to stimulate the fish, and bioluminescence was observed to persist for about 1 second after the flashlight was turned off. The possibility that this emission was the result of phosphorescence or some other such phenomenon was considered and tested, without receiving any support. The luminescence in response to light appears to derive from the emission of the luminous organ. The response was reliable—neither fatigue nor failures were noted, and it was independent of the duration of the exposure, up to 2 minutes.

The observation of light-induced bioluminescence strongly supports the hypothesis that luminescence is used to match the background light intensity. However, experimental studies of the effect of intensity of irradiation upon the intensity of emission will be needed in the evaluation of the proposed hypothesis. Another important but also unresolved question is posed by the fact that the fish are apparently bottom dwellers, where the silhouette-concealing mechanism would seem to be of limited value (*15*). If the proposed hypothesis is correct, it would be expected that in deeper water these fish spend some part of their life off the bottom. Knowledge concerning the natural history, ecology, and especially the behavior of these fish is needed to evaluate this question.

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References and Notes

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2. E. N. Harvey, *Bioluminescence* (Academic Press, New York, 1952); in *Physiology of Fishes*, M. E. Brown, Ed. (Academic Press, New York, 1957), vol. 2, pp. 345–366.
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5. Alternatively, but not so ideal in practice, the organism might match its emission to that of the background by vertical migration.
6. The converse situation, in which bioluminescence is restricted to or favored during the nighttime, is known to hold for certain other luminous organisms, such as dinoflagellates [J. W. Hastings and B. M. Sweeney, *Biol. Bull.* **115**, 440 (1958), and Harvey (*2*)].
7. J. W. Harms, *Spec. Z. Wiss. Zool.* **131**, 157 (1928); Y. Haneda, *Palao Trop. Biol. Sta. Stud.* **2**(1), 29 (1940); *Pacific Sci.* **4**, 214 (1950); J. W. Hastings and G. Mitchell, *Biol. Bull.*, in press.
8. Harvey [(*2*), p. 528] states that "Control appears to be by chromatophores, as it is necessary to handle the fish or remove it from the water before the luminescence is displayed." Harvey's failure to mention the shutter mechanism, which was clearly described by Haneda (along with the possible involvement of chromatophores) was presumably an oversight.
9. Much but presumably not all of the ventral surface emits light. It is likely that the anterior and posterior extremities may still be visible. Nevertheless, the silhouette would be substantially interrupted. Another matter of concern relates to intensity as a function of viewing angle. E. J. Denton [*Phil. Trans. Roy. Soc. London Ser. B* **258**, 285 (1970)] has recently shown that the even more complicated optical arrangements associated with the photophores in hatchet fish have ingenious features that would enable the fish to match their background regardless of the angle of view. See also, E. Denton, *Sci. Amer.* **224**, 64 (Jan. 1971).
10. J. E. Tyler and R. C. Smith, *Measurements of Spectral Irradiance Underwater* (Gordon and Breach, New York, 1970).
11. The fish used in these experiments were collected within Sek Harbor, near Madang, New Guinea, at the head of Bostrem Bay, by trawling on the bottom at depths of 3 to 10 m. Collections by Drs. Haneda and Paxton were also made off the coast at the mouth of the Ramu River at depths up to 50 m.
12. A number of different species of *Leiognathus*, but principally *L. equulus* and *L. splendens*, were employed in these studies. The experiment was carried out both with a single fish and with several (up to 12) fish in the tank.
13. G. W. Mitchell and J. W. Hastings, *Anal. Biochem.* **39**, 243 (1971).
14. Similar observations were made in the field by Drs. John Paxton and J. M. Bassot. Captured fish were released within a fenced area in shallow water and observed visually during the night. No light emission was noted.
15. The fact that the fish occur on the bottom during the day in shallow water is not necessarily inconsistent with the proposed hypothesis. At an ambient light intensity higher than that which can be matched by the bioluminescence, the fish would presumably be driven to the bottom, attempting to move to greater depths.
16. Supported in part by a grant from NSF to the University of California, Scripps Institution of Oceanography, and in part by a contract with the Office of Naval Research, N00014-67-A-0001. I am deeply indebted to Dr. John B. Buck, chief scientist of Program C, *Alpha Helix*, 1969, for his assistance during the expedition in ways too numerous to mention. I am grateful to Drs. John Paxton and George Mitchell for assistance with the experiments, and to Dr. Y. Haneda for identifying the fish. My thanks to Howard Davis and Baxter O'Brien for help in the collection of the fish.

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Lunar Gravity Analysis from Long-Term Effects

Abstract. *The global lunar gravity field was determined from a weighted least-squares analysis of the averaged classical element of the five Lunar Orbiters. The observed-minus-computed residuals have been reduced by a factor of 10 from a previously derived gravity field. The values of the second-degree zonal and sectorial harmonics are compatible with those derived from libration data.*

The results given here represent an extension and refinement of previous work by Lorell (*1*). The second-degree zonal and sectorial harmonics determined here are in agreement with values obtained by libration data given by Jeffreys (*2*) and Koziel (*3*). A comparison is also made between our work and that of Muller and Sjogren (*4*).

The lunar gravity potential Φ is represented by the spherical harmonic expansion

$$\Phi = \frac{\mu}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left[\left(\frac{R}{r}\right)^n \times P_n^m(\sin\phi)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right]\right\}$$

where μ is the gravitational constant of the moon, adopted as 4902.78 km^3/sec^2; R is the mean equatorial radius of the moon taken as 1738.09 km; P_n^m $(\sin\phi)$ is the associated legendre polynomial of order m and degree n in sine of lunar latitude ϕ; λ is the lunar longitude; and r is the radial distance of the orbiter from the moon. The harmonic coefficients C_{nm} and S_{nm} have numerical values that are determined from the data.

With the vast quantity of tracking data, a direct reduction of the data becomes a formidable undertaking, even for the high-speed computers of today. Therefore, the radar data were compressed into normal points consisting of five mean orbital parameters, a, e, i, Ω, and ω, averaged over an anomalistic period. A weighting matrix describing the statistics and correlations between the mean elements was associated with each normal point. A complete description of these matrices and data has been given (*5*). Lorell (*1*) was limited to an 8th-degree, 4th-order (8-4) model because of computer limitations. His computer program (*6*) computed the averaged orbital elements and produced the partial derivatives necessary for differential correction by the technique of finite differences. Having access to a third-generation computer, we were able not only to extend the solution to the 15th degree but also to use variational equations to compute the partial derivatives.

Included in the equations of motion were effects of the harmonic coefficients, the point mass perturbations of

Table 1. Estimates of lunar gravity harmonics based on Lunar Orbiters 1 to 5.

Harmonic	Unnormalized coefficient	Normalized coefficient
$J(2)$	$0.1996 \times 10^{-3} \pm 0.20 \times 10^{-5}$	$0.8928 \times 10^{-4} \pm 0.89 \times 10^{-6}$
$C(2,1)$	$0.8171 \times 10^{-5} \pm 0.24 \times 10^{-5}$	$0.6329 \times 10^{-5} \pm 0.19 \times 10^{-5}$
$S(2,1)$	$-0.7213 \times 10^{-5} \pm 0.71 \times 10^{-5}$	$-0.5587 \times 10^{-5} \pm 0.55 \times 10^{-5}$
$C(2,2)$	$0.2359 \times 10^{-4} \pm 0.53 \times 10^{-5}$	$0.3655 \times 10^{-4} \pm 0.82 \times 10^{-5}$
$S(2,2)$	$0.4538 \times 10^{-5} \pm 0.60 \times 10^{-5}$	$0.7030 \times 10^{-5} \pm 0.94 \times 10^{-5}$
$J(3)$	$0.5878 \times 10^{-5} \pm 0.29 \times 10^{-5}$	$0.2221 \times 10^{-5} \pm 0.11 \times 10^{-5}$
$C(3,1)$	$0.3001 \times 10^{-4} \pm 0.27 \times 10^{-5}$	$0.2778 \times 10^{-4} \pm 0.25 \times 10^{-5}$
$S(3,1)$	$0.1421 \times 10^{-5} \pm 0.32 \times 10^{-5}$	$0.1316 \times 10^{-5} \pm 0.30 \times 10^{-5}$
$C(3,2)$	$0.4698 \times 10^{-5} \pm 0.28 \times 10^{-5}$	$0.1375 \times 10^{-4} \pm 0.83 \times 10^{-5}$
$S(3,2)$	$0.5748 \times 10^{-6} \pm 0.17 \times 10^{-5}$	$0.1682 \times 10^{-5} \pm 0.50 \times 10^{-5}$
$C(3,3)$	$0.4847 \times 10^{-5} \pm 0.22 \times 10^{-5}$	$0.3476 \times 10^{-4} \pm 0.16 \times 10^{-4}$
$S(3,3)$	$-0.2919 \times 10^{-5} \pm 0.13 \times 10^{-5}$	$-0.2093 \times 10^{-4} \pm 0.96 \times 10^{-5}$
$J(4)$	$-0.1195 \times 10^{-4} \pm 0.17 \times 10^{-5}$	$-0.3985 \times 10^{-5} \pm 0.59 \times 10^{-6}$
$C(4,1)$	$-0.2226 \times 10^{-5} \pm 0.25 \times 10^{-5}$	$-0.2347 \times 10^{-5} \pm 0.26 \times 10^{-5}$
$S(4,1)$	$0.3299 \times 10^{-5} \pm 0.41 \times 10^{-5}$	$0.3478 \times 10^{-5} \pm 0.44 \times 10^{-5}$
$C(4,2)$	$-0.2418 \times 10^{-5} \pm 0.33 \times 10^{-5}$	$-0.1081 \times 10^{-4} \pm 0.15 \times 10^{-4}$
$S(4,2)$	$-0.2389 \times 10^{-5} \pm 0.38 \times 10^{-5}$	$-0.1068 \times 10^{-4} \pm 0.17 \times 10^{-4}$
$C(4,3)$	$0.2306 \times 10^{-6} \pm 0.10 \times 10^{-5}$	$0.3859 \times 10^{-5} \pm 0.17 \times 10^{-4}$
$S(4,3)$	$-0.6222 \times 10^{-6} \pm 0.11 \times 10^{-5}$	$-0.1041 \times 10^{-4} \pm 0.19 \times 10^{-4}$
$C(4,4)$	$-0.4547 \times 10^{-6} \pm 0.40 \times 10^{-6}$	$-0.2152 \times 10^{-4} \pm 0.19 \times 10^{-4}$
$S(4,4)$	$0.4248 \times 10^{-6} \pm 0.43 \times 10^{-6}$	$0.2010 \times 10^{-4} \pm 0.20 \times 10^{-4}$
$J(5)$	$0.4544 \times 10^{-5} \pm 0.26 \times 10^{-5}$	$0.1370 \times 10^{-5} \pm 0.79 \times 10^{-6}$
$C(5,1)$	$-0.2455 \times 10^{-5} \pm 0.38 \times 10^{-5}$	$-0.2867 \times 10^{-5} \pm 0.45 \times 10^{-5}$
$S(5,1)$	$-0.6925 \times 10^{-5} \pm 0.33 \times 10^{-5}$	$-0.8087 \times 10^{-5} \pm 0.38 \times 10^{-5}$
$C(5,2)$	$0.8880 \times 10^{-6} \pm 0.12 \times 10^{-5}$	$0.5487 \times 10^{-5} \pm 0.76 \times 10^{-5}$
$S(5,2)$	$-0.3178 \times 10^{-6} \pm 0.97 \times 10^{-6}$	$-0.1964 \times 10^{-5} \pm 0.60 \times 10^{-5}$
$C(5,3)$	$-0.7774 \times 10^{-6} \pm 0.61 \times 10^{-6}$	$-0.2353 \times 10^{-4} \pm 0.18 \times 10^{-4}$
$S(5,3)$	$0.1159 \times 10^{-5} \pm 0.40 \times 10^{-6}$	$0.3511 \times 10^{-4} \pm 0.12 \times 10^{-4}$
$C(5,4)$	$0.1027 \times 10^{-6} \pm 0.22 \times 10^{-6}$	$0.1319 \times 10^{-4} \pm 0.29 \times 10^{-4}$
$S(5,4)$	$0.6296 \times 10^{-7} \pm 0.28 \times 10^{-6}$	$0.8086 \times 10^{-5} \pm 0.36 \times 10^{-4}$
$C(5,5)$	$-0.3044 \times 10^{-8} \pm 0.56 \times 10^{-7}$	$-0.1236 \times 10^{-5} \pm 0.23 \times 10^{-4}$
$S(5,5)$	$-0.3012 \times 10^{-7} \pm 0.96 \times 10^{-7}$	$-0.1223 \times 10^{-4} \pm 0.39 \times 10^{-4}$
$J(6)$	$-0.1088 \times 10^{-5} \pm 0.55 \times 10^{-5}$	$-0.3018 \times 10^{-6} \pm 0.15 \times 10^{-5}$
$C(6,1)$	$-0.6373 \times 10^{-5} \pm 0.46 \times 10^{-5}$	$-0.8100 \times 10^{-5} \pm 0.59 \times 10^{-5}$
$S(6,1)$	$0.5342 \times 10^{-5} \pm 0.10 \times 10^{-4}$	$0.6789 \times 10^{-5} \pm 0.12 \times 10^{-4}$
$C(6,2)$	$-0.6917 \times 10^{-6} \pm 0.18 \times 10^{-5}$	$-0.5560 \times 10^{-5} \pm 0.14 \times 10^{-4}$
$S(6,2)$	$-0.1157 \times 10^{-5} \pm 0.17 \times 10^{-5}$	$-0.9300 \times 10^{-5} \pm 0.14 \times 10^{-4}$
$C(6,3)$	$-0.2041 \times 10^{-6} \pm 0.32 \times 10^{-6}$	$-0.9845 \times 10^{-5} \pm 0.15 \times 10^{-4}$
$S(6,3)$	$-0.3484 \times 10^{-6} \pm 0.51 \times 10^{-6}$	$-0.1680 \times 10^{-4} \pm 0.25 \times 10^{-4}$
$C(6,4)$	$0.4620 \times 10^{-7} \pm 0.15 \times 10^{-6}$	$0.1220 \times 10^{-4} \pm 0.42 \times 10^{-4}$
$S(6,4)$	$-0.9870 \times 10^{-7} \pm 0.17 \times 10^{-6}$	$-0.2607 \times 10^{-4} \pm 0.45 \times 10^{-4}$
$C(6,5)$	$-0.7564 \times 10^{-8} \pm 0.47 \times 10^{-7}$	$-0.9372 \times 10^{-5} \pm 0.58 \times 10^{-4}$
$S(6,5)$	$-0.1654 \times 10^{-7} \pm 0.45 \times 10^{-7}$	$-0.2049 \times 10^{-4} \pm 0.55 \times 10^{-4}$
$C(6,6)$	$-0.7277 \times 10^{-8} \pm 0.10 \times 10^{-7}$	$-0.3123 \times 10^{-4} \pm 0.44 \times 10^{-4}$
$S(6,6)$	$0.2373 \times 10^{-8} \pm 0.10 \times 10^{-7}$	$0.1018 \times 10^{-4} \pm 0.44 \times 10^{-4}$
$J(7)$	$-0.1779 \times 10^{-4} \pm 0.68 \times 10^{-5}$	$-0.4594 \times 10^{-5} \pm 0.17 \times 10^{-5}$
$C(7,1)$	$0.1324 \times 10^{-5} \pm 0.44 \times 10^{-5}$	$0.1809 \times 10^{-5} \pm 0.61 \times 10^{-5}$
$S(7,1)$	$-0.1579 \times 10^{-5} \pm 0.42 \times 10^{-5}$	$-0.2158 \times 10^{-5} \pm 0.58 \times 10^{-5}$
$C(7,2)$	$-0.1293 \times 10^{-6} \pm 0.10 \times 10^{-5}$	$-0.1298 \times 10^{-5} \pm 0.10 \times 10^{-4}$
$S(7,2)$	$0.1152 \times 10^{-6} \pm 0.39 \times 10^{-6}$	$0.1156 \times 10^{-5} \pm 0.39 \times 10^{-5}$
$C(7,3)$	$-0.1805 \times 10^{-6} \pm 0.26 \times 10^{-6}$	$-0.1281 \times 10^{-4} \pm 0.19 \times 10^{-4}$
$S(7,3)$	$0.2440 \times 10^{-6} \pm 0.19 \times 10^{-6}$	$0.1732 \times 10^{-4} \pm 0.13 \times 10^{-4}$
$C(7,4)$	$-0.3523 \times 10^{-8} \pm 0.74 \times 10^{-7}$	$-0.1659 \times 10^{-5} \pm 0.35 \times 10^{-4}$
$S(7,4)$	$0.1295 \times 10^{-7} \pm 0.65 \times 10^{-7}$	$0.6102 \times 10^{-5} \pm 0.31 \times 10^{-4}$
$C(7,5)$	$0.6061 \times 10^{-8} \pm 0.18 \times 10^{-7}$	$0.1712 \times 10^{-4} \pm 0.51 \times 10^{-4}$
$S(7,5)$	$0.5582 \times 10^{-8} \pm 0.36 \times 10^{-7}$	$0.1577 \times 10^{-4} \pm 0.10 \times 10^{-3}$
$C(7,6)$	$-0.2837 \times 10^{-9} \pm 0.62 \times 10^{-8}$	$-0.4088 \times 10^{-5} \pm 0.90 \times 10^{-4}$
$S(7,6)$	$0.1227 \times 10^{-8} \pm 0.68 \times 10^{-8}$	$0.1767 \times 10^{-4} \pm 0.98 \times 10^{-4}$
$C(7,7)$	$0.2454 \times 10^{-10} \pm 0.14 \times 10^{-8}$	$0.1322 \times 10^{-5} \pm 0.76 \times 10^{-4}$
$S(7,7)$	$-0.8131 \times 10^{-10} \pm 0.17 \times 10^{-8}$	$-0.4383 \times 10^{-5} \pm 0.92 \times 10^{-4}$
$J(8)$	$0.5967 \times 10^{-5} \pm 0.97 \times 10^{-5}$	$0.1447 \times 10^{-5} \pm 0.23 \times 10^{-5}$
$C(8,1)$	$-0.8040 \times 10^{-5} \pm 0.44 \times 10^{-5}$	$-0.1170 \times 10^{-4} \pm 0.65 \times 10^{-5}$
$S(8,1)$	$0.6208 \times 10^{-5} \pm 0.56 \times 10^{-5}$	$0.9035 \times 10^{-5} \pm 0.82 \times 10^{-5}$
$C(8,2)$	$0.2650 \times 10^{-6} \pm 0.14 \times 10^{-5}$	$0.3227 \times 10^{-5} \pm 0.17 \times 10^{-4}$
$S(8,2)$	$-0.1767 \times 10^{-6} \pm 0.14 \times 10^{-5}$	$-0.2151 \times 10^{-5} \pm 0.17 \times 10^{-4}$
$C(8,3)$	$-0.6771 \times 10^{-7} \pm 0.26 \times 10^{-6}$	$-0.6697 \times 10^{-5} \pm 0.26 \times 10^{-4}$
$S(8,3)$	$-0.1437 \times 10^{-6} \pm 0.33 \times 10^{-6}$	$-0.1421 \times 10^{-4} \pm 0.33 \times 10^{-4}$
$C(8,4)$	$0.1925 \times 10^{-7} \pm 0.37 \times 10^{-7}$	$0.1475 \times 10^{-4} \pm 0.28 \times 10^{-4}$
$S(8,4)$	$-0.1757 \times 10^{-7} \pm 0.43 \times 10^{-7}$	$-0.1346 \times 10^{-4} \pm 0.33 \times 10^{-4}$
$C(8,5)$	$0.4844 \times 10^{-9} \pm 0.14 \times 10^{-7}$	$0.2676 \times 10^{-5} \pm 0.82 \times 10^{-4}$
$S(8,5)$	$0.2391 \times 10^{-8} \pm 0.11 \times 10^{-7}$	$0.1321 \times 10^{-4} \pm 0.65 \times 10^{-4}$
$C(8,6)$	$0.1252 \times 10^{-8} \pm 0.33 \times 10^{-8}$	$0.4483 \times 10^{-4} \pm 0.12 \times 10^{-3}$
$S(8,6)$	$-0.2390 \times 10^{-9} \pm 0.34 \times 10^{-8}$	$-0.8558 \times 10^{-5} \pm 0.12 \times 10^{-3}$
$C(8,7)$	$-0.4523 \times 10^{-10} \pm 0.79 \times 10^{-9}$	$-0.8872 \times 10^{-5} \pm 0.15 \times 10^{-3}$
$S(8,7)$	$-0.5813 \times 10^{-10} \pm 0.77 \times 10^{-9}$	$-0.1140 \times 10^{-4} \pm 0.15 \times 10^{-3}$
$C(8,8)$	$-0.3178 \times 10^{-11} \pm 0.13 \times 10^{-9}$	$-0.2493 \times 10^{-5} \pm 0.10 \times 10^{-3}$
$S(8,8)$	$0.7798 \times 10^{-11} \pm 0.14 \times 10^{-9}$	$0.6117 \times 10^{-5} \pm 0.11 \times 10^{-3}$
$J(9)$	$0.3206 \times 10^{-5} \pm 0.15 \times 10^{-4}$	$0.7356 \times 10^{-6} \pm 0.35 \times 10^{-5}$
$J(10)$	$-0.1367 \times 10^{-5} \pm 0.74 \times 10^{-5}$	$-0.2984 \times 10^{-6} \pm 0.16 \times 10^{-5}$
$J(11)$	$0.7311 \times 10^{-5} \pm 0.13 \times 10^{-4}$	$0.1524 \times 10^{-5} \pm 0.27 \times 10^{-5}$
$J(12)$	$-0.1251 \times 10^{-4} \pm 0.85 \times 10^{-5}$	$-0.2503 \times 10^{-5} \pm 0.17 \times 10^{-5}$
$J(13)$	$0.3315 \times 10^{-4} \pm 0.74 \times 10^{-5}$	$0.6381 \times 10^{-5} \pm 0.14 \times 10^{-5}$
$J(14)$	$-0.1044 \times 10^{-4} \pm 0.51 \times 10^{-5}$	$-0.1939 \times 10^{-5} \pm 0.96 \times 10^{-6}$
$J(15)$	$0.2977 \times 10^{-4} \pm 0.44 \times 10^{-5}$	$0.5347 \times 10^{-5} \pm 0.80 \times 10^{-6}$

the earth and the sun, and solar radiation pressure. It should be noted that we found it necessary to expand the disturbing function for the earth to third and fourth order in the ratio of the orbiter selenocentric distance and the earth selenocentric distance (*7*).

We have solved for a 15th degree, 8th order (15-8) model, and the resulting normalized and unnormalized harmonic coefficients, together with their standard deviations, are shown in Table 1. Corresponding contour graphs of equivalent surface density distribution, represented by departures from a reference sphere with contour lines at 400-m intervals, are shown in Fig. 1. Multiplying the contour values by 0.11 will give the variations in units of milligals.

Figure 1 shows a large positive area near the center of the far side, which indicates a large "mascon" or a system of "mascons" in this region. The existence and location of such a farside anomaly had already been proposed by Campbell, O'Leary, and Sagan (*8*), and the results presented here are consistent with their analysis. To demonstrate the degree of improvement over the 8-4 model (*1*), the residuals are compared in Table 2.

We base our statistics on the *a posteriori* data fit, which reflects unmodeled effects such as mascons. The resulting standard deviations are about 1000 times larger than the formal statistics based only on the Doppler noise. The values of C_{20} and C_{22} are consistent with f and g, which have been obtained by observations of the physical librations (*2, 3*) where f, g, C_{20}, and C_{22} are related by

$$\beta = \frac{C-A}{B} = 0.000627 \pm 0.000001$$

$$f = \frac{C-B}{A}\frac{1}{\beta} = -1 + \frac{2C_{20}}{C_{20} - 2C_{22}} = 0.633 \pm 0.011$$

$$g = \frac{3}{2}\frac{C}{Ma^2} = \frac{-3C_{20}}{\beta(1+f)} = 0.5956 \pm 0.001$$

where M is the lunar mass and a is the mean lunar radius.

The principal moments of inertia are A, B, and C. Adopting the observed value of β to be 0.000627 and using the values C_{20} and C_{22} from our model, we find

$$f = 0.617 \pm 0.067$$

$$g = 0.590 \pm 0.028$$

Since the even-degree coefficients of the same order, and the odd-degree coefficients of the same order, are highly correlated, the individual C_{nm} and S_{nm}

Table 2. Residuals for Jet Propulsion Laboratory's 8-4 model (mod 3) versus the 15-8 model.

Arc	Number of points		$a(m)$		e ($\times 10^6$)		i (deg)		Ω (deg)		ω (deg)		$\Omega + \omega$ (deg)		Time span			Inclination (deg)
	Mod 3	15-8	Mod 3	15-8	Mod 3	15-8	Mod 3	15-8	Mod 3	15-8	Mod 3	15-8	Mod 3	15-8	Day	Hour	Minute	
Average residuals																		
IA	20	20	0.26	−0.79	−46	0.106	−0.027	0.015	−0.061	0.176	0.057	−0.184	−0.004	−0.008	6	11	37	12
IB	55	66	−2.02	9.34	−175	6.48	−0.033	−0.116	−0.501	0.311	0.445	−0.325	−0.056	−0.014	57	5	58	12
IIC	40	42	17.85	45.5	−1264	−40.8	0.044	0.047	0.538	0.218	−0.700	−0.223	−0.162	−0.005	126	1	18	18
IID	11	11	−25.85	9.32	231	81.7	0.005	−0.200	−0.756	0.186	0.622	−0.168	−0.134	0.016	73	16	10	17
IIE	4	4	1.78	4.01	244	21.3	−0.003	−0.019	1.587	−0.012	−1.889	−0.001	−0.302	−0.014	94	9	22	16
IIIB	56	59	−79.55	−1.04	140	−9.19	−0.195	−0.064	−0.389	0.054	0.394	−0.058	0.005	−0.005	50	18	32	21
IIIC	7	7	3.74	−20.1	−909	−61.8	0.141	−0.161	1.616	−0.193	−1.298	0.171	0.318	−0.022	94	22	25	21
IIID	4	4	−0.75	6.93	1023	29.5	−0.076	−0.014	−0.349	0.482	0.100	−0.520	−0.249	−0.038	44	8	40	21
IIIE	18	18	41.94	29.4	−2514	277.0	−0.024	0.022	2.074	0.021	−2.158	−0.022	−0.084	−0.005	39	7	1	21
IVC	10	10	−24.61	−21.6	66	4.50	0.058	−0.009	0.020	−0.012	0.079	0.081			31	22	29	85
VC	34	35	−26.71	−14.0	910	30.3	0.148	−0.054	−0.155	0.013	0.092	−0.074			53	3	17	85
VD	30	32	−64.09	22.9	−1102	29.8	0.445	−0.030	0.308	0.010	−0.431	−0.054			111	20	20	85
Total	289	308																
Standard deviations																		
IA			5.41	5.37	64	21.2	0.026	0.027	0.205	0.186	0.190	0.197						
IB			63.97	71.3	358	176.0	0.268	0.283	0.751	0.763	0.790	0.800						
IIC			56.90	58.8	1292	102.0	0.231	0.203	0.822	0.714	0.802	0.733						
IID			58.03	58.3	586	240.0	0.412	0.412	1.054	0.694	1.076	0.706						
IIE			9.93	5.29	218	75.8	0.044	0.0578	2.239	0.430	2.600	0.439						
IIIB			103.71	78.0	408	109.0	0.392	0.401	0.984	0.822	1.009	0.855						
IIIC			42.75	91.3	1240	115.0	0.153	0.193	1.628	0.519	2.069	0.492						
IIID			6.13	9.36	692	186.0	0.073	0.029	0.498	0.591	0.439	0.642						
IIIE			57.99	57.7	2061	601.0	0.217	0.192	1.053	0.477	1.191	0.466						
IVC			97.43	92.5	235	86.7	0.316	0.296	0.079	0.019	0.166	0.099						
VC			95.39	92.2	655	25.2	0.568	0.537	0.174	0.100	0.704	0.579						
VD			101.95	100.0	1147	219.0	0.497	0.457	0.318	0.068	0.512	0.364						

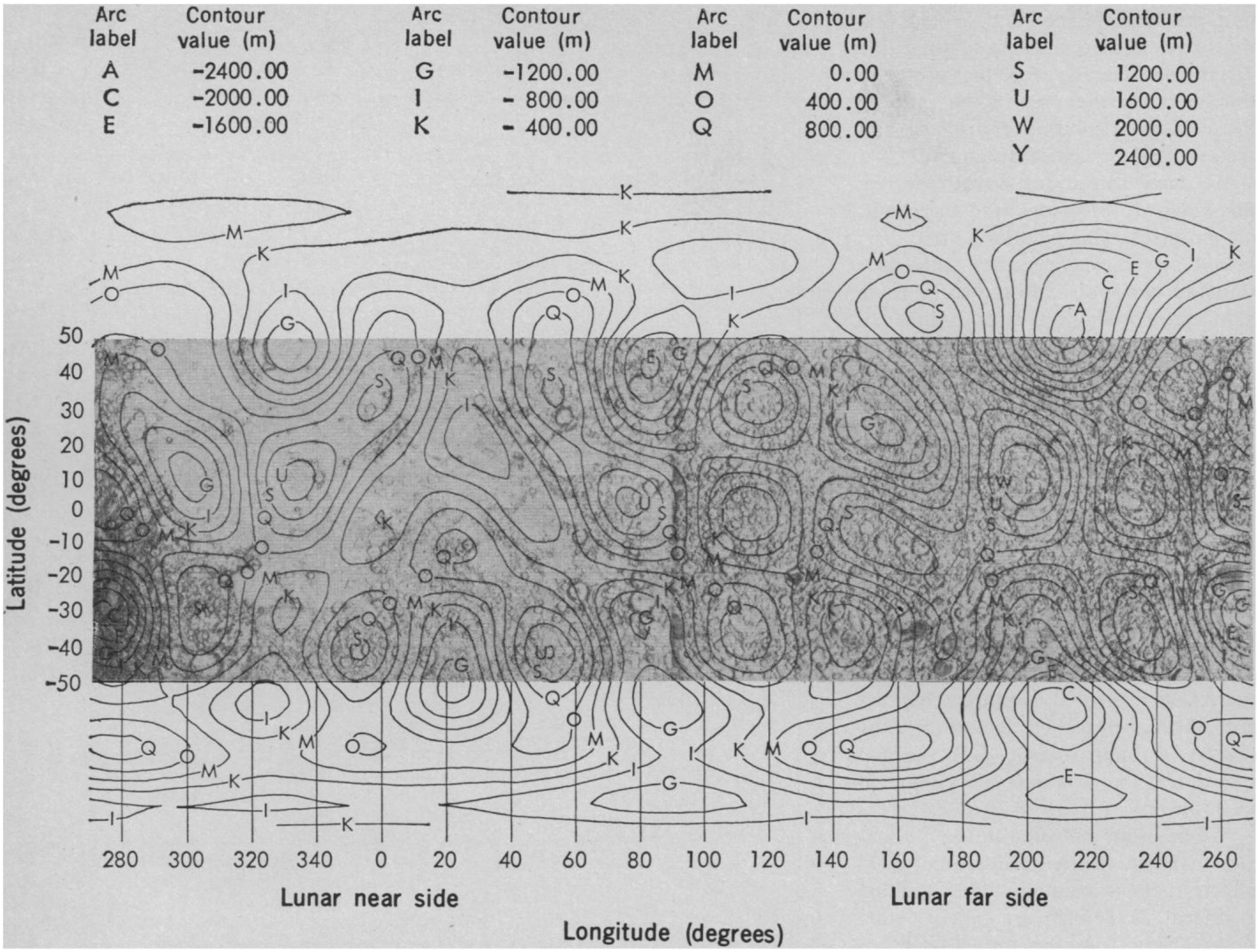

Fig. 1. Lunar gravity field contours. (The contour value multiplied by 0.11 is approximately 1 mgal.)

are not so meaningful as all the coefficients taken collectively.

The validity of the 15-8 model was checked by using the original Doppler data to predict over an arc of 14 revolutions (36 hours) for a low circular orbit (Lunar Orbiter 3). The maximum disagreement at the end of 36 hours was 50 hz. There appears to be no tendency for the orbit to drift; that is, there is no secular trend in the Doppler residuals. The only detectable error is a gradual increase in amplitude of the periodic error.

To compare our work with the work done by Muller and Sjogren (*4*), we adjoined a mascon expansion to our 15-8 model. The result of this experiment was to show that the works are completely consistent. The adjoined model breaks up the gross nearside features of the 15-8 model into clear, mascon-type features [as presented in (*4*)]. The high region between Imbrium and Serenitatis breaks into two, and Crisium, Humorum, and Nectaris appear in their appropriate locations. The far side remains relatively unaffected, indicating a small correlation between low-order gravity harmonics and mascon-type features.

The resolving power of the 15-8 model is limited to about 40 degrees of latitude or longitude; that is, two adjacent high regions described by the model cannot be closer than 40 degrees. Therefore, the mascon-type features, which are more local, are glossed over.

In summary, we have generated a global model for the moon which is consistent with the gross features derived from physical libration data and which does not contradict the fine features (that is, mascons) derived by direct mapping of the Doppler data. An independent check of the model's validity is its ability to predict orbital parameters better than any model to date. Considering that the biases and other systematic effects have been essentially removed from the normal point data residuals, we feel that this model represents about the limit of information extractable from the Lunar Orbiter data by our method.

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