 2: $(M_{\oplus} + M_{☾})^{-1} = 328{,}900 \pm 1$. The uncertainty is determined by the estimated inaccuracies, several parts in 10^6, in the values for R_e and g_e, expressed in units consistent with those used for A and k. The contribution of A and μ to the uncertainty are negligible.

The earth-moon mass ratio (μ^{-1}) shown in Table 2 was determined from the radar data through the effect on echo delays of the monthly motion of the earth about the center of mass of the earth-moon system. The distance to the moon is known to sufficient precision from lunar radar measurements (*17*). Our value for μ^{-1} is consistent with the slightly more accurate determination made from spacecraft tracking data, with the use of the same principle of barycentric rotation (*18*).

The results for Mars, like those for Venus, are gratifying; the agreement with the far more accurate Mariner 4 determination (*19, 13*) is better than could be expected. The relevant computer experiments disclosed, for example, that from radar data alone $M_{♂}^{-1} = 3{,}100{,}000 \pm 800$, whereas from the inner planet optical measurements $M_{♂}^{-1} = 3{,}096{,}500 \pm 1{,}300$ (formal standard errors). The mass of Mars, coupled with the radar value 3394 ± 6 km (*20*) for the mean equatorial radius and the dynamical value for the oblateness (*21*), yields an average density intermediate between the values for the earth and the moon (see Table 2).

Jupiter's inverse mass is slightly higher than the values in Table 1, which rely heavily on determinations based on resonance effects on the orbits of minor planets. Over 30 computer experiments were performed estimating the mass of Jupiter from all the outer planet data, from observations of each of the other outer planets separately, from the inner planet data, and with various auxiliary parameters alternately deleted from the estimation procedure. Almost invariably the inverse mass came out above 1047.30, ranging from that value to 1047.64. The formal standard error for the total data set was 0.03.

Interestingly, the radar data alone yielded a value of 1047.46 ± 0.06 for the inverse mass of Jupiter. The precise

Table 3. Potential accuracy of planetary mass determinations from pulsar observations of barycentric motion.

Planet	Amplitude of barycentric motion (light sec)	Potential uncertainty limit on inverse mass ($M_{\odot}^{-1}$)
Mercury	0.00003	2,000,000
Venus	0.0009	5,000
Earth + moon	0.0017	2,000
Mars	0.00025	100,000
Jupiter	25	0.0004
Saturn	13	0.02
Uranus	0.45	0.6
Neptune	0.8	0.3
Pluto*	0.004	9,000

* Based on inverse mass of 4,000,000 and distance of 30 A.U.

Mars data obtained at Haystack during the 1967 and 1969 oppositions are primarily responsible for the small standard error. Thus, modern inner planet radar data spanning only 2 years can almost compete with several centuries of outer planet meridian circle observations in the estimation of Jupiter's mass. The earth-Mars radar and radio tracking data can even be used to estimate the longitude of Jupiter with considerable precision, almost to 1 second of arc.

With Saturn we found the inverse situation. Approximately 30 computer experiments—analogous to those for Jupiter's mass—yielded values well below the Table 1 entry of 3501.6, the spread being between 3498.1 and 3499.3 for the more accurate determinations with the formal standard error 0.15 for the ensemble of data. Our values are most influenced by the Jupiter observations and are consistent with inferences (*22*) made from a study of the orbits of minor planets, or dying comets, that are influenced by Saturn. The inner planet radar data alone yielded a reasonable value for Saturn's inverse mass: 3499 ± 3. As for Jupiter, our results for Saturn's mass from the various computer experiments varied over intervals a few times the formal standard error. For this reason the estimated uncertainty in Table 2 is taken to be about three times the formal standard error obtained from the analysis of the total data set.

The inverse mass estimates for Uranus and Neptune adhered less well to the trend found for Jupiter and Saturn. The 20-odd computer experiments that involved these planets yielded spreads of more than five times the formal standard errors, which were 20 and 10 for the total data set for Uranus and Neptune, respectively, and were determined primarily by the Saturn and Uranus data, respectively. Our "best" estimates, given in Table 2, are compatible with some shown in Table 1; however, we are skeptical about the reality of most of the uncertainties in Table 1. Although the error estimates from the classical determinations depend on satellite observations as well as on planetary perturbations, the discordance among individual results (*23*) only enforces our skepticism. The average density for Neptune (≈ 1.65 g/cm^3) is substantially different from its standard value (*21*) because of a recent 15 percent downward revision of the estimate of its radius (*24*). Uranus' average density is about 1.60 g/cm^3.

Pluto, because of its small size and great distance from the sun, presents a severe challenge to dynamical astronomy in regard to the estimation of its mass. Moreover, due to the unusual Pluto-Neptune orbit resonance (*25*), Pluto never approaches Neptune more closely than 18 astronomical units. To compound the problem there is uncertainty regarding the influence of a possible comet belt beyond the orbit of Pluto. The classical determination of Pluto's mass which led to an unbelievably large value (>50 g/cm^3) for its average density, yielded the value given in the first appropriate row in Table 1. This value has been suspect for many years for many reasons (*23*). More recently, the study undertaken at the United States Naval Observatory (USNO) (*26*) yielded the value for the inverse mass given in the third row, $1{,}812{,}000 \pm 50{,}000$. The best available radius determination (*27*) then allows an average density of about 8 g/cm^3 to be deduced for Pluto. We find, however, that the data don't seem to warrant more than the placement of a lower bound of about 2,000,000 on the inverse mass. When all of the outer planet data were analyzed (*28*) and solutions obtained only for the initial conditions and masses of these planets, we found for Pluto's inverse mass $1{,}500{,}000 \pm 210{,}000$ (formal standard error); when the equator, equinox, and declination biases were added to the parameter set, the corresponding value was $-4{,}200{,}000 \pm 260{,}000$! Further, when all the data were analyzed simultaneously or broken into single-planet subsets, solutions for some reasonable parameter sets also yielded negative values for Pluto's mass. These differential corrections, being relatively large, may not correspond to the converged least squares solutions. But iterating the linearlized solutions to insure that the true least squares solutions were obtained would have entailed more hours of computer time than were available to us. For the same reason, we were unable to obtain the residuals predicted by our differential correction results. However, the postfit residuals from our base solution, described above, exhibited no obvious systematic trends above the measurement noise level. The extreme sensitivity of the differential correction for Pluto's mass to the choice of data and parameter sets indicates the difficulty in extracting a meaningful value for it from the existing outer planet observations (*29*).

The fact that the sum of squares of the residuals is decreased by the addition of Pluto's mass to the parameter set is, in itself, not significant. Any additional parameter, whether or not it has physical meaning, may reduce the residuals. In Pluto's case, the conspicuous asymmetry about the minimum on the graph of the sum of squares of residuals plotted against planet mass obtained at the USNO (*26*) indicates that the value at the minimum may represent more of an upper bound on the mass than a reliable determination. At the very least, the uncertainty quoted in Table 1 strikes us as substantially undervalued in view of the far larger formal standard errors obtained from our analyses. These formal errors are determined by the mass (*30*), by the standard deviations associated with the optical observations and by the partial derivatives. The latter were determined from numerical integrations of the variational equations, and the former were found empirically for each observation series and represent the actual root-mean-square scatter of the residuals after deletion of "blunder points" (*31*). Moreover, the distributions of the optical data residuals were distinctly non-Gaussian. Invariably, the tails of the distributions were overpopulated relative to predictions based on Gaussians that best fit the residuals. (In some cases, if not cut off, these tails would wag the dog in virtue of their disproportionately greater influence on the least squares solutions.) Unmodeled systematic effects on the data undoubtedly cause the non-Gaussian distributions.

These results and inferences, when coupled with our ignorance of the mass environment past Neptune's orbit, lead us to the conclusion that Pluto's mass, and hence its average density, cannot be determined reliably from existing data. Our entry for Pluto in Table 2 is based partly on our numerical experiments and partly on a priori density considerations: current theories of planet and satellite formation from the solar nebula make it seem unlikely that a body formed in the outer reaches has a density much higher than 3 g/cm^3. The uncertainty given in Table 2 for Pluto's inverse mass is based on the conclusion that 2,000,000 is the approximate lower bound set by the data.

Potential Improvements in Precision

Future spacecraft missions to Mercury and to the outer planets hold the best promise for improving the system of planetary masses. Ground-based radar measurements might contribute usefully to estimates of Jupiter's and Saturn's masses through observations of their larger satellites—if advanced radar facilities commensurate with present technology were implemented. With the existing systems, radar observations are confined to the inner planets. Even when extended over the next two oppositions of Mars—including the extremely favorable 1971 close approach —and augmented by radio tracking data from Mars orbiters, such observations will probably not yield an uncertainty for Jupiter's mass below the 10 parts per million claimed for results from analyses of asteroids in orbital resonance with Jupiter. For the immediate future, radar offers substantial improvement only for the mass of Mercury. Continued radar observations of Venus, with emphasis on topography determination as discussed above, may yield a two- or threefold reduction in our uncertainty for $M_{☿}$. Such measurements are being made with Haystack, with Jet Propulsion Laboratory's 210-foot-diameter Goldstone antenna, and with Cornell's Arecibo radar. The planned 1973 Venus-Mercury flyby mission will yield a mass value with fractional errors of a few parts per million; nonetheless, the prior availability of an improved radar result will be important to maximize the chances of reencounters between the probe and Mercury. Reducing the uncertainty in $M_{☿}$ to 0,1 percent will allow more precise navigation before the first encounter and consequently will reduce expenditure of the midcourse-maneuver fuel needed to insure reencounter.

Another possibility is the use of pulsar observations to infer outer planet masses. From measurements within the solar system we can not discern motions about its barycenter. However, pulsars represent an external reference system with respect to which such motions can be detected. The effect a planet has on the position of the barycenter varies, of course, directly with its mass and, in essence, with its distance from the sun. The outer planets are favored on both counts, except for Pluto which is favored on only one. The difficulty in the estimation of planetary masses from their effects on the earth's barycentric motion stems from several sources, such as (i) the intrinsic variability of the pulsar "clocks"; (ii) the time extent of each pulse (in general, the longer the pulse, the larger the error in the estimate of its time of arrival); and (iii) the long periods of the outer planets, which make their effects over the short term difficult to distinguish from, say, proper motions of the pulsars. Although we have not made a careful analysis of the potential of pulsar data for outer planet mass determinations, we can gain some insight from order of magnitude estimates. Using the radar "template-matching" techniques proposed for pulsars (*32*), various observers have deduced arrival times for pulses from the Crab Nebula (NP 0532) with uncertainties as small as a few microseconds (*33*). The Crab pulsar, unfortunately, appears unique in several related respects. It is the only one visible optically, its pulse length is the shortest so far observed, and its beat is subject to irregular changes (*34*). Nonetheless, for several more mature pulsars such as CP 0328, it appears that high-frequency or multi low-frequency radio observations can yield arrival times with uncertainties of a few tens of microseconds (*35, 36*). In Table 3, as an illustration, we list the amplitude of the barycentric contribution of each outer planet and the uncertainty in the inverse mass determination, with the oversimplified assumption that the fractional error is equal to 10 μsec (the assumed standard error in pulse arrival time) divided by the amplitude. For Jupiter the pulsar technique looks especially promising, and for Pluto, even though long-term observations may be required, we are fortunate that it is now near perihelion where its angular velocity and, hence, the variation in its contribution to the barycentric position will be maximal.

Despite the possible usefulness of radar and pulsar observations, spacecraft radio tracking data offer the best possibility during the next decades for the determination of the system of planetary masses with errors uniformly below a part in a million. Such accuracy should be sufficient for all foreseeable scientific purposes (*37*).

References and Notes

1. G. Colombo, private communication.
2. Recent United States Naval Observatory meridian circle data were sent to us in machine-readable form by R. L. Duncombe. All of the Eros observations were kindly provided in similar form by J. H. Lieske.
3. The epoch for the integration was actually in 1968; the backward direction for the major part of the 220-year period was chosen because the latest measurements are the most accurate and the earliest ones the least accurate.
4. The lunar coordinates from 1850 to 1970, with a 300-day gap in 1895, were provided to us on magnetic tape by the Jet Propulsion Laboratory and represent part of their Development Ephemeris No. 19. We evaluated Brown's lunar theory as given in "The Improved Lunar Ephemeris 1952–1959," *Joint Supplement to the American Ephemeris and Nautical Almanac* (Government Printing Office, Washington, D.C., 1954), p. 283, in order to fill the gap and to extend the tabulation back to 1750. Our values agreed at several check points within at least 0.01 arcsecond with the independent evaluations of the theory given in the ILE.
5. U.S. Naval Observatory, *Explanatory Supplement to the American Ephemeris and Nautical Almanac* (Government Printing Office, Washington, D.C., 1961).
6. D. Brouwer, *Astron. J.* **57**, 133 (1952); U.S. Naval Observatory, *American Ephemeris and Nautical Almanac for 1970* (Government Printing Office, Washington, D.C., 1968). Since 1956, the difference between A1 (atomic) time and universal time has been used, as provided by the USNO. The differences between A1 time and coordinate time are negligible for the present application.
7. The corrections of transcription errors had already undergone several iterations; practical considerations prevented a final one from being performed.
8. One exception was for the Greenwich observations (1750–1830), for which we used G. B. Airy's reductions. The others involved the two asteroids: Eros observations had already been adjusted to a common reference system [J. H. Lieske, *Astron. J.* **73**, 628 (1968)]; the relatively few observations of Icarus were distributed among too many different observatories to make feasible a separation into series [see I. I. Shapiro, W. B. Smith, M. E. Ash, *Phys. Rev. Lett.* **20**, 1517 (1968); J. H. Lieske and G. W. Null, *Astron. J.* **74**, 297 (1969); I. I. Shapiro, W. B. Smith, M. E. Ash, S. Herrick, *ibid.*, in press].
9. Unfortunately, only for the radar data was it feasible to compute postfit residuals. The corresponding computations for the optical data required too much computer time, and we had to rely on examination of the residuals from the base solution to discern any trends. Similarly, iteration of the solution to the normal equations through reintegration and recomputation of the O-C values was impossible. With a few exceptions (see text), the differential corrections were not large compared to the corresponding formal standard errors in the estimates; thus, our results do not differ appreciably from the true least squares values.
10. See W. B. Smith, R. P. Ingalls, I. I. Shapiro, M. E. Ash [*Radio Sci.* **5**, 411 (1970)] for a discussion of surface height variations, and see I. I. Shapiro, G. H. Pettengill, M. E. Ash, M. L. Stone, W. B. Smith, R. P. Ingalls, R. A. Brockelman [*Phys. Rev. Letters* **20**, 1265 (1968)] for further discussion of the dependence of the echo time delay on the radar scattering law. The slow rotation of Venus makes difficult the separation of these effects from orbital ones in the data processing. In principle, however, the separation is possible because the topography, whatever it is, will probably remain constant for the duration of the radar experiments. From a repetition of measurements at each of a number of different aspects of Venus during each of its four rotations (as seen by an earth observer) between inferior conjunctions, it would seem that the systematic effects of topography could be separated. The realization of such a result in practice is inhibited by the fact that Venus does not present exactly the same "face" toward the earth in successive rotations. Although the longitude of the subradar point on Venus repeats, the latitude does not. Because of the orientation of Venus' spin axis and the noncoplanarity of the orbits of the earth and Venus, the subradar point varies in latitude with the severest variations occurring at successive close approaches where the echo signals are strongest. The near commensurability of the orbital periods of the earth and Venus insures that during successive 8-year periods, however, the aspects will repeat almost precisely; hence only a very long-range radar observing program could yield orbital data that are free from systematic errors induced by topography.
11. M. E. Ash, I. I. Shapiro, W. B. Smith, *Astron. J.* **72**, 338 (1967).
12. J. D. Anderson and L. Efron, *Bull. Amer. Astron. Soc.* **1**, 231 (1969).
13. We have also estimated Venus' and Mars' masses from the Mariner 5 and Mariner 4 data, respectively, and obtained values virtually identical with the appropriate entries in Table 1 (L. D. Friedman, M. E. Ash, I. I. Shapiro, W. B. Smith, in preparation). These results are, of course, not surprising as they are based on the same radio tracking data. The software, on the other hand, is independent.
14. M. E. Ash, D. B. Campbell, R. B. Dyce, R. P. Ingalls, R. Jurgens, G. H. Pettengill, I. I. Shapiro, M. A. Slade, W. B. Smith, T. W. Thompson, *Science* **160**, 985 (1968); W. G. Melbourne, D. O. Muhleman, D. A. O'Handley, *ibid.*, p. 987.
15. These values also encompass the results obtained by Schubert and Zech, Rabe, Rabe and Francis, and Lieske [see (*40*)]. Each of these authors processed the data under somewhat different assumptions, which may account for the spread in their results.
16. D. Brouwer, *Bull. Astron.* **25**, 241 (1965).
17. B. S. Yaplee, S. H. Knowles, A. Shapiro, K. J. Craig, D. Brouwer, *ibid.*, p. 81.
18. See, for example, J. D. Anderson, L. Efron, S. K. Wong, *Science* **167**, 277 (1970).
19. G. W. Null, *Bull. Amer. Astron. Soc.* **1**, 356 (1969).
20. I. I. Shapiro, *Radio Sci.* **5**, 473 (1970).
21. C. W. Allen, *Astrophysical Quantities* (Athlone, London, 1963), p. 143. For Mars' oblateness, see also G. A. Wilkins in *Mantles of the Earth and Terrestrial Planets*, S. K. Runcorn, Ed. (Interscience, London, 1967), p. 77.
22. Results from the four post-1960 Saturn mass determinations used in Table 1 are summarized by B. Marsden in *The Motion, Evolution of Orbits, and Origin of Comets*, B. Marsden, G. A. Chebotarev, E. I. Kazimirchak-Polonskaya, Eds. (Reidel, Dordrecht, in press). Two are based on the motion of Jupiter and two on those of minor planets.
23. See, for example, D. Brouwer and G. M. Clemence, in *Planets and Satellites*, G. P. Kuiper and G. M. Middlehurst, Eds. (Univ. of Chicago Press, Chicago, 1961), p. 57.
24. J. E. Bixby and T. C. Van Flandern, *Astron. J.* **74**, 1220 (1969).
25. See, for example, C. J. Cohen, E. C. Hubbard, C. Oesterwinter, *ibid.* **72**, 973 (1967).
26. R. L. Duncombe, W. J. Klepczynski, P. K. Seidelmann, *ibid.* **73**, 830 (1958). The error estimate was given by W. J. Klepczynski, P. K. Seidelmann, and R. L. Duncombe in a paper presented at the IAU Symposium No. 9 (in press). *Note added in proof*: After that symposium, at which our results were also presented (see Table 2), P. K. Seidelmann, W. J. Klepczynski, R. L. Duncombe, and E. S. Jackson [*Astron. J.* **76**, 488 (1971)] analyzed the observations of Neptune recently rereduced by Jackson (*Astronomical Papers of the American Ephemeris*, in press), and obtained a new estimate of Pluto's inverse mass: 3,000,000 ± 500,000 (mean error). However, examination of the behavior of the residuals given in table 2 of their paper and other considerations (see text) indicate that the reliability of this newer value and its tenfold larger mean error may also be open to question.
27. G. P. Kuiper, *Publ. Astron. Soc. Pac.* **62**, 133 (1950); I. Halliday, R. H. Hardie, O. Franz, J. B. Priser, *Astron. J.* **70**, 676 (1965).
28. We did not include the two prediscovery observations of Neptune in 1795 by Lalande. Because of our experience with discordances among other 18th-century data we consider the use of such a small sample to be fraught with the danger of obtaining misleading results [see, however, D. Rawlins, *Astron. J.* **75**, 856 (1970)].
29. Similar difficulties were encountered several years ago at the U.S. Naval Weapons Laboratory (C. Oesterwinter, private communication).
30. The formal standard error for the mass is the primary quantity calculated; the corresponding value for the inverse mass is related to it by the equation $\sigma(M^{-1}) = \sigma(M)/M^2$. For Pluto, $\sigma(M) \simeq 5 \times 10^{-8}$ which led to $\sigma(M^{-1}) \simeq 2 \times 10^5$ for the value of $M^{-1} \simeq 1.8 \times 10^6$ determined by the USNO (*26*) and used in our base solution. Note also that very large negative values of the inverse mass are equivalent to very large positive ones. Both imply that the estimate of the mass is indistinguishable from zero.
31. These standard errors for inner planet observations were about 1 arcsec and for outer planets 0.5 arcsec for the modern data; the errors grow progressively larger farther into the past, reaching about 3 to 5 arcsec in the 18th century. The 20th-century USNO data are the best on a root-mean-square basis.
32. C. C. Counselman and I. I. Shapiro, *Science* **162**, 352 (1968).
33. See, for example, P. Horowitz, thesis, Harvard University (April 1970); D. W. Richards, G. H. Pettengill, C. C. Counselman, J. M. Rankin, *Astrophys. J. Lett.* **160**, L1 (1970).
34. P. E. Boynton, E. J. Groth, R. B. Partridge, D. T. Wilkinson, *Int. Astron. Union Circ. No. 2179* (1969).
35. The inner planet radar data allow the errors in the earth's orbit relative to the sun to be reduced enough to have no significant effect on the accuracy of the estimate of the barycentric motion from the pulsar data.
36. See, for example, P. E. Reichley, G. S. Downs, G. A. Morris, Jr., *Astrophys. J. Lett.* **159**, L35 (1970).
37. One must remember, of course, that foresight has not usually been numbered among scientists' greatest talents.
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39. W. G. Melbourne, J. D. Mulholland, W. L. Sjogren, F. M. Sturms, *Technical Report 32-106* (Jet Propulsion Laboratory, Pasadena, 1968).
40. J. Schubart and G. Zech, *Nature* **214**, 900 (1967); E. Rabe *Astron. J.* **72**, 852 (1967); E. Rabe and M. Francis, *ibid.*, p. 856; J. H. Lieske, *ibid.* **73**, 628 (1968).
41. Jupiter mass determinations from a total of seven different minor planets and two satellites of Jupiter are summarized by N. S. Chernykh in *The Motion, Evolution of Orbits, and Origin of Comets*, B. Marsden, G. A. Chebotarev, E. I. Kazimirchak-Polonskaya, Eds. (Reidel, Dordrecht, in press). These and the recent results from three additional minor planets given by H. Scholl (*Celest. Mech.*, in press), were used in Table 1.
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44. J. R. Gill and B. L. Gault, *Astron. J.* **73**, S95 (1968).
45. P. K. Seidelmann, R. L. Duncombe, W. J. Klepczynski, *ibid.* **74**, 776 (1969).
46. D. Brouwer, *Mon. Notic. Roy. Astron. Soc.* **115**, 221 (1955).

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